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MATHEMATICS COMPETITIONS

Journal of the

WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

AUSTRALIAN MATHS TRUST

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For WFNMC Standing Committees please refer to ABOUT WFNMC section of the WFNMC website `http://www.wfnmc.org/`.

From the President

Dear readers,

I hope that you are already aware of our upcoming congress, which is planned for 22-28 July 2026 in Kuala Lumpur, Malaysia. Since this is just a little over a year away as I write this, the time has come to start thinking seriously about the preparations.

As usual at our congresses, we plan to have plenary talks, as well as problem sessions, divided into four thematic groups. Each of these groups will have a Chair, who will give the main group talk, as well as organising the group program. Of course, there will also be a social program, including an excursion, a gala dinner, and perhaps some regionally appropriate surprizes.

If you are interested in delivering a talk please write to the Chairman of the Program Committee, Krzysztof Ciesielski at Krzysztof.Ciesielski(at)im.uj.edu.pl

To give you an idea for possible topics, tentative titles of the sessions at the moment are

Topic Group A: Building Bridges between Problems of Mathematical Research and Competitions

Topic Group B: Creating Problems and Problem Solving

Topic Group C: Competitions around the World

Topic Group D: Technological Applications in Mathematics Competitions

More details of the congress will be added to our website at wfnmc.org as they become available, so be sure to watch that space. If you are able to come, you will surely not be disappointed. I hope to see you there!

Also, please do not forget that your contributions to this journal are always welcome, as are nominations for the Erdős Award, which we hope to be able to award to deserving candidates at the congress.

So, be sure to mark the congress dates in your calendars right now!

Robert Geretschläger

Editor's Page

Dear Competitions enthusiasts, readers of our *Mathematics Competitions* journal!

Mathematics Competitions is the right place for you to publish and read the different activities about competitions in Mathematics from around the world. For those of us who have spent a great part of our life encouraging students to enjoy mathematics and the different challenges surrounding its study and development, the journal can offer a platform to exhibit our results as well as a place to find new inspiration in the ways others have motivated young students to explore and learn mathematics through competitions. In a way, this learning from others is one of the better benefits of the competitions environment.

Following the example of previous editors, I invite you to submit to our journal *Mathematics Competitions* your creative essays on a variety of topics related to creating original problems, working with students and teachers, organizing and running mathematics competitions, historical and philosophical views on mathematics and closely related fields, and even your original literary works related to mathematics.

Just be original, creative, and inspirational. Share your ideas, problems, conjectures, and solutions with all your colleagues by publishing them here. We have formalized the submission format to establish uniformity in our journal.

Submission Format
FORMAT: should be LaTeX, TeX, or for only text articles in Microsoft Word, accompanied by another copy in pdf. However, the authors are strongly recommended to send article in TeX or LaTeX format. This is because the whole journal will be compiled in LaTex. Thus your Word document will be typeset again. Texts in Word, if sent, should mainly contain non-mathematical text and any images used should be sent separately.
START: with the title centered in Large format (roughly 14 pt), followed on the next line by the author(s)' name(s) in italic 12 pt.
MAIN TEXT: Use a font from the Times New Roman family or 12 pt in LaTex.
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INCLUDE: your high-resolution small photo and a concise professional summary of your works and titles.
ILLUSTRATIONS: must be inserted at about the correct place of the text of your submission in one of the following formats: jpeg, pdf, tiff, eps, or mp. Your illustration will not be redrawn.
The resolution of your illustrations must be at least 300 dpi, or, preferably, done as vector illustrations.
If a text is embedded in illustrations, use a font from the Times New Roman family in 11 pt.
In figures, the letters used in labeling points, distances, etc. must be written in the same font as in the text that refers to these figures. Note that if the manuscript is prepared in Word, then mathematical symbols will be compiled with the use of the math mode in LaTex.
Furthermore, a version of the figure in a LaTex-compatible graphic language (such as TikZ, PSTricks or Asymptote) must be included with the submission to insure compatibility with the text.
REFERENCES: Every reference must be cited in the paper. References should be ordered alphabetically by author. If any webpages are mentioned as webpages, not as sources of articles, they should be in the end of the reference list. Any cited webpage should include a retrieval date.

A recommended citation style is the following:

[1] A. Soifer, *Mathematics, its history, and mathematical olympiads: a golden braid*, Mathematics Competitions 35(2022), No. 2, 8–23.
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[3] https://www.imo-official.org/. Retrieved 1 July, 2004.

Please submit your manuscripts to María Elizabeth Losada at
`director.olimpiadas@uan.edu.co`

We are counting on receiving your contributions, informative, inspired and creative. Best wishes,

Maria Elizabeth Losada
EDITOR

WFNMC Mini-Conference in Sydney, 2024

Krzysztof Ciesielski

Krzysztof Ciesielski is a professor at the Jagiellonian University in Kraków. In the years 2008-2016 he was a member of the Raising Public Awareness of Mathematics Committee of the European Mathematical Society, in 2017-2025 he was the Polish representative in the International Commission on Mathematical Instruction of the International Mathematical Union, in 2012-2019 and since 2022 he has been a member of the Council of the European Mathematical Society. He is active in several mathematical competitions in Poland, among others Mathematical Olympiad and Kangaroo. He has been a member of Editorial Boards of several journals, including The Mathematical Intelligencer, The European Mathematical Society Newsletter and Mathematics Competitions. He is currently the Senior Vice President of the WFNMC.

The International Commission on Mathematical Instruction (ICMI) is a worldwide organization devoted to research and development in mathematical education at all levels. It was founded in 1908 and has been a commission of the International Mathematical Union since 1952. ICMI introduced the notion of affiliation to ICMI of multi-national organizations active in mathematics education. At the moment, there are 9 ICMI Thematic Affiliate Organizations and 8 ICMI Regional Affiliate Organizations. The World Federation of National Mathematics Competitions has been an ICMI Thematic Affiliate Organization since 1994 and only three Thematic Organisations obtained the status of Affiliate Organisation earlier.

The Congresses of WFNMC are organised every four years, in years that are even but not divisible by 4. In other even years, the quadrennial International Congress on Mathematical Education (ICME) is organised by ICMI. The WFNMC uses this opportunity to organise a one-day satellite mini-conference affiliated to ICME. This happened also in 2024 and the WFNMC Mini-Conference associated with 15th ICME took place in Sydney, Australia, on 6th July.

About sixty participants took part in the mini-conference, some of them online. The conference was coordinated by Krzysztof Ciesielski and Robert Geretschläger.

During the Opening Ceremony of the mini-conference, Erdős Awards for a significant role in the development of mathematical challenges were presented. The 2024 Erdős Awards were given to Angelo Di Pasquale from Australia and Matjaž Željko from Slovenia. Matjaž Željko received the Medal and diploma during Opening Ceremony, Angelo Di Pasquale was given it two weeks later, during International Mathematical Olympiad in Bath, UK.

Then, fourteen talks were presented, some of them online.

Omar Colón (Puerto Rico) in *Mathematical Team Competition (COMATEQ): an opportunity to compete internationally* presented details and achievements of the Mathematical Team Competition that was some years ago created by Puerto Rico Mathematics Olympiad developed through universities that organize pre-university mathematical Olympiads. The talk was a joint work with Luis Cáceres (Puerto Rico).

M. Suhaimi Ramly (Malaysia) in *WFNMC–10 in Kuala Lumpur 2026: Some Preliminary Information* provided some preliminary details on the following WFNMC Congress that would take place in Kuala Lumpur in 2026, including the tentative program and logistical arrangements.

Hidetoshi Fukagawa (Japan) in *Maxima and Minimum problems in Traditional Japanese Mathematics* introduced sixteen interesting problems of "Maxima and Minima problems" for students which are available to use in any mathematics competitions. They were connected with times when Japan was almost completely cut off from the western world and people of all social classes, from farmer to samurai, enjoyed studying mathematics and constructed Traditional Japanese Mathematics.

Lukas Donner (Germany) in *Which test-wiseness based strategies are used by Austrian winners of the Mathematical Kangaroo?* talked about test-wiseness strategies at the Mathematical Kangaroo and about some of the results and implications of the comparison of those strategies used by successful participants and novices (i.e. further participants). This was joint work with Jakob Kelz, Evita Lerchenberger, Elisabeth Stipsits, David Stuhlpfarrer (all of them Austria).

Robert Geretschläger (Austria) in *On a certain type of triangle problems* showed some surprising area relationships that appear among the triangles created when the sides on a triangle are divided into thirds.

Mike Clapper (Australia) in *Calibrating the Australian Maths Competition* outlined the methodologies that have been used in Australian Maths Competition. AMC had demonstrably improved from 2014, as earlier the trend to underestimate difficulties appeared. The database was shown, the questions they started to avoid were highlighted. This was joint work with Andrew Kepert (Australia).

Alexander Soifer (USA) in *New and Old Problems and Conjectures around the Chromatic Number of the Plane* presented his now classic 2002 conjectures about the chromatic number of $n-$ dimensional Euclidean space, and consequently of the plane. New conjectures and open problems may lead the field to new discoveries. They come from the recently published "The New Mathematical Coloring Book" written by the speaker.

Hnin Set Aye (Myanmar) in *Smart Solutions for Math Educators: Embracing AI in the Classroom* spoke about artificial intelligence solutions in math classrooms that move beyond traditional teaching. She underlined how using AI can change the way of teaching math, preparing both educators and students for a future shaped by technology and innovation.

Krzysztof Ciesielski (Poland) in *From here to dimensionality – an untypical solution of a typical olympic problem* talked about one problem that appeared in the Polish Mathematical Olympiad and its surprising solution where three dimensions helped in the solution of a two-dimensional problem.

Nicolás Atanes Santos (Spain) in *In the style of Eureka, the Mathematics League in Spain* spoke about the Mathematics League, a new competition organized in Spain with the aim of promoting mathematics, maintaining its continuity, and uniting mathematics students.

Evgeny V. Khinko (Russia) in *Tournament of Towns and its electronic marking system now* talked about the electronic system for elaboration of the papers of the International Mathematical Tournament of Towns. It existed in various forms since the 80s of the 20th century, but the system has been recently developing and now can give more opportunities for participants, local organizers and jury.

Sergey Dorichenko (Russia) in *Polynomials, curves and geometry* considered some approaches to geometric problems, using some algebraic methods, but almost without calculations. One of the applications was an original proof of the famous Pascal theorem.

Stephanie Schiemann and Robert Wöstenfeld (Germany) in *Math Advent Calendar – Math puzzles for Kids and Classes* talked about "Mathe im Advent" – Math Advent Calendar. This is designed to foster overall mathematical competences like problem solving, pattern recognition, logic and strategic thinking. The youngsters get in touch with modern mathematics and their applications, many areas which they normally never discover within the school curriculum. The factors that MiA's success is based on were underlined.

Luis Cáceres (a coauthor of the first talk) in *Spiral Teaching Methodology in Mathematics Olympiads* talked about spiral learning. The speaker emphasized that with the use of the teaching and learning strategy through the spiral model students' retention of concepts and skills increases significantly. The same thing happens in Mathematics Olympiads. Examples of this methodology in Mathematical Olympiads were presented

The conference was organised with the great help of Laura Stuart, Nathan Ford and Peter Taylor.

As usual, the General Assembly of ICMI took place the day prior to the opening of an ICME; it was on 7th July. The General Assembly consists of the national ICMI representatives (one representative of each ICMI country), representatives of ICMI Thematic Adhering Organizations and the ICMI Executive Committee. This time as many as four General Assembly delegates were simultaneously members of WFNMC Executive. Robert Geretschläger and Meike Akveld were there as the representatives of WFNMC, Kiril Bankov was the Country Representative of Bulgaria and Krzysztof Ciesielski was the Country Representative of Poland. In the agenda of the General Assembly short quadrennial reports of Thematic Adhering Organizations are always included. This year the WFNMC report was presented by Geretschläger.

