Notes on an Agenda for Research and Action for WFNMC

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María Falk de Losada is a world famous authority in the work with mathematically gifted pupils. She was, first of all, one of the founders of the Mathematical Olympiad in Colombia. She has worked as a professor at the Universidad Antonio Narino in Bogotá for many years. In 2008 she was elected at the WFNMC mini-conference in Monterrey as the president of WFNMC.

Abstract. Two fundamental lines of research and action for the World Federation of National Mathematics Competitions are proposed complementing the traditional areas of problem creation and competition design. Links to the work of other international groups and to areas of research and development such as curriculum design and teacher education are discussed. Certain precepts to be adopted by WFNMC are proposed.

In the thoughts that we will try to convey this morning, there are at least two lines of research and action that we see as fundamental to the work of our Federation.

The first has to do with the academic planning, organization and carrying out of mathematical olympiads and similar events designed to allow each student who so wishes to pursue his or her optimal level of development in doing mathematics, which we define as solving problems that for him or her are new and original.

Every year new and exciting competitions are organized to cover new areas seen as crucial, be these by age group, by geographical region or by the type of problems—content, level of difficulty, format—included. In 2008 in Latin America we were told about the Brazilian Math Olympiad for Public Schools that had more than 18 million participants. In 2009 the Colombian Math Olympiads founded an interuniversity competition for Latin America similar to the IMC, with the participation of universities from 6 countries in its first version, and a second version to be held in Brazil in 2010. WFNMC has been instrumental in giving support and recognition to those doing this work, as well as serving as a multiplier through its congress and journal to inform others whose own work can benefit from analyzing the design, the activities and the problems posed.

Peru has been able to obtain the backing of the Ministry of Education for its math Olympiad, which has meant an increase to about 3 million students in the event. This is another important area—that of significantly increasing the number of students who benefit from the Olympiad experience—where the work of WFNMC continues to be influential.

Similar activity abounds in virtually all geographical regions around the globe and in the days to come we will hear from several speakers how new events have given new impulse to students and sparked favorable changes in math education in their countries or regions.

Our Federation should continue to be instrumental in bringing the Olympiad experience to students in all corners of the globe: accompanying new national, transnational and regional events, approaching new groups such as those organizing the primary or elementary school Olympiad on the international level, as well as the organizers of the IMC and similar university competitions, to extend the reach of mathematical competitions to ensure that they also enrich the mathematical experience of the youngest students and undergraduates alike.

WFNMC also serves as an avenue for competitions covering new academic territory. Two years ago we learned about an optimization competition in Russia named *Construct, Investigate, Explore* held in the same mode as distance learning and serving students from the sixth form to postgraduate level. In this congress we will hear about an International Internet Competition for University Students, the Australian Intermediate Mathematics Olympiad, a Statistics Olympiad in Iran and several others.

One area that must be strengthened is that of formal research to supplement our professional appreciation of the impact of competition activity.

The Federation should look to encourage its members to engage in research that can provide solid evidence of the impact of competitions on the student and on the educational system, as well as on the field of mathematics; research that provides a foundation for practice in a variety of ways.

The American Mathematical Society, for example, published in 2008 a study done by Jim Gleason, using models developed by psychologists, and thus pertinent and acceptable to educators in other areas of research in mathematics education such as PME (International Group for the Psychology of Mathematics Education), that shows that in fact designing an Olympiad with a first round that includes multiple-choice questions—as many of the popular Olympiads do—is a process that responds to the objectives and corresponds to the aims that such events profess, and that the great majority of the problems posed in a popular (unnamed) nationwide competition are in fact well designed to fulfill its aims and objectives.

The Mathematical Sciences Research Institute of UC Berkeley commissioned a study to analyze the educational history of students with outstanding results in the Putnam Competition, a case study whose 2005 report by Steve Olson revealed several critical moments in the attraction of young students to the field of mathematics.

There is no doubt that this line of research and action is seen, by IMU for example, and by all those present, as vital to the continuing task of attracting talented young people to the field of mathematics, not only enriching the lives of these future mathematicians, but also thus furthering mathematics itself. The afternoon of talks given by former IMO medalists at the 50$^{\text{th}}$ IMO last year in Bremen showed brilliantly the key contribution being made.

During the time Petar Kenderov and I had the opportunity to serve on the ICMI Executive Committee, this role of mathematics competitions was highlighted by a concern of IMU referred to as "the pipeline issue". The math education community represented by ICMI has slowly focused on the issue, coming up with an intermediate case study (eight countries) that has yielded some very useful information. There will be a report at the IMU meeting in India in August. Nevertheless, the question of whether or not there is a real decline in the number of people choosing to follow a career in mathematics has not been answered by the study; and under the current design and objectives no attempt to gather complete information will be made.

As we all know this is the same concern that prompted Hilbert in the early twentieth century to draw up his famous list of problems yet to be solved. Young people must know that there are still open questions in mathematics so that they will find the field not only attractive but irresistible. Certainly events such as the IMO have led a large number of talented young students to devote themselves to doing mathematics. Its power and beauty are exposed; the student feels himself part of the sublime human endeavor that is mathematics.

The Federation must be active in proposing and supporting research regarding the solidity of practice and the depth and extent of impact of competitions, and in publishing its findings. Our journal must reflect these aspects of the work of the Federation as well as the fine job it has done offering articles on the design, academic planning, problem creation, organization and realization of competitions.

As we have seen in the past and will see again in the days to come, the problems posed in math competitions often link the work of the Federation and its members to research in mathematics. This is one of the most important aspects that link the Federation to research; the original problems created for math competitions often correspond to new results, that is to say, results of effectively doing research in elementary mathematics. Another important link stems from new results in research on the frontier of mathematics that can and have lead to the formulation of original problems for the highest level competitions, such as the IMO. More than a dozen of the talks given at this congress speak to results of this nature.

WFNMC can and should stress this aspect of its work and the work of its members before the international community, making clear its links to important areas of research in mathematics and mathematics education on many different levels.

There is a second line of research and development that we wish to suggest for future action of the Federation, that is, the designing, planning, organizing and carrying out of research relating to the nature of mathematical thinking and of how the experience of the average math class can be brought closer to developing the mathematical thinking of students in ways that will be personally satisfying and fun, and enable the student to leave all options open when making life choices, from career to personal finance to exercising the rights of a citizen.

This line of development will link the work of the Federation with other areas of research in mathematics education, and will involve looking at both teacher education and the curriculum.

The idea that *all* students can enjoy challenging and enriching experiences in mathematics is not new.

When beginning to prepare this talk, I remembered Plato's dialogue *Meno* in which Plato wishes to gain adherents to his explanation of how learning is possible, and in which Socrates' actions have been famously taken to illustrate what we call the Socratic method, however I wish to look at them as an example of mathematics teaching involving challenges.

In the *Meno*, the pupil's condition as a slave is meant to assure us that he has no prior mathematical knowledge, and that all for him is new. The problem set by Socrates is to draw (construct) a square whose area is twice the area of a given square.

The slave begins by suggesting that we double the length of the side and Socrates, drawing in the sand or dust shows that the resulting figure has four times the area of the original.

(We reproduce on the next pages from a copy of Plato's *Collected Dialogues and Letters* the only illustrations in more than 1700 pages of text.)

what I can since you ask me. I see you have a large number of retainers b
here. Call one of them, anyone you like, and I will use him to demonstrate it to you.

MENO: Certainly. [*To a slave boy.*] Come here.

SOCRATES: He is a Greek and speaks our language?

MENO: Indeed yes—born and bred in the house.

SOCRATES: Listen carefully then, and see whether it seems to you that he is learning from me or simply being reminded.

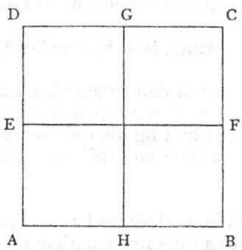
MENO: I will.

SOCRATES: Now boy, you know that a square is a figure like this?

(*Socrates begins to draw figures in the sand at his feet. He points to the square* ABCD.)

BOY: Yes.

SOCRATES: It has all these four sides equal? c



BOY: Yes.

SOCRATES: And these lines which go through the middle of it are also equal? [EF, GH.]

BOY: Yes.

SOCRATES: Such a figure could be either larger or smaller, could it not?

BOY: Yes.

SOCRATES: Now if this side is two feet long, and this side the same, how many feet will the whole be? Put it this way. If it were two feet in this direction and only one in that, must not the area be two feet taken once?

BOY: Yes.

SOCRATES: But since it is two feet this way also, does it not be- d
come twice two feet?

BOY: Yes.

SOCRATES: And how many feet is twice two? Work it out and tell me.

BOY: Four.

The slave's second suggestion is to increase the sides a distance of half the length of the given side.

In each instance Socrates disproves the conjecture using diagrams or proofs without words, and then finally he draws the diagonal of the given square and leads the slave to see that (again proof without words) the square constructed on the diagonal as side fulfills the requirements of the problem posed.

The problem is fresh, requiring thought; the attempts by the slave to solve it are respected but firmly shown to be in error; the solution arrived at through questioning is diagramatic, conclusive and elegant.

We are not trying to promote the Socratic method, but rather illustrate how 2400 years ago it was thought that anybody—even a slave with no prior knowledge—can solve interesting and challenging problems in mathematics if and when that person is given a chance.