ICME was officially opened on 8th July, the Closing Ceremony was on 14th July. ICME organisers invited WFNMC to organise a 90-minute special session that would present WFNMC to a wider maths education audience. This session was held on 12nd July and consisted of three 30-minute talks. Robert Geretschläger presented the current state of competitions and maths educations research in the area, both of which appear to be booming right now. The topic of Peter Taylor's talk was on the history of the WFNMC, especially with respect to the Australian background, connections to the American Mathematical Trust and the beginning of Australian mathematics competitions and Kangaroo. The Kangaroo competition was the topic of the third lecture, presented by Meike Akveld. She presented this competition, its history and development as well as several details behind it. It was remarkable to include in this session in the country of the kangaroos the talk about Kangaroo.

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Properties of partition of squares and cubes

Arno Mikaelyan, Nairi Sedrakyan, Hayk Sedrakyan

Arno Mikaelyan is an N29 state non-profit organization, researcher in Yerevan, Armenia. The author is known for the trapezoid law (see mathematical reflections, issue 6, 2024).

Nairi Sedrakyan is an Erdős Award 2022 winner mathematician involved in international Olympiads, including American Mathematics Competitions and IMO, having been a jury member and problem selection committee member of the International Mathematical Olympiad, the Zhautykov International Mathematical Olympiad and the International Olympiad of Metropolises.

Hayk Sedrakyan is an IMO medal winner, Professor of Mathematics in Paris and a professional Math Olympiad Coach in the greater Boston area, USA. PhD from the UPMC-Sorbonne University, Paris. Author of problem solving and Olympiad style mathematics books published globally.

Abstract
In this paper we investigate novel properties of partition of squares and cubes. We provide the answer to the following open-ended question: into how many non-overlapping squares can a square be divided? We also prove that a cube cannot be divided into 14 non-overlapping cubes. We also investigate the uniqueness of squares.

Main results and proofs

Consider four identical paper squares. Each of them is cut into several squares (see Fig. 1). Let us

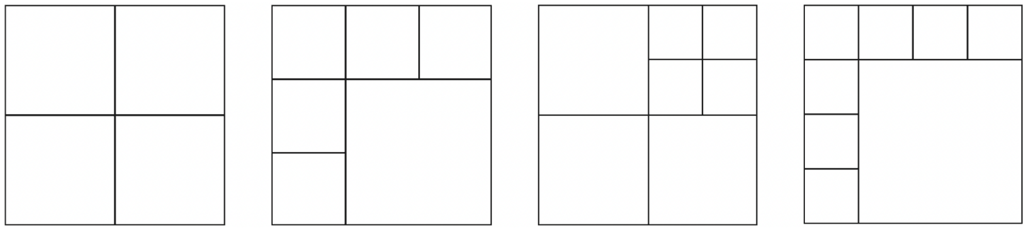

Figure 1

prove the following lemma.

Lemma 1. Let *ABCD* be a square divided into non-overlapping squares, so that each of these squares has exactly two sides laying on the sides of square *ABCD*. Then, square *ABCD* is cut into four identical squares.

Proof. Consider figure 2. We have

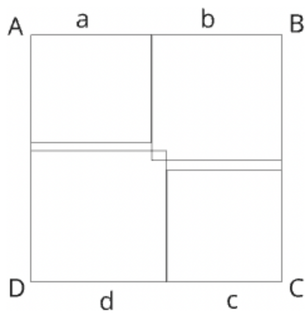

Figure 2

$$a+b=c+d=2m,$$

and

$$\max(a,b) \geq m, \ \ \max(c,d) \geq m.$$

If $\max(a,b) > m$, or $\max(c,d) > m$, then the squares with sides $\max(a,b)$ and $\max(c,d)$ will have overlapping parts, which is not possible.

Thus, it follow that

$$\max(a,b) = \max(c,d) = m.$$

We get

$$a=b=c=d=m.$$

This ends the proof of Lemma 1.

Remark 1. It follows from Lemma 1 that a square can be cut into four squares in a unique way (see Fig. 1).

Remark 2. It follows from Lemma 1 that it is impossible to cut a square into five squares.

Lemma 2. A square can be cut into six, seven, or eight squares in a unique way (Fig. 1).

Proof by a contradiction argument. Assume that the square is cut into six squares. According to Lemma 1, four of these six squares (those containing the vertices of the square) do not form an "edge" on two adjacent sides of the original square but do form one on the other two sides (see Fig. 3). If the points M and N do not coincide, then either two of the squares have an overlapping

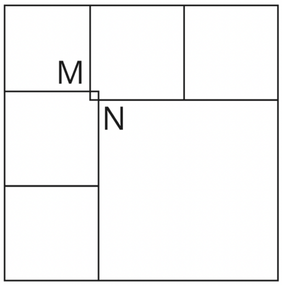

Figure 3

part. We find that M and N coincide. This ends the proof of the statement.

Now, let us prove that a square can be cut into seven squares in a unique way. Consider the four squares covering the vertices of the square, two of which do not form an "edge" on one side of the initial square. Because we get that we have at least eight squares. The remaining three sides form an "edge" (Fig. 4).

If the points M and N do not coincide, then the two shaded squares that cover these points have

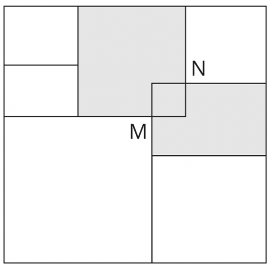

Figure 4

an overlapping part, which is impossible. If they coincide, then again it is impossible.

When it creates an "edge" only on two adjacent sides (Fig. 5).

If $a \neq b$, then again the points M and N must coincide. This is impossible. This ends the proof of the statement.

If $a = b$, then we get Figure 6. This ends the proof of the statement for seven squares.

The case for eight squares, can be done in a similar way.

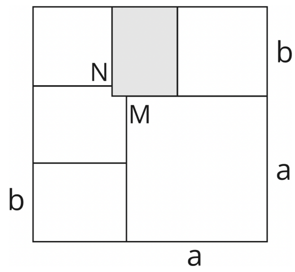

Figure 5

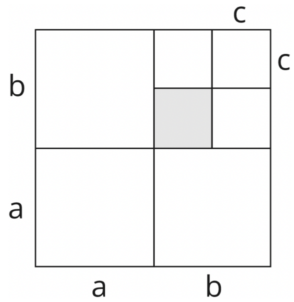

Figure 6

If $a > b$, the points M and N do not coincide and the two shaded squares covering these points have an overlapping part, which is impossible (see Fig. 7).

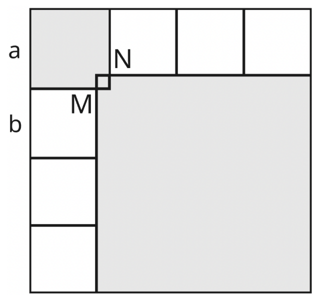

Figure 7

If $a < b$, the points M and N again do not coincide and the two shaded squares covering these points have overlapping part, which is impossible (see Fig. 8)

If $a = b$, then we get Figure 9.
This ends the proof of a statement for eight squares.

Lemma 3. If a square is cut into n squares ($n > 8$), the division is not unique.

Definition for unique. The solution is **unique up to rotation and reflection**. That is, any other solution is just a flipped or rotated version of the original.

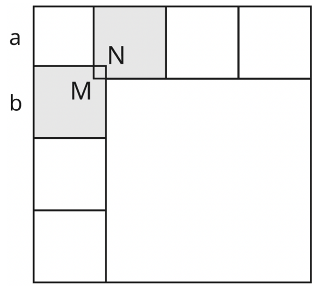

Figure 8

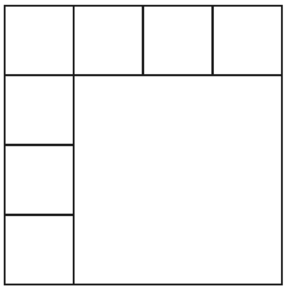

Figure 9

Proof. As we know from lemma 2 "a square can be cut into six, seven, eight squares in a **unique** way".
Let us investigate the case for nine squares.
Let us use the square for the case of six and divide any of the already divided squares into four identical squares.

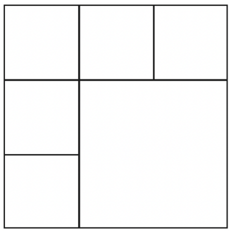

Figure 10: case of 6 squares

We get

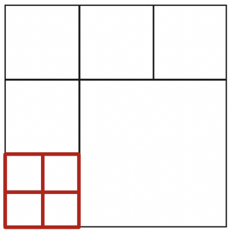

Figure 11: 1st case of 9 squares

Now let us choose any **other** square from case 6

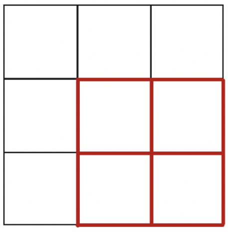

Figure 12: 2nd case of 9 squares

This ends the proof of the statement for nine squares.
Proving the case for 10 squares can be done in a similar way using the square cut into seven squares.

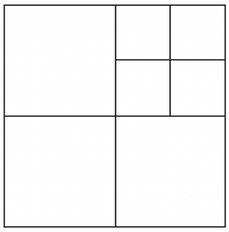

Figure 13: case of 7 squares

Let us choose any two **different** squares from case 7 and cut them into four identical squares separately. We get

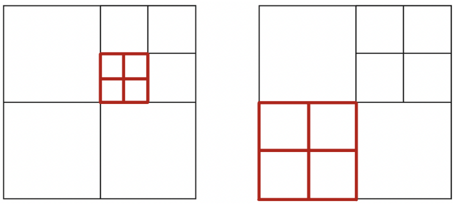

Figure 14: cases of 10 squares

This ends the proof of the statement for ten squares.

Now, let us prove the case for eleven squares.
Let us cut any two **different** squares from case 8.

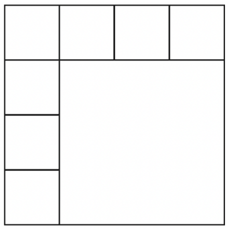

Figure 15: case of 8 squares

We get

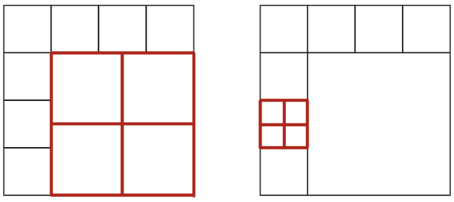

Figure 16: cases of 11 squares

This ends the proof of the statement for eleven squares.
All the other cases can be done in the same manner (12 comes from 9, 13 comes from 10 and so on).

We covered the two-dimensional case (squares) and proved that a square can be cut into any natural number of squares except 2, 3, 5 squares. Moreover, we proved that this partition is unique for the cases of 4, 6, 7, 8 squares (unique with respect to rotation and reflection). We also proved that the partition is not unique if the number of parts is more than 8.

The next logical step would be to investigate some of these questions for the three-dimensional case. Of course, for the three-dimensional case things get much more complicated and some of these results do not hold true. We provide a counter-example showing that, for example, the general statement for the two-dimensional case (that a partition to more than 8 parts is possible) does not hold true in the three-dimensional case.

Lemma 4 (counter-example for the three-dimensional case). Prove that it is impossible to divide a cube into 14 non-overlapping cubes.
(*Nairi Sedrakyan*) (IZhO-2025)

Proof. Assume that a large cube with a side length of a is divided into several small cubes (see below). By the term *face characteristic* of a cube, we mean the number of cubes having faces on that face of the initial large cube. According to lemma 1, the *characteristics* of two opposite faces of a cube cannot simultaneously be 4 (see below for the explanation).

According to lemma 2, the *characteristics* of two faces of a cube cannot be 4 and 6, or 6 and 6.

If the *characteristics* of any two faces of a cube are simultaneously 4, then we get 6 cubes of an edge length of $\frac{a}{2}$ and one rectangular parallelepiped of the size $\frac{a}{2} \times \frac{a}{2} \times a$ (see the figure below). We divide this rectangular parallelepiped into several cubes as well. Among the cubes containing

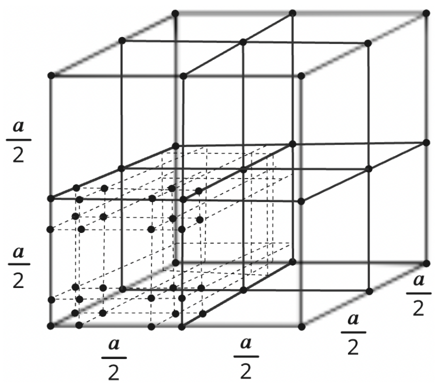

Figure 17: Cubes and Parallelepiped

the vertices of this rectangular parallelepiped (see the figure above), if there are no overlaps, then their number is not less than $8+1=9$, which is impossible. And, if there are overlaps, then we get 7 cubes of an edge length $\frac{a}{2}$, and the 8^{th} cube must be divided into at least 8 cubes, which is also impossible.

Thus, it follows that the sum of all *characteristics* is not less than

$$4+5\cdot 7=39,$$

which is impossible. On the other hand, the sum of all *characteristics* is not greater than

$$8\cdot 3+6\cdot 2=36,$$

which is impossible.

More of these type of problems can be found in the references.

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Generalizations of Napoleon's and Van Aubel's Theorems

Max Wolfensberger, Hayk Sedrakyan

Max Wolfensberger is a student at Ransom Everglades in Miami, FL who has high interest in Mathematics, Physics and Technology. At age 13 he was a finalist at the German Math Olympiad (2021). He has engaged in research and in 2023 at the International Math Circle presented an alternative to Heron's Formula. In Physics he was a presenter at APS Global Physics Conference on Cost Effective Optical tweezers research. He also was winner at Codemania Hackathon in Miami.

Hayk Sedrakyan is an IMO medal winner, Professor of Mathematics in Paris and a professional Math Olympiad Coach in the greater Boston area, USA. PhD from the UPMC-Sorbonne University, Paris. Author of problem solving and Olympiad style mathematics books published globally.

Abstract
This paper explores a process, which is a generalized form of Napoleon's and Van Aubel's Theorems that allows to create equilateral triangles and squares centered around the centroid of any $3n$-gon or $4n$-gon. We also investigate a recursive property of these generalizations, which allows to create an infinite sequence of circumscribed equilateral triangles and squares. Additionally, we introduce an extended generalization of Napoleon's theorem, which constructs a regular hexagon centered at the centroid of any $3n$-gon. This work not only expands the understanding of classical theorems but also paves the way for future explorations in geometric construction and its recursive properties.

Introduction

The motivation of this paper was Napoleon's theorem, which states: if equilateral triangles are constructed on the sides of a triangle, either all outside or all inside, the lines connecting their centers form an equilateral triangle (see the figure, Case $n = 1$).

Cool Shape Process:

$3n$-gon case represents the generalized Napoleon's Theorem.
$4n$-gon case represents the generalized Van Aubel's Theorem.

Step 1.

- **For any $3n$-gon:** Construct regular equilateral triangles on each side (either all inside or outside). The centroids of these equilateral triangles will form a cool $3n$-gon.

- **For any $4n$-gon:** Construct squares on each side (either all inside or outside). Find the centroids and connect the midpoints of adjacent centroids to form a cool $4n$-gon.

Step 2.

- For any cool $k \cdot n$-gon, where $k = 3$ or $k = 4$:
  - Create k shapes by connecting every k vertices, each with n vertices. Repeat this process until every vertex is connected to a n-gon. The centroids of these k shapes will form a regular k-gon that has the same centroid as the original $k \cdot n$-gon.

Generalized Napoleon's Theorem ($k = 3$ Case)

Original Case ($n = 1$)
Napoleon's Theorem states that if equilateral triangles are constructed externally on the sides of any triangle, the centroids of these equilateral triangles themselves form an equilateral triangle.

Generalized Van Aubel's Theorem ($k = 4$ Case)

Original Case ($n = 1$)
Van Aubel's theorem states that the two line segments connecting the centers of opposite squares are of equal lengths and are perpendicular to each other. From a result shown in the Petr-Douglas-Neumann Theorem, the center points of the four squares form a square.

Recursive property of generalizations

We explore a recursive property related to equilateral triangles and squares constructed via the generalized Napoleon's theorem (any $3n$-gon), and the generalized Van Aubel's theorem ($4n$-gon).

When the generalized Napoleon's theorem is applied to a $3n$-gon, the resulting inner equilateral triangle appears in the center of the shape. By applying the theorem again to the shape formed by the apices of the equilateral triangles formed in the first construction, a second equilateral triangle is generated. This new triangle perfectly circumscribes the original inner equilateral triangle.

For any $4n$-gon where Van Aubel's Theorem is applied, you can construct a square that perfectly circumscribes the square formed by the initial application of the theorem. After applying Van Aubel's theorem, you can use the midpoints of the constructed squares to create a new $4n$-gon. If you then apply the Generalized Van Aubel's Theorem to this new $4n$-gon, it produces a square that perfectly circumscribes the original square.

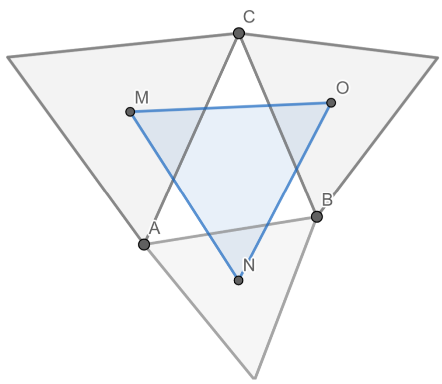


Figure 1: *
Case $n = 1$

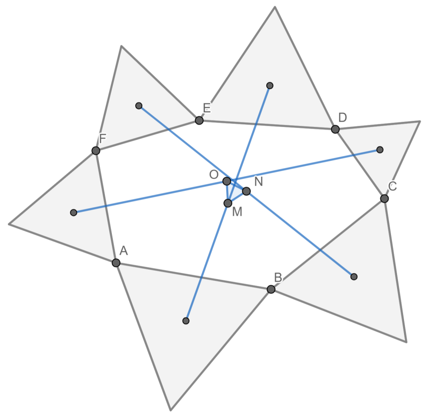


Figure 2: *
Case $n = 2$

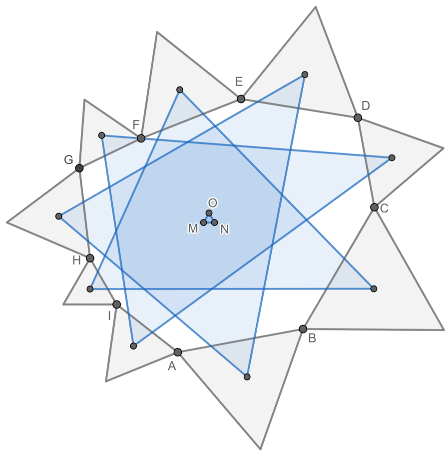


Figure 3: *
Case $n = 3$

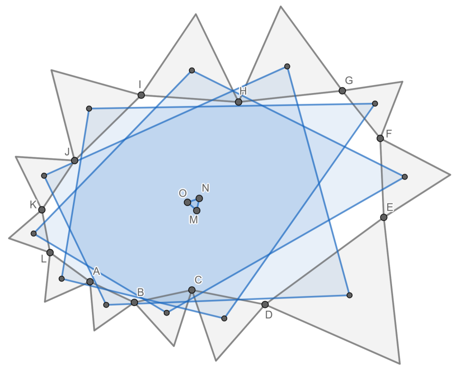


Figure 4: *
Case $n = 4$

Extended Generalization of Napoleon's Theorem for Constructing Regular Hexagons

This theorem extends the generalized Napoleon's theorem to construct a regular hexagon centered on the centroid of any $3n$-gon. First, by applying the generalized Napoleon's theorem to a $3n$-gon, an equilateral triangle is formed in the center of the shape. Then, applying the same theorem to the $3n$-gon formed by the centroids of equilateral triangles constructed on the inside or outside of the original $3n$-gon results in another equilateral triangle. These two equilateral triangles overlap with a 60-degree rotation, creating a regular hexagon centered at the centroid of the original $3n$-gon.

Proofs:

Every result from this paper can be proven with coordinate geometry, but we only provide the proof for the $n = 2$ case of the Generalized Napoleons Theorem.

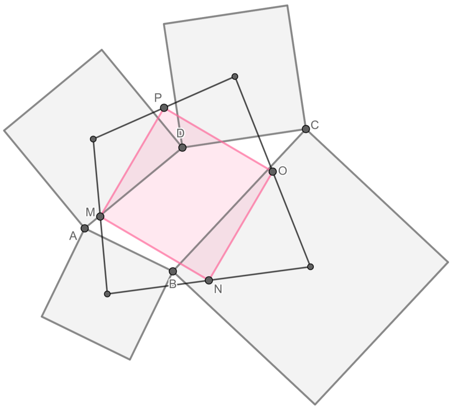


Figure 5: *
Case $n = 1$

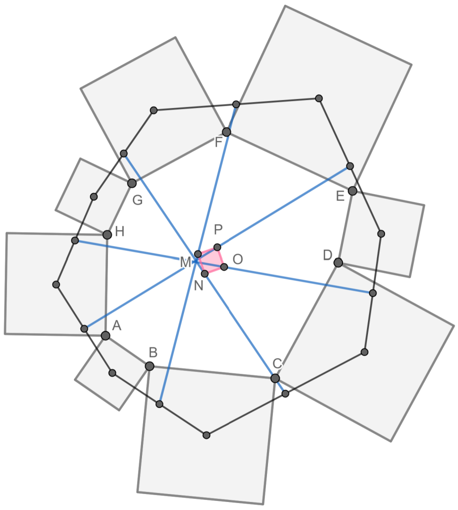


Figure 6: *
Case $n = 2$

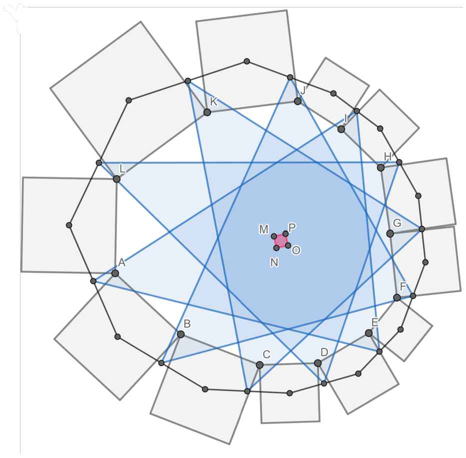


Figure 7: *
Case $n = 3$

Proof for Generalized Napoleon's Theorem ($n = 2$ hexagon case):

We will prove this case with coordinate geometry.