What have we here? An excellent problem, a superstar teacher, a superlative way of exhibiting the solution that instantly convinces the learner of its correctness.

Every child shows his or her originality and creativity outside the math classroom; how must we set our goals and build the possibility of their realization into our schools so that every child is given the chance to show his or her creativity within the confines of the mathematics classroom as well?

When we speak of giving a child or youngster the opportunity to meet more challenging mathematics as an essential component of school mathematics, we are in fact proposing three things: give the child/student exposure to a mathematical situation or problem that is beyond what he or she has already met and practiced, provide tools to grasp the problem and think about it, assist the child to find ways of expressing his or her thoughts, progress and solutions.

We cite four fundamental and interrelated reasons for establishing a role for the Federation in giving renewed impulse to the evolution of mathematics as presented to the student in school and indeed even at university—although we will develop only one of them—under the

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SOCRATES: Now could one draw another figure double the size of this, but similar, that is, with all its sides equal like this one?

BOY: Yes.

SOCRATES: How many feet will its area be?

BOY: Eight.

SOCRATES: Now then, try to tell me how long each of its sides will be. The present figure has a side of two feet. What will be the side of the double-sized one?

BOY: It will be double, Socrates, obviously.

SOCRATES: You see, Meno, that I am not teaching him anything, only asking. Now he thinks he knows the length of the side of the eight-foot square.

MENO: Yes.

SOCRATES: But does he?

MENO: Certainly not.

SOCRATES: He thinks it is twice the length of the other.

MENO: Yes.

SOCRATES: Now watch how he recollects things in order—the proper way to recollect.

You say that the side of double length produces the double-sized figure? Like this I mean, not long this way and short that. It must be equal on all sides like the first figure, only twice its size, that is, eight feet. Think a moment whether you still expect to get it from doubling the side.

BOY: Yes, I do.

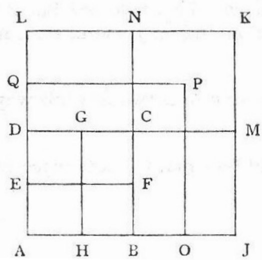
SOCRATES: Well now, shall we have a line double the length of this [AB] if we add another the same length at this end [BJ]?

BOY: Yes.

SOCRATES: It is on this line then, according to you, that we shall make the eight-foot square, by taking four of the same length?

BOY: Yes.

SOCRATES: Let us draw in four equal lines [*i.e., counting* AJ *and adding* JK, KL, *and* LA *made complete by drawing in its second half* LD], using the first as a base. Does this not give us what you call the eight-foot figure?



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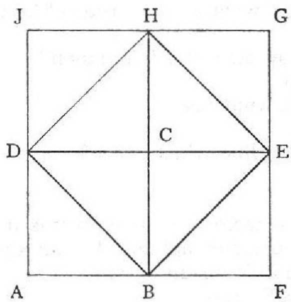

BOY: Yes.

SOCRATES: Now we can add another equal to it like this? [BCEF.]

BOY: Yes.

SOCRATES: And a third here, equal to each of the others? [CEGH.]

BOY: Yes.

SOCRATES: And then we can fill in this one in the corner? [DCHJ.]

BOY: Yes.

SOCRATES: Then here we have four equal squares?

BOY: Yes. e

SOCRATES: And how many times the size of the first square is the whole?

BOY: Four times.

SOCRATES: And we want one double the size. You remember?

BOY: Yes.

SOCRATES: Now does this line going from corner to corner cut each of these squares in half? 85

BOY: Yes.

SOCRATES: And these are four equal lines enclosing this area?

premise that math competitions and competition mathematics have shown how careful planning, analysis, original problems and non standard representations can counteract the separation that some sectors have attempted to foster between elementary mathematics and school mathematics, between mathematics and math education.

These four reasons concern the rights of the student, the nature of mathematics itself, the way technology is changing the way we do mathematics—and indeed the way we think, and the needs of the knowledge-based society.

Several countries and school systems have long since taken note of the fundamental change that must take place. Many have built a rich mathematical culture. For example, professor Konstantinov will be talking during this Congress of the Russian system under the title "Math Classes in Russia".

Other systems are beginning to take note of the desirability of structuring a more challenging curriculum for all.

In the USA the state of Connecticut has put forth the following precept:

> Every student needs and deserves a rich and rigorous mathematics curriculum that is focused on the development of concepts, the acquisition of basic and advanced skills and the integration of problem solving experiences. The Department of Education encourages educators to provide such challenging mathematics opportunities to foster the growth of intelligent, thoughtful and mathematically literate members of society.