First, we graph the six points of the original hexagon with variable coordinates:

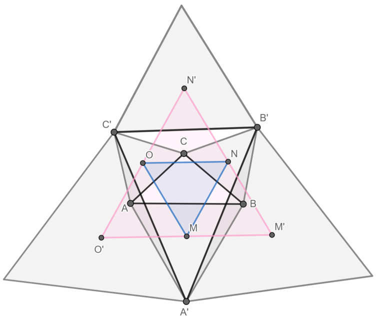


Figure 8: *
3n-gon Case, $n = 1$

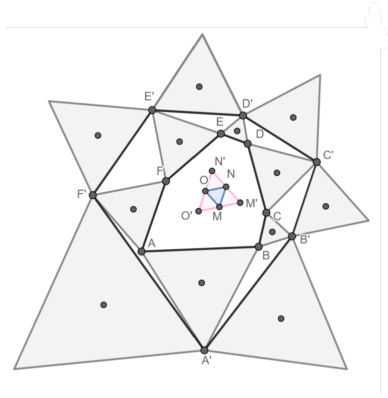


Figure 9: *
3n-gon Case, $n = 2$

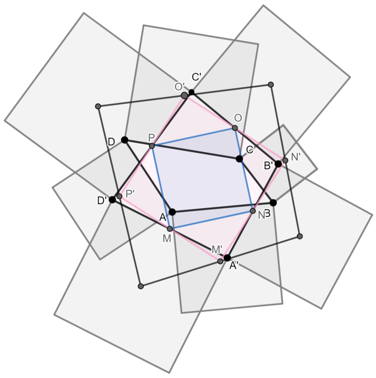
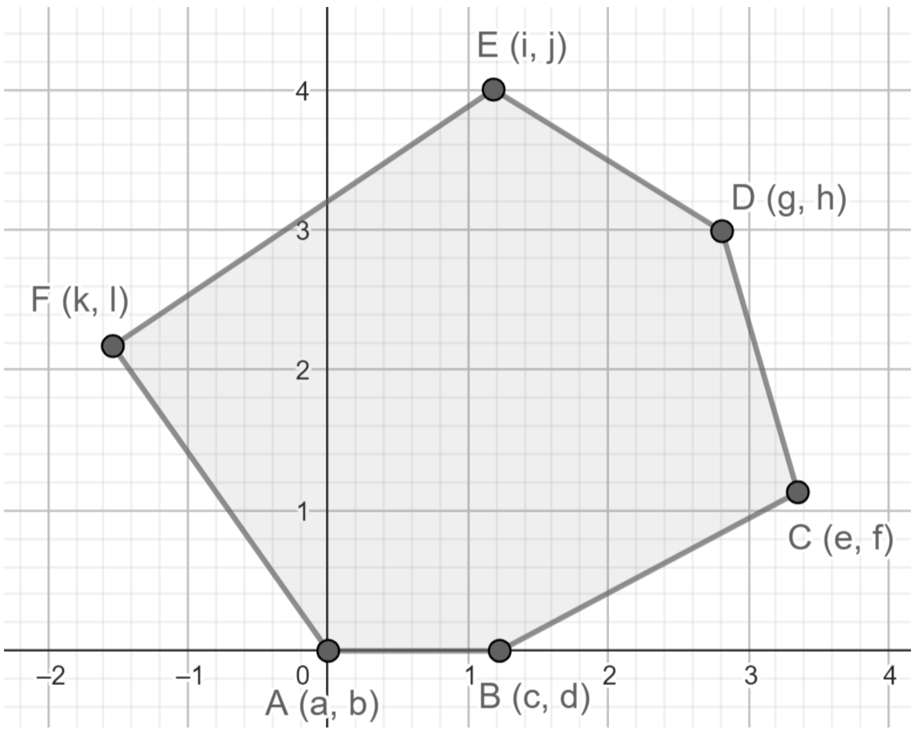


Figure 10: *
4n-gon Case, $n = 1$



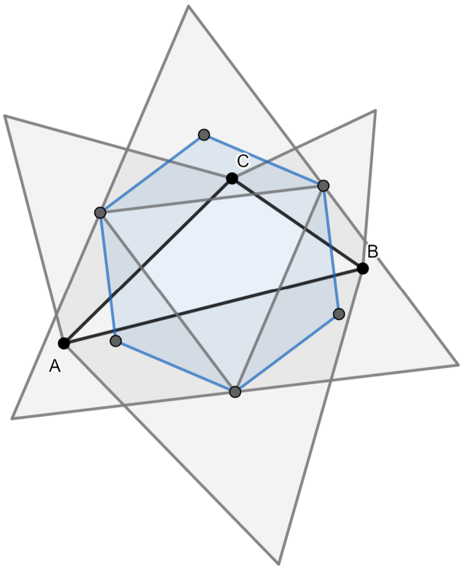


Figure 11: *
Case $n = 1$

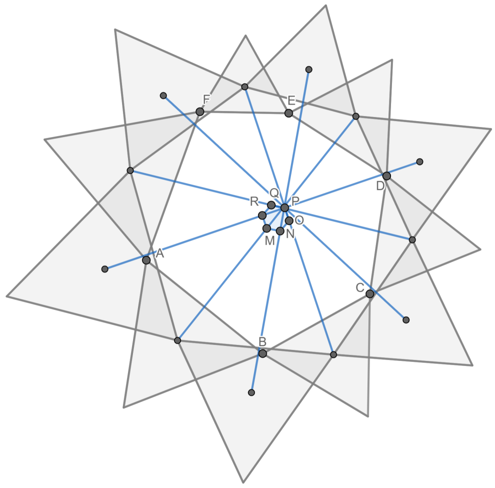
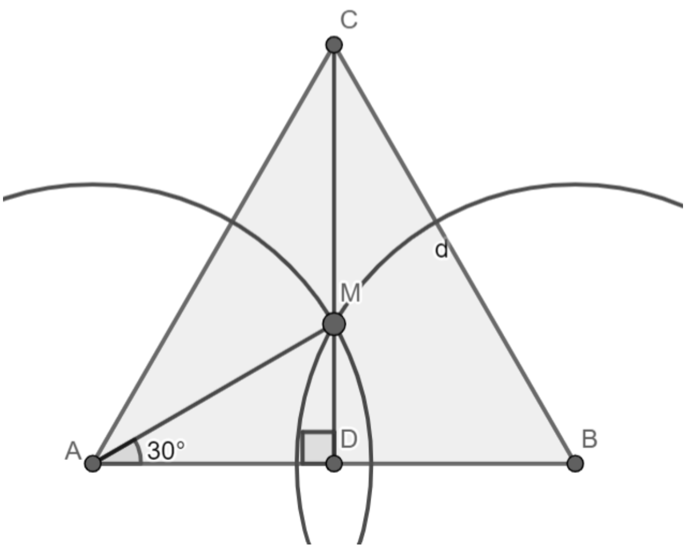


Figure 12: *
Case $n = 2$

Now we need to find the midpoint of an equilateral triangle facing outwards on each side. First of all, imagine there is an equilateral triangle ABC.



We want to find the center given two points, as that is what we need for the hexagon.
We know that M is 30° up from AB. We also know that $AD = \frac{1}{2}AB$
We can then solve for AM:

$$\cos 30^\circ = \frac{AD}{AM}$$

$$AM = \frac{2AD}{\sqrt{3}}$$

We know that $AD = \frac{1}{2}AB$ so:

$$AM = \frac{AB}{\sqrt{3}}$$

We also know that the intersection of two circles on points A and B will intersect at the center of the triangle, point M.

Now let's prove how to find the coordinates of point M given the coordinates of A and B.
We know from the distance theorem that the length of AB will be:

$$AB = \sqrt{(c-a)^2 + (d-b)^2}$$

Now we can solve for AM:

$$AM = \sqrt{\frac{(c-a)^2 + (d-b)^2}{3}}$$

Now we can use the formula of a circle where (x_1, y_1) is the center of the circle and R is the radius:

$$(x - x_1)^2 + (y - y_1)^2 = R^2$$

We assign $R = AM$, so if you plug in the points $A(a,b)$ and $B(c,d)$ for the center of the two circles, you get the equation:

$$(x-a)^2 + (y-b)^2 = \frac{(c-a)^2 + (d-b)^2}{3} = (x-c)^2 + (y-d)^2$$

Now we will solve this equation step by step, to get the coordinates for the intersects of the two circles:
Consider the first equality:

$$(x-a)^2 + (y-b)^2 = \frac{(c-a)^2 + (d-b)^2}{3}$$

First, expand both sides:

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 = \frac{c^2 + d^2 - 2ac - 2bd + a^2 + b^2}{3}$$

Consider the second equality:

$$(x-c)^2 + (y-d)^2 = \frac{(c-a)^2 + (d-b)^2}{3}$$

Expand both sides:

$$x^2 + y^2 - 2cx - 2dy + c^2 + d^2 = \frac{c^2 + d^2 - 2ac - 2bd + a^2 + b^2}{3}$$

Now we subtract both equations from each other, leaving us with:

$$-2ax-2by+a^2+b^2_2cx+2dy-c^2-d^2=0$$

Rearranging:

$$y(2b-2d)+c^2+d^2=x(2c-2a)+a^2+b^2$$

Slope intercept form:

$$y=x\left(\frac{c-a}{b-d}\right)+\frac{a^2+b^2-c^2-d^2}{2b-2d}.$$

The two intersect points lie on this line. Now we need to find the two points:

$$(x-a)^2+(y-b)^2=\frac{(c-a)^2+(d-b)^2}{3}.$$

Now we substitute for y using the equation from before:

$$(x-a)^2+\left(x\left(\frac{c-a}{b-d}\right)+\frac{a^2+b^2-c^2-d^2}{2b-2d}-b\right)^2=\frac{(c-a)^2+(d-b)^2}{3}.$$

If you solve this equation, you get two x values:

$$x_1=\frac{3a-\sqrt{3}(b-d)+3c}{6},$$

$$x_2=\frac{3a+\sqrt{3}(b-d)+3c}{6}.$$

Now, plug in to find the respective y values:

$$y_1=\frac{3a-\sqrt{3}(b-d)+3c}{6}\left(\frac{c-a}{b-d}\right)+\frac{a^2+b^2-c^2-d^2}{2b-2d}$$

$$y_1=\frac{\sqrt{3}a-\sqrt{3}c+3b+3d}{6}$$

$$y_2=\frac{3a+\sqrt{3}(b-d)+3c}{6}\left(\frac{c-a}{b-d}\right)+\frac{a^2+b^2-c^2-d^2}{2b-2d}$$

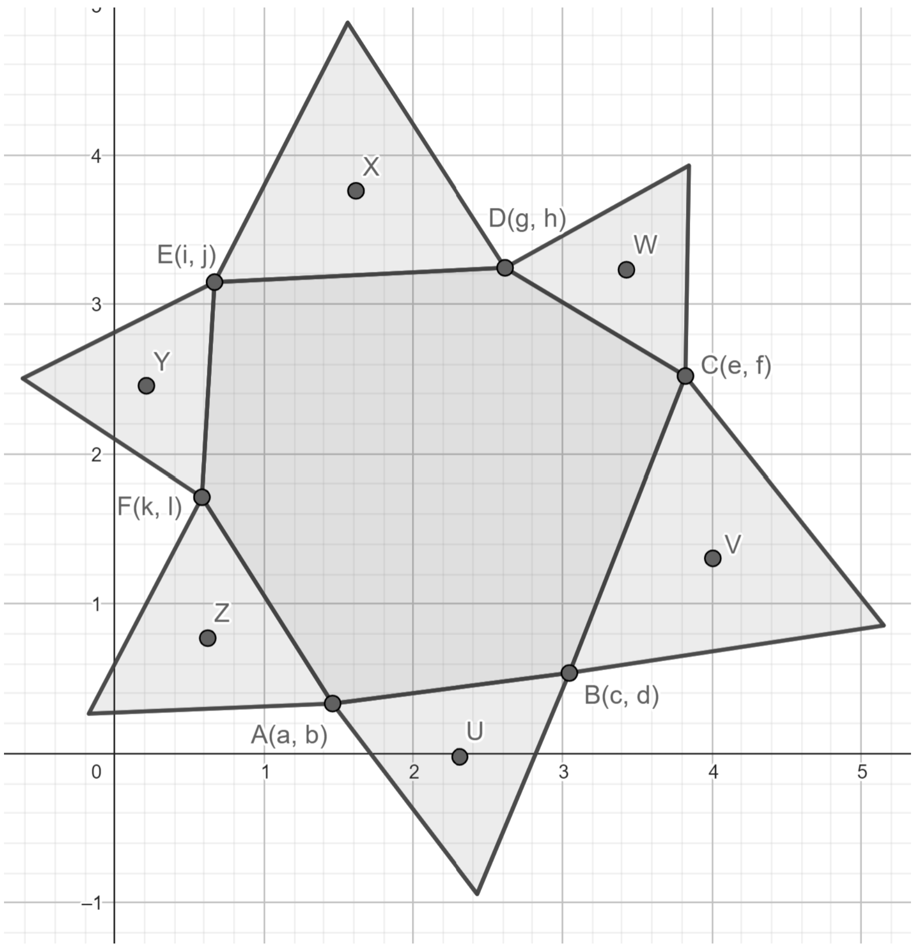
$$y_2=\frac{-\sqrt{3}a+\sqrt{3}c+3b+3d}{6}$$

So for two points A(a, b) and C(c, d), the two coordinates that represent the centers of the equilateral triangles are:

$$M_1\left(\frac{3a-\sqrt{3}(b-d)+3c}{6},\frac{\sqrt{3}a-\sqrt{3}c+3b+3d}{6}\right)$$

$$M_2\left(\frac{3a+\sqrt{3}(b-d)+3c}{6},\frac{-\sqrt{3}a+\sqrt{3}c+3b+3d}{6}\right)$$

Now let's find the coordinates of each point (For Now we will just do the midpoints on the outside of the hexagon, but it also works for the inside)



$$U\left(\frac{3a-\sqrt{3}(b-d)+3c}{6},\frac{\sqrt{3}a-\sqrt{3}c+3b+3d}{6}\right)$$

$$V\left(\frac{3c-\sqrt{3}(d-f)+3e}{6},\frac{\sqrt{3}c-\sqrt{3}e+3d+3f}{6}\right)$$

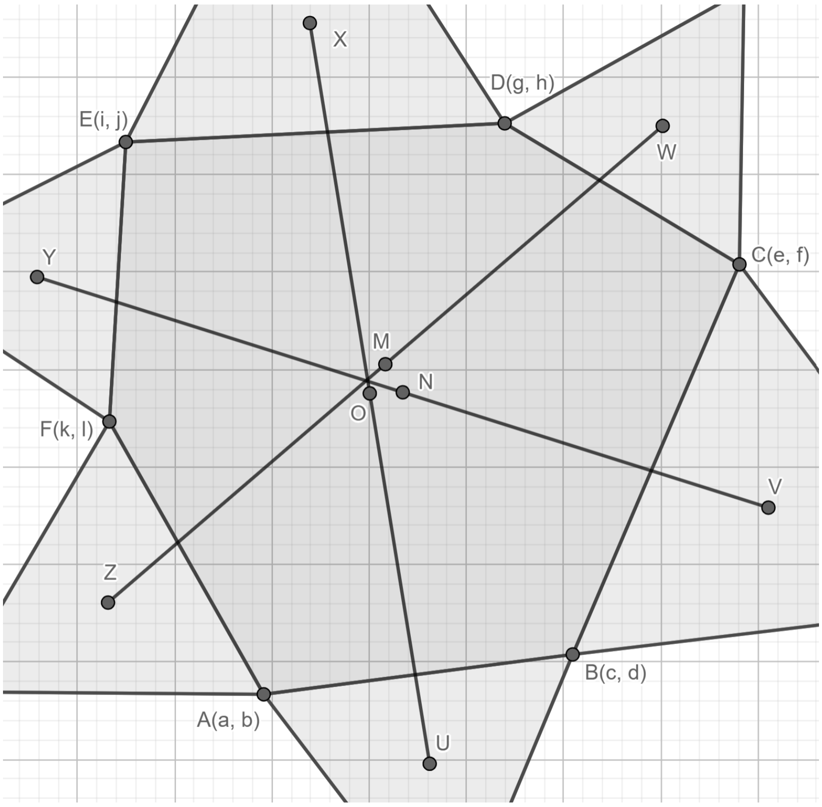
$$W\left(\frac{3e-\sqrt{3}(f-h)+3g}{6},\frac{\sqrt{3}e-\sqrt{3}g+3f+3h}{6}\right)$$

$$X\left(\frac{3g-\sqrt{3}(h-j)+3i}{6},\frac{\sqrt{3}g-\sqrt{3}i+3h+3j}{6}\right)$$

$$Y\left(\frac{3i-\sqrt{3}(j-l)+3k}{6},\frac{\sqrt{3}i-\sqrt{3}k+3j+3l}{6}\right)$$

$$Z\left(\frac{3k-\sqrt{3}(l-b)+3a}{6},\frac{\sqrt{3}k-\sqrt{3}a+3l+3b}{6}\right)$$

Now to find the midpoint of UX, VY and WZ we need to use the midpoint formula.



$$O\left(\frac{3(a+c+g+i)-\sqrt{3}(b-d+h-j)}{12}, \frac{3(b+d+h+j)+\sqrt{3}(a-c+g-i)}{12}\right)$$

$$N\left(\frac{3(c+e+i+k)-\sqrt{3}(d-f+j-l)}{12}, \frac{3(d+f+j+l)+\sqrt{3}(c-e+i-k)}{12}\right)$$

$$M\left(\frac{3(e+g+k+a)-\sqrt{3}(f-h+l-b)}{12}, \frac{3(f+h+l+b)+\sqrt{3}(e-g+k-a)}{12}\right)$$

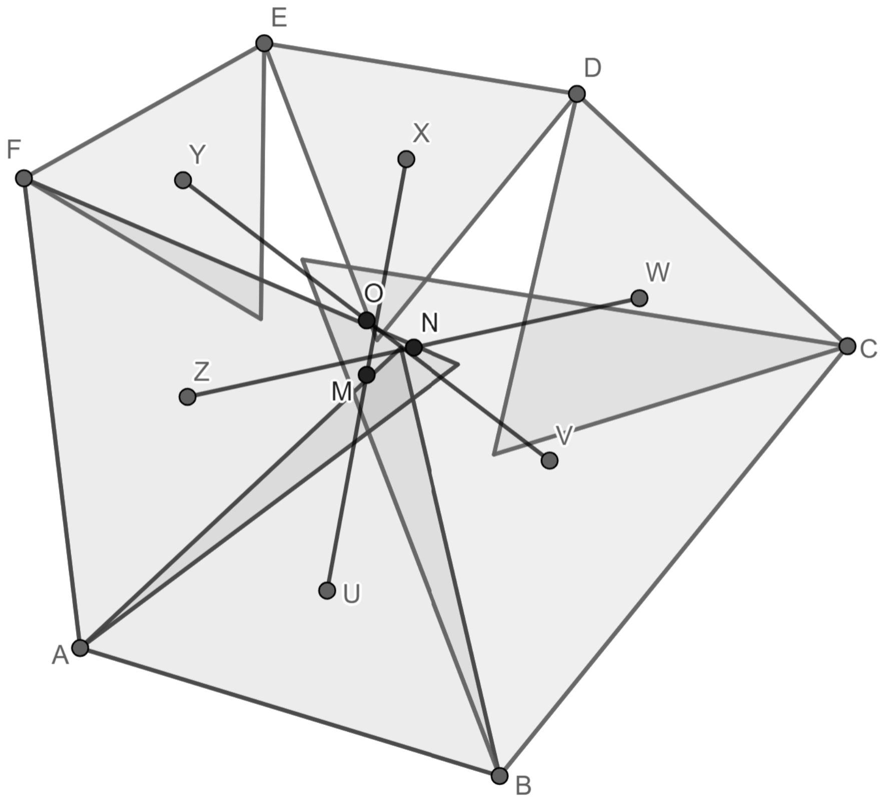
Now we can calculate the distance between O,M and N, and if they are equal, then OMN is an equilateral triangle.

$$ON^2 = \frac{\left(3(a-e+g-k)-\sqrt{3}(b-2d+h-2j+f+l)\right)^2+\left(3(b-f+h-l)+\sqrt{3}(a-2c+g-2i+e+k)\right)^2}{144}$$

$$NM^2 = \frac{\left(3(c-a-g+i)-\sqrt{3}(-2f-2l+d+j+h+b)\right)^2+\left(3(d-b-h+j)+\sqrt{3}(a-2e+g-2k+c+i)\right)^2}{144}$$

$$OM^2 = \frac{\left(3(c-e+i-k)-\sqrt{3}(2b-d+2h-j-f-l)\right)^2+\left(3(d-f+j-l)+\sqrt{3}(2a-c+2g-i-e-k)\right)^2}{144}$$

Now for the Inner case:



$$U\left(\frac{3a+\sqrt{3}(b-d)+3c}{6},\frac{-\sqrt{3}a+\sqrt{3}c+3b+3d}{6}\right)$$

$$V\left(\frac{3c+\sqrt{3}(d-f)+3e}{6},\frac{-\sqrt{3}c+\sqrt{3}e+3d+3f}{6}\right)$$

$$W\left(\frac{3e+\sqrt{3}(f-h)+3g}{6},\frac{-\sqrt{3}e+\sqrt{3}g+3f+3h}{6}\right)$$

$$X\left(\frac{3g+\sqrt{3}(h-j)+3i}{6},\frac{-\sqrt{3}g+\sqrt{3}i+3h+3j}{6}\right)$$

$$Y\left(\frac{3i+\sqrt{3}(j-l)+3k}{6},\frac{-\sqrt{3}i+\sqrt{3}k+3j+3l}{6}\right)$$

$$Z\left(\frac{3k+\sqrt{3}(l-b)+3a}{6},\frac{-\sqrt{3}k+\sqrt{3}a+3l+3b}{6}\right)$$

Now Using the midpoint formula, we get that:

$$M=\left(\frac{3(a+c+g+i)+\sqrt{3}(b-d+h-j)}{12},\frac{3(b+d+h+j)-\sqrt{3}(a-c+g-i)}{12}\right)$$

$$O\left(\frac{3(c+e+i+k)+\sqrt{3}(d-f+j-l)}{12},\frac{3(d+f+j+l)-\sqrt{3}(c-e+i-k)}{12}\right)$$

$$N\left(\frac{3(e+g+k+a)+\sqrt{3}(f-h+l-b)}{12}, \frac{3(f+h+l+b)-\sqrt{3}(e-g+k-a)}{12}\right)$$

Now we can calculate the distances, and they will all be equal:

$OM^2 = \left(\frac{3(e+k-a-g)+\sqrt{3}(2d-2j-(b+h))}{12}\right)^2 + \left(\frac{3(f+l-b-h)-\sqrt{3}(2c-2i-e+k-a+g)}{12}\right)^2$

$ON^2 = \left(\frac{3(g+a-c-i)+\sqrt{3}(2f-2l-(d+j))}{12}\right)^2 + \left(\frac{3(h+b-d-j)-\sqrt{3}(2e-2k+g-a-c+i)}{12}\right)^2$

$MN^2 = \left(\frac{3(k+a-c-i)+\sqrt{3}(2h-2b-(d+j))}{12}\right)^2 + \left(\frac{3(f+l-d+j)-\sqrt{3}(2g+2a-e-k-c+i)}{12}\right)^2$

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Novel Menelaus' Type Theorems

Max Wolfensberger, Hayk Sedrakyan

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Abstract
This paper presents novel generalizations of Menelaus' Theorem, extending its classical geometric insights to quadrilaterals and polygons with a scribe-center. The first main result, Theorem 1 (Wolfensberger-Sedrakyan), establishes a relationship between the segments formed by angle bisectors and intersecting lines in a quadrilateral, leading to a proportionality product equal to unity. The second result, Theorem 2 (Wolfensberger-Sedrakyan), further relates the intersection points of diagonals and sides in a quadrilateral, providing an enhanced framework that incorporates cevian lengths into Menelaus-like relationships. Additionally, a conjecture is proposed for even-sided polygons with a scribe-center, extending the proportionality principles to higher-order polygons. Rigorous proofs are provided, utilizing classical geometric methods such as the Law of Sines and Menelaus' Theorem, along with illustrative examples and diagrams. This work contributes to the rich field of geometric theorems, offering new perspectives and inviting further exploration of related topics.