What is interesting about the Connecticut statement is that it stresses the importance of a challenging mathematical education for the student and future citizen. This is an essential emphasis, it is what *every student needs and deserves*. Furthermore, in a recent (Summer, 2010) article *Ed.* The Magazine of the Harvard Graduate School of Education thought it appropriate to emphasize that access to a more exacting mathematics curriculum—albeit in the form of better ways of teaching algebra—should be thought of as *a new civil right*. Let's glance back at the *Meno*. Can the fact that the pupil is a slave allude to the belief that challenging mathematics education is the right of all?

The latter is essentially the gist of ICMI Study 16 in which many members of WFNMC took part, and Peter Taylor will be speaking to the topic during this Congress.

Just a few words that come to mind. In Study 16 several case studies of educational systems developing their commitment to a much more challenging mathematics curriculum were highlighted, among them that of Singapore. This is not the forum for discussing "Singapore Math" as it has been baptized, but to emphasize that its strength in treating more challenging problems for all students lies in the appropriate and, at the elementary level, graphical or visual representation of mathematical concepts and facts, a representation that permits the student to think about the concepts and facts without the intermediation of mathematical symbols that for the student unfortunately can come to resemble Hilbert's formalist philosophy—the manipulation of symbols devoid of meaning according to explicit rules of transformation.

The parallels with the method of Socrates in the *Meno* are nothing less than striking, but the message is clear. Present a challenging problem, provide the tools to grasp it and think about it, and accompany the student to find ways of expressing the solution in a convincing manner.

Another important area of research in this line of thought involves developing the capacity of all students to think mathematically. When debating new topics for its studies a few years ago, ICMI rejected a proposal to focus on the topic of mathematical thinking as too vague and research in the area too underdeveloped, choosing the topic of proof and proving instead. Thus we have an area of research—the development of mathematical thinking—prime for the intervention of WFNMC and its members. We have seen how problem solving, algorithms and formal mathematical proof all stem from the same thinking strategy, that of finding a way—however ingenious—to base each new step or result on one previously solved or proved. Several of the presentations we will hear in the days to come speak to analysis of mathematical thinking and strategies of problem solution.

Clearly, we hesitate to make a sweeping proposal having lived through the furor of the new math just some 50 years ago; we prefer to use the word evolution and not revolution; we insist nevertheless that curriculum change is long overdue.

1 The royal road and the curriculum

However, this evolution towards a more challenging math curriculum has not had the impact it merits on the situation and the policies related to mathematics education in general.

Again I wish to return to a famous incident in Greek mathematics, one that is frequently quoted but not necessarily interpreted fully with respect to its implications. We have all heard the tale of Menaecmus' saying to Alexander the Great words to the effect that "there is no royal road to geometry".

Now a recent headline in the *Los Angeles Times* read:

> America keeps looking for one simple solution for its education shortcomings. There isn't one.

Unfortunately, I believe that the great majority of teachers as well as those responsible for educational policy are still looking for the royal road. A fix-it, something that future teachers can digest in an instant and that can be transmitted to pupils without difficulty, but above all effortlessly.

I have not encountered an appropriate parallel, but I will venture this one. Many curricular proposals and textbooks present an excessively algorithmic approach to school mathematics, a sort of predigested or previously blended input, much like baby food, leaving totally unrecognizable the source elements in all their richness, color, shape, texture.

WFNMC and its members can participate in convincing society as a whole and political leaders in particular that there truly is no royal road to geometry; and it would be a tragedy if there were, supporting its efforts with research related to the success that more challenging mathematics has had in Singapore and in many other countries and regions of the world. Mathematics is captivating because it requires an effort, a sustained effort, an inspired effort, to do mathematics on every level of expertise. Proposals, projects, plans, design of more challenging mathematical curricula must be a focus of WFNMC and its members, jointly with research permitting a clear evaluation of their impact.

2 Teachers

If we look at analyses in several different contexts we can see that outstanding student performance almost always leads us back to extraordinary teachers, and this is true both for students performing on the

Olympiad level and for students taking part in almost any mathematical activity measuring their performance.

This is stated unequivocally in the MSRI report alluded to earlier.

WFNMC and its members know how to work with teachers on an extracurricular basis, we can and should learn to work intensely and with inspiration on transforming pre-service and in-service teacher education, allowing teachers to experience the power and beauty of mathematics so aptly embodied in challenges in the elementary mathematics that forms the backdrop for school mathematics, and we must document the results obtained through research.

3 This may well be the moment to develop some precepts and work toward their implementation.

Precepts

- Every child has a right to confront mathematics that is challenging enough to develop his mathematical thinking and to elicit his creativity in response.
- Challenging mathematics is the only learning experience that is true to the nature of mathematics itself and to that kind of thinking that can be said to be mathematical.
- Students' mathematical growth is directly related to the education, talent and dedication of the teacher. Teachers who have not had themselves the experience of confronting challenging mathematics and of thinking mathematically will not be able to open the door to such experience for their students.

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