Theorem 1

Theorem 1a (Wolfensberger-Sedrakyan). Let $ABCD$ be a quadrilateral. Let X, Y be the intersection points of lines AB and CD, and lines BC and AD, respectively. Let the angle bisector of $\angle Y$ intersect AB at M_1, and CD at M_3. Let the angle bisector of $\angle X$ intersect AD at M_4, and BC at M_2, then

$$\frac{AM_1}{M_1B} \cdot \frac{BM_2}{M_2C} \cdot \frac{CM_3}{M_3D} \cdot \frac{DM_4}{M_4A} = 1.$$

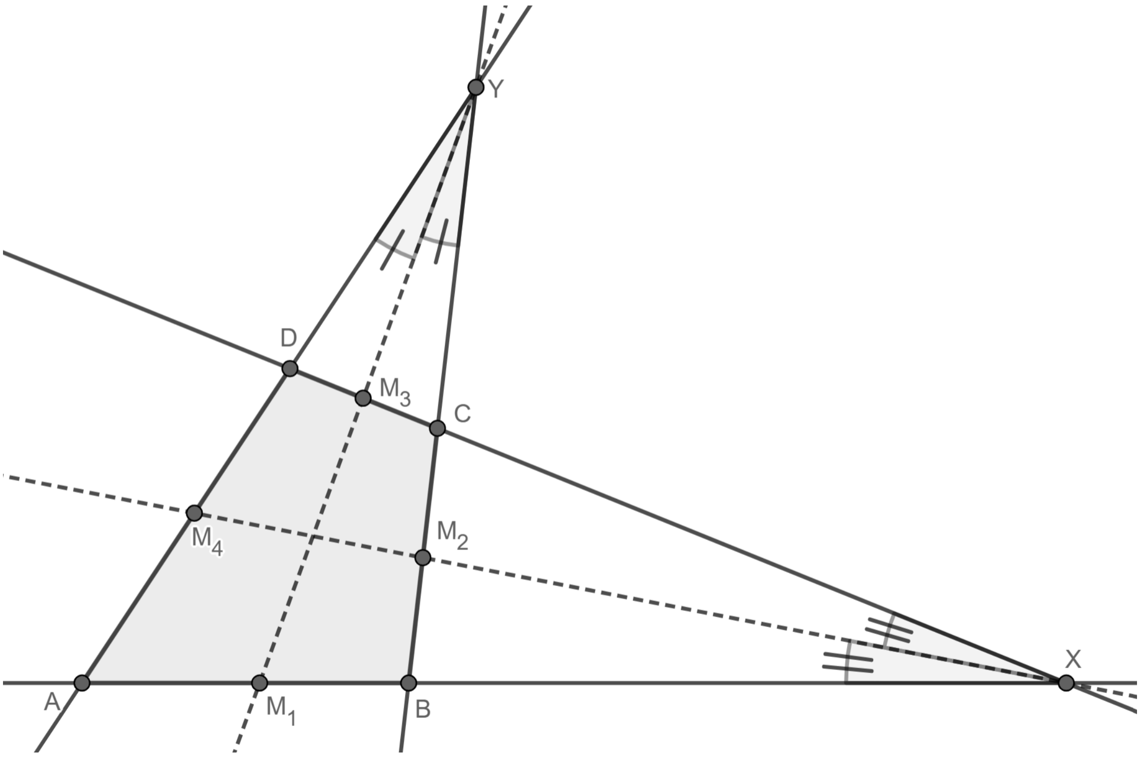

Figure 1: Illustration for Theorem 1a.

Theorem 1b. Alternate Form of Theorem 1a. For each segment AB, BC, CD, and DA, construct a circle external to $ABCD$ that is tangent to the respective segment and the extensions of its two adjacent sides. Connecting the centers of opposing circles gives you the angle bisectors as in Theorem 1a. Now, the same identity holds:

$$\frac{AM_1}{M_1B} \cdot \frac{BM_2}{M_2C} \cdot \frac{CM_3}{M_3D} \cdot \frac{DM_4}{M_4A} = 1.$$

Theorem 2

Relation to Theorem 1

From Figure 1, according to angle bisector theorem, we have:

$$\frac{AM_1}{M_1B} = \frac{AY}{BY}$$

$$\frac{BM_2}{M_2C} = \frac{BX}{CX}$$

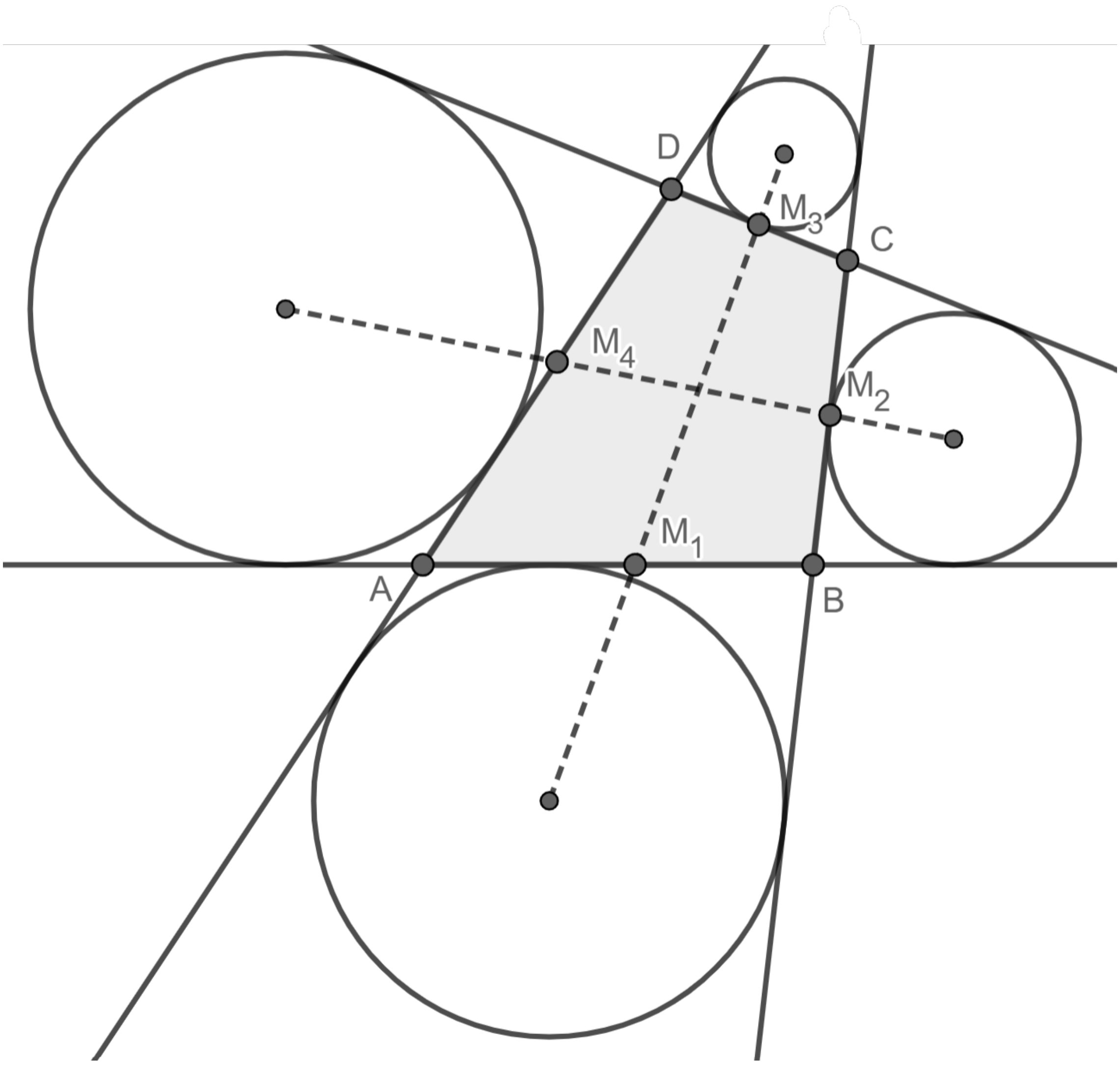

Figure 2: Illustration for Theorem 1b.

$$\frac{CM_3}{M_3D} = \frac{CY}{DY}$$
$$\frac{DM_4}{M_4A} = \frac{DX}{AX}$$

Substituting into Theorem 1, we get:

Theorem 2 (Wolfensberger-Sedrakyan). Consider any quadrilateral $ABCD$, let X be the intersection point of lines AB and CD, and Y be the intersection point of AD and BC (see the figure). Then the following holds:

$$\frac{AY}{BY} \cdot \frac{CY}{DY} \cdot \frac{DX}{AX} \cdot \frac{BX}{CX} = 1$$

Remark. This is the same drawing as in Menelaus' Theorem, however Menelaus' Theorem doesn't include the length of the cevian, and our formula does.

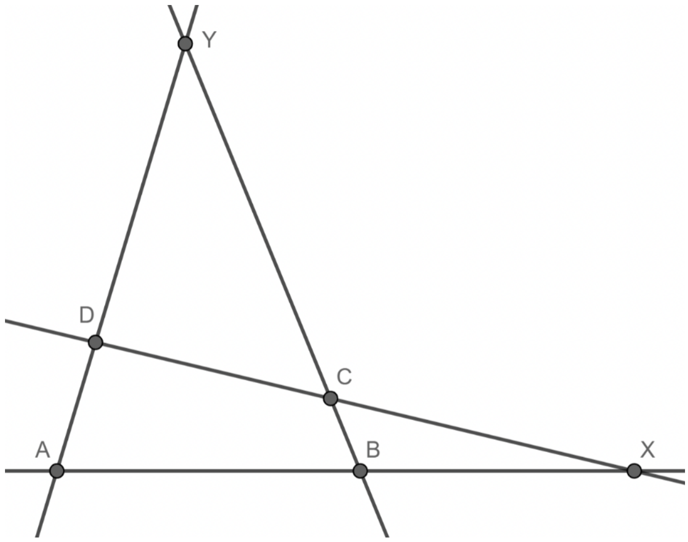

Figure 3: Illustration for Theorem 2.

Proof 1. for Theorem 1 and Theorem 2

The proof of Theorem 2 will be presented first. Since Theorem 1 is closely linked to Theorem 2 through the Angle Bisector Theorem, its validity follows directly from this proof. (see Theorem 2 Relation to Theorem 1 section)

Consider the following drawing.

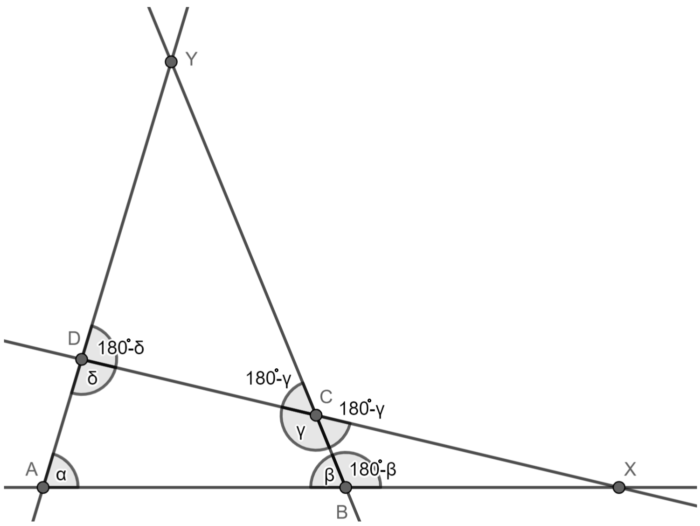

Figure 4: Drawing for Proof 1.

From triangle ABY, according to the law of sines we have

$$\frac{AY}{\sin\beta} = \frac{BY}{\sin\alpha}$$

From triangle CDY, according to the law of sines we have

$$\frac{CY}{\sin(180^\circ - \delta)} = \frac{DY}{\sin(180^\circ - \gamma)}$$

From triangle ADX, according to the law of sines we have

$$\frac{DX}{\sin\alpha} = \frac{AX}{\sin\delta}$$

From triangle BCX, according to the law of sines we have

$$\frac{BX}{sin(180^\circ - \gamma)} = \frac{CX}{sin(180^\circ - \beta)}$$

This can be rewritten since $sin(x) = sin(180^\circ - x)$:

$$\frac{AY}{BY} = \frac{\sin\beta}{\sin\alpha}, \quad \frac{CY}{DY} = \frac{\sin\delta}{\sin\gamma}, \quad \frac{DX}{AX} = \frac{\sin\alpha}{\sin\delta}, \quad \frac{BX}{CX} = \frac{\sin\gamma}{\sin\beta}$$

When you multiply these four, you get this:

$$\frac{AY}{BY} \cdot \frac{CY}{DY} \cdot \frac{DX}{AX} \cdot \frac{BX}{CX} = \frac{\sin\beta}{\sin\alpha} \cdot \frac{\sin\delta}{\sin\gamma} \cdot \frac{\sin\alpha}{\sin\delta} \cdot \frac{\sin\gamma}{\sin\beta} = 1.$$

Proof 2. Alternate Proof for Theorem 2

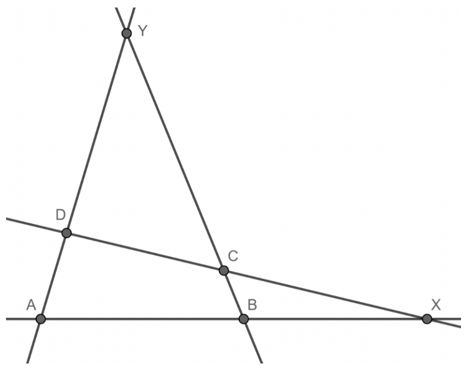

Figure 5: Illustration for Theorem 2 and its proof.

We will apply Menelaus' Theorem twice to prove Theorem 2:

Step 1: Apply Menelaus' Theorem to Triangle ABY

For triangle ABY, consider the transversal DX. By Menelaus' Theorem, we have:

$$\frac{AX}{BX} \cdot \frac{BC}{CY} \cdot \frac{DY}{DA} = 1.$$

Step 2: Apply Menelaus' Theorem to Triangle DCY

For triangle DCY, consider the transversal AX. By Menelaus' Theorem, we have:

$$\frac{DX}{CX} \cdot \frac{BC}{BY} \cdot \frac{AY}{AD} = 1.$$

Step 3: Combine the Two Applications Multiply the reciprocal of the first equation with the second equation:

$$\left(\frac{BX}{AX} \cdot \frac{CY}{BC} \cdot \frac{DA}{DY}\right) \cdot \left(\frac{DX}{CX} \cdot \frac{BC}{BY} \cdot \frac{AY}{AD}\right) = 1.$$

Step 4: Simplify the Expression

Cancel out the common terms:

$$\frac{BX}{AX}\cdot\frac{CY}{DY}\cdot\frac{DA}{AD}\cdot\frac{DX}{CX}\cdot\frac{BC}{BC}\cdot\frac{AY}{BY}=1.$$

After simplification, this leaves us with:

$$\frac{AY}{BY}\cdot\frac{CY}{DY}\cdot\frac{DX}{AX}\cdot\frac{BX}{CX}=1.$$

This completes the proof of Theorem 2, and thus Theorem 1.

Conjecture: Ceva's-like Theorem for Polygons with a Scribe-Center

*Scribe-Center Definition. The scribe-center of a polygon is the point of concurrency of the line segments connecting the centers of opposite circles inscribed around the polygon. These inscribed circles are constructed such that each circle is tangent to one side of the polygon and the extensions of its two adjacent sides. A polygon possesses a scribe-center if and only if all such segments concur at a single point.

*Conjecture Statement. Let P be a polygon with n sides, where n is even, and assume P has a scribe-center. For each side i of the polygon (denoted XY), extend the polygon's sides and construct inscribed circles tangent to XY and the extensions of its adjacent sides. Let the centers of opposite inscribed circles be connected by line segments. These segments intersect the polygon's sides at points M_i.

For each side XY, let XM_i and YM_i denote the segments of XY divided by M_i, measured in the clockwise and counterclockwise directions, respectively. Define the ratio:

$$R_i=\frac{XM_i}{YM_i}.$$

Then, the product of these ratios over all n sides of the polygon is conjectured to equal 1:

$$\prod_{i=1}^{n}R_i=\prod_{i=1}^{n}\frac{XM_i}{YM_i}=1.$$

Specific Case Example: Hexagon ($n=6$)

For a hexagon with a scribe-center, the conjecture implies:

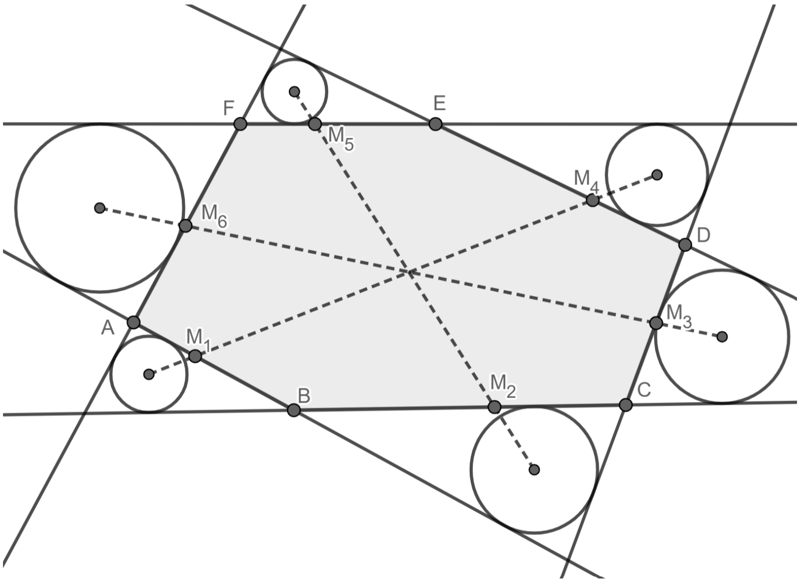

Figure 6: Illustration of the conjecture for a hexagon ($n = 6$).

$$\frac{AM_1}{M_1B} \cdot \frac{BM_2}{M_2C} \cdot \frac{CM_3}{M_3D} \cdot \frac{DM_4}{M_4E} \cdot \frac{EM_5}{M_5F} \cdot \frac{FM_6}{M_6A} = 1.$$

Possible Directions for Proof

1. **Investigate the Properties of Polygons with a Scribe-Center**
   - Begin by exploring the geometric properties and constraints of polygons that possess a scribe-center.
   - Identify any symmetries or recurring patterns that arise from the construction of inscribed circles and their centers.
   - Examine how the scribe-center affects the relationships between the segments and their points of intersection on the polygon's sides.
2. **Establish Connections to the Conjecture**
   - Utilize these properties to derive relationships or invariants that directly contribute to the proof of the conjecture.
   - Consider leveraging the concurrency of segments at the scribe-center and how it influences the ratios of segment lengths.
3. **Alternative Approach to Proving the Conjecture**
   - Coordinate geometry could be used to formalize relationships, though this might become computationally intensive.

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Mathematical Competitions and Gender Equity: Performance and Preferences Among Ninth-Grade Participants

Mark Applebaum

Professor Mark Applebaum has authored 16 books and over 80 scholarly articles published in multiple languages, including English, Hebrew, Russian, Spanish, Arabic, Ukrainian, and Polish. His research interests include the popularization of mathematics, gifted education, the development of creative and critical thinking, math competitions, and STEM education. He has presented at over 30 international conferences, often serving as a keynote speaker or member of scientific committees. Prof. Applebaum leads various educational initiatives, notably as CEO of the Kangaroo Israel Math Contest, and holds prominent positions in international organizations, including IGMCG, the EMS Education Committee, and MACAS. Since 2023, he has headed the Integrative STEM Education M.Ed. program at Kaye Academic College and actively contributes to national and international committees advancing mathematics education.

Abstract

This study investigates gender differences in mathematical performance and task preferences among ninth-grade students participating in a mathematics competition. A total of 71 students (39 girls and 32 boys) from a highly ranked school engaged in a competition featuring eight multiple-choice mathematical tasks derived from the International Kangaroo Math Contests. The study examined gender-based differences in overall performance, the selection of the most challenging and preferred tasks, and the relationship between task preferences and success. Results revealed that female students significantly outperformed male students in pattern recognition and common sense reasoning tasks, supporting the notion that structured, applied reasoning tasks align with female students' strengths. Additionally, while both genders demonstrated similar patterns in selecting challenging and preferred tasks, a significant correlation emerged between task enjoyment and success, emphasizing the role of positive engagement in mathematical achievement. Conversely, no significant correlation was found between selecting a task as the most challenging and successfully solving it, indicating that perceived difficulty does not necessarily predict lower performance. These findings contribute to the broader discourse on gender dynamics in mathematics competitions, demonstrating that competition format, question structure, and confidence-building approaches can influence performance. The results underscore the importance of designing inclusive educational policies that foster engagement and create supportive, low-anxiety competitive environments to promote equitable participation and success across genders.

Theoretical Background

Understanding gender disparities in mathematical competitions is essential for addressing broader inequities in STEM education. Traditional research has often suggested that boys outperform girls in competitive mathematical settings (Gneezy et al., 2003). However, contemporary findings indicate that gender gaps in mathematics performance are not innate but rather influenced by sociocultural, psychological, and structural factors (Hyde et al., 2008; Ceci & Williams, 2020). Research highlights that competition environments significantly affect gender-based performance differences. Niederle and Vesterlund (2010) argue that competitive pressures may distort actual gender differences in mathematical ability, as women tend to underperform relative to their skills in highly competitive environments while excelling in non-competitive or structured settings. This aligns with findings from Gneezy, Niederle, and Rustichini (2003), who demonstrated that women perform well in single-sex competitions but struggle in mixed-gender tournaments due to confidence and stereotype-related pressures. Gender differences in mathematics performance have been widely examined across educational settings, including mathematical competitions. Historically, male dominance in competitive mathematics has been reported, but recent research suggests that when competition environments are structured to be supportive and anxiety-reducing, female students perform at comparable or superior levels to their male counterparts (Leder & Forgasz, 2008; Hyde & Mertz, 2009). Online mathematical competitions further support this perspective, showing that girls excel in structured, low-pressure problem-solving formats (Freiman & Applebaum, 2011). For instance, the Virtual Mathematical Marathon demonstrated that when given extended opportunities to engage with complex problems, boys and girls performed similarly, countering traditional narratives of male superiority in competitive mathematics (Applebaum et al., 2013). Early research attributed gender disparities in mathematics to biological factors such as brain structure and hormonal differences (Halpern, 1997; Moir & Jessel, 1989). However, more recent studies emphasize the dominant role of environmental and cultural influences in shaping mathematical performance (Halpern et al., 2007; Spelke, 2005). Cross-cultural analyses reveal that gender differences vary significantly based on societal attitudes toward STEM education (Stoet & Geary, 2013). In countries with strong gender equity initiatives, performance gaps are minimal or even reversed (Hyde et al., 2008; Wang & Degol, 2017). Similarly, research on Israeli students highlights how competition format, instructional strategies, and national education policies influence gender disparities in performance (Applebaum et al., 2020).

Psychological factors, including self-efficacy and stereotype threat, have been widely examined in gender-related mathematical performance. Studies indicate that stereotype threat—the fear of confirming negative gender-based stereotypes—can adversely affect female students in mathematical competitions (Spencer et al., 1999). However, interventions such as growth mindset approaches and exposure to positive role models have been effective in mitigating these effects (Good et al., 2003; Dweck, 2006). Additionally, attribution theory suggests that boys and girls perceive success and failure differently in mathematical contexts: boys tend to attribute success to innate ability and failure to external factors, whereas girls often attribute success to effort and failure to a lack of ability (Bandura, 1997; Schunk & Gunn, 1986). These perceptions significantly impact confidence levels and long-term persistence in mathematics (Lloyd et al., 2005). Studies have shown that when girls are encouraged to view mathematical success as a reflection of ability rather than effort alone, their performance and engagement improve significantly (Eccles & Jacobs, 1986).

The structure of mathematical competitions significantly influences gender-based performance differences. Research suggests that high-stakes, time-limited competitions tend to favor boys due to their greater confidence and risk-taking behavior (Reuben et al., 2015). However, when competitions allow for extended problem-solving, collaboration, and strategic thinking, female students perform at levels equal to or above their male counterparts (Ellison & Swanson, 2018). Evidence from online mathematics competitions further supports the notion that structured and interactive learning environments enable girls to demonstrate strong mathematical reasoning (Applebaum et al., 2020). These findings align with self-determination theory, which posits that competence, autonomy, and motivation are essential drivers of academic performance (Deci & Ryan, 2000). Therefore, ensuring inclusive competition structures that emphasize engagement over speed can help bridge gender disparities in mathematics achievement.

Educational policy and competition design also play a pivotal role in fostering gender equity in mathematics. Research highlights the importance of confidence-building programs, stereotype threat reduction strategies, and a growth mindset approach in increasing female participation in mathematics (Good et al., 2003; Hyde et al., 2008). Moreover, incorporating collaborative and team-based components into competitions has been shown to increase female engagement and overall performance (Slavin, 1995). Studies on mathematics education in Israel further emphasize that curricular adjustments and the inclusion of female role models in STEM encourage female students to pursue advanced mathematics studies (Applebaum et al., 2021).

Given these insights, this study examines how gender differences manifest in a school-based mathematics competition. Specifically, it explores how task difficulty, competition format, and individual confidence levels interact to shape performance outcomes. By incorporating findings from online and traditional mathematics competitions, this research contributes to the broader discussion on gender equity in mathematics education and provides insights into inclusive competition frameworks that foster participation and success across genders.

Methodology and Research Questions

This study was conducted with 39 girls and 32 boys from two ninth-grade classes at a highly ranked school known for its exceptional mathematics performance on national tests and its high socioeconomic demographic. The participants were contenders in the school's annual mathematics championship, having prepared over several months by practicing various non-standard problems on a specialized online problem-solving platform. This preparation aimed to ensure that all students, regardless of gender, had exposure to similar competition-style tasks before the event. The competition consisted of eight multiple-choice questions, each with five answer options, including one correct answer. Students received 1 point for each correct response and 0 points for incorrect responses. These tasks (see Appendix 1) were previously used in the International Kangaroo Math Contests and were officially sanctioned by the Kangaroo International Committee. The competition lasted 60 minutes. Afterward, students completed a brief questionnaire designed to capture their perceptions of task difficulty and enjoyment. Specifically, they were asked to identify which of the eight tasks they found most challenging and which they enjoyed the most.

To analyze the data, independent samples t-tests were conducted to compare the mean scores of male and female participants across each task and the overall competition score. This statistical method was selected to determine whether performance differences between the two groups were statistically significant. Additionally, correlation analyses were conducted to explore the relationships between task preference and success rates, as well as differences in perception between genders. These analyses provided deeper insights into whether students' enjoyment of tasks correlated with their performance outcomes and whether perceived difficulty influenced success rates differently for boys and girls.

The following research questions guided this study:

1. Performance in Non-Standard Tasks: Do boys outperform girls in solving non-standard mathematical tasks?
2. Perception of Challenge: Are there gender differences in the selection of the most challenging question?
3. Preference for Tasks: Are there gender differences in the selection of the 'loveliest' question, or the task students found most enjoyable?
4. Relationship Between Challenge, Preference, and Success:
    - Is there a relationship between selecting a task as the most challenging and a student's success in solving that task?
    - Is there a relationship between choosing a task as the 'loveliest' and the student's success in solving that task?
    - Do these relationships differ by gender?

By addressing these questions, this study aims to uncover insights into gender dynamics, motivational factors, and the relationship between confidence, task perception, and mathematical performance. The findings contribute to broader discussions on educational equity, learning environments, and strategies for fostering gender-inclusive participation in mathematics competitions. Additionally, the study's results can inform the design of future competitions that support equitable participation and enhance student engagement with mathematical problem-solving, regardless of gender.

Data Presentation and Analysis

This study involved 71 ninth-grade students (39 girls and 32 boys) who participated in a mathematics competition consisting of eight multiple-choice tasks. These tasks assessed geometry, logical reasoning, pattern recognition, and spatial ability. Each task had five possible answers, with only one being correct.

To examine gender-based performance differences, mean scores were calculated for each task and the overall competition score. Independent samples t-tests were used to determine whether these differences were statistically significant.

To enhance clarity, the results are presented in Table 1 below:

Table 1. Students' Performance Related to Gender

Task	Mathematical Skill	Girls' Mean (SD)	Boys' Mean (SD)	p-value
1	Geometry and Proportion	0.974 (0.160)	0.938 (0.246)	0.469
2	Common Sense	0.718 (0.456)	0.781 (0.420)	0.545
2	Patterns	0.769 (0.427)	0.531 (0.507)	0.039*
4	Spatial Ability	0.590 (0.498)	0.656 (0.483)	0.571
5	Geometry and Constructions	0.462 (0.505)	0.313 (0.471)	0.203
6	Spatial Ability	0.256 (0.442)	0.313 (0.471)	0.610
7	Common Sense	0.821 (0.389)	0.500 (0.508)	0.005*
8	Logical Reasoning	0.513 (0.506)	0.344 (0.483)	0.155
Overall	All Tasks Combined	0.510 (0.135)	0.437 (0.160)	0.046*

*indicates statistical significance

Key Insights

- Pattern recognition (Task 3) and common-sense reasoning (Task 7) showed a statistically significant advantage for girls. These results align with studies suggesting that female students excel in structured and applied reasoning tasks.
- Overall, girls had a higher mean total score, with statistical significance ($p = 0.046$). This challenges traditional assumptions of male dominance in competitive mathematics and suggests a shift in performance trends.
- Spatial ability and logical reasoning tasks showed no significant gender differences, suggesting that problem-solving skills in these areas were evenly distributed among participants.

These results emphasize the importance of both cognitive and environmental factors in shaping mathematical performance. The next section will explore these findings concerning previous research and their implications for fostering gender-equitable participation in mathematics competitions.

To gain deeper insight into student experiences, participants were asked to identify which task they found most challenging and which they enjoyed the most. The results (see Table 2) indicated that Task N6, which required spatial ability, was perceived as the most difficult by 33 students, including 21 girls and 12 boys. Despite being labelled as challenging, approximately 33% of these students successfully solved it, demonstrating that perceived difficulty does not always correlate with poor performance.

Conversely, Task N1, focusing on geometry and proportion, emerged as the most enjoyable task. A total of 20 students, with a nearly even distribution between boys and girls, identified it as their favorite. Interestingly, 95% of those who favored this task answered it correctly, highlighting a strong relationship between enjoyment and success.

Overall, no significant gender differences were observed in task preference. Both boys and girls demonstrated similar patterns in their perceptions of challenge and enjoyment, suggesting that familiarity with task types, confidence, and problem-solving approach may play a more crucial role than gender in influencing mathematical engagement and performance.

These findings underscore the connection between engagement and success in mathematical tasks. Students tend to perform better when they find tasks enjoyable, emphasizing the importance of fostering a positive learning environment in mathematics education. Encouraging students to develop confidence in their abilities and engage with mathematical challenges enjoyably may enhance overall performance and motivation.

Furthermore, the study highlights the disparity between perception and actual performance. While students may identify a task as particularly challenging, many can still solve it successfully. This suggests that promoting resilience, persistence, and effective problem-solving strategies can help students navigate mathematical challenges with greater confidence, ultimately fostering a more inclusive and supportive learning environment.

Table 2. Summary of Most Challenging and Lovely Tasks by Gender and Success Rate

Task: Skill Needed	Most Challenging Total*	Most Challenging Girls*	Most Challenging Boys*	Most Lovely Total*	Most Lovely Girls*	Most Lovely Boys*
N1:*Geometry Proportion*	0 (-)	0 (-)	0 (-)	20 (19)	9 (9)	11 (10)
N2:*Common Sense*	2 (1)	2 (1)	0 (-)	6 (5)	3 (2)	3 (3)
N3:*Patterns*	12 (6)	8 (4)	4 (2)	7 (6)	5 (5)	2 (1)
N4:*Spatial Ability*	2 (2)	1 (1)	1 (1)	13 (8)	5 (4)	8 (4)
N5:*Geometry Constructions*	4 (2)	1 (1)	3 (1)	0 (-)	0 (-)	0 (-)
N6:*Spatial Ability*	33 (11)	21 (6)	12 (5)	5 (2)	4 (2)	1 (0)
N7:*Common Sense*	13 (8)	3 (3)	10 (5)	6 (5)	4 (4)	2 (1)
N8:*Logics*	5 (3)	3 (1)	2 (2)	14 (10)	9 (6)	5 (4)

* in parenthesis is the success rate
Note: The numbers represent the number of students who selected a task as most challenging or lovely, with the number of students who successfully solved the task in parentheses.
Most Challenging Task: Task N6 (Spatial Ability) was identified as the most challenging for most students.

The relationship between task selection and performance provides further insight into how boys and girls approach mathematical challenges. A strong correlation was found between selecting Task N1 as the most enjoyable and successfully solving it, with 19 out of 20 students answering correctly. This highlights the impact of positive task perception on performance - when students feel confident or engaged, they are more likely to succeed.

In contrast, no significant correlation was observed between selecting a task as the most challenging and successfully solving it. This suggests that students may overestimate the difficulty of certain problems or underestimate their abilities, leading to a disconnect between perception and actual performance.

While certain tasks, such as Tasks 3 and 7, showed notable gender differences in performance - favoring female participants - overall, both boys and girls exhibited similar task preferences and success rates. This suggests that confidence, familiarity with task types, and individual problem-solving strategies may play a more significant role than gender alone in determining mathematical performance. Rather than innate ability, factors such as prior exposure to problem-solving methods, task engagement, and self-efficacy likely contribute to performance differences. These findings reinforce the need for educational strategies that prioritize confidence-building, problem-solving resilience, and equitable exposure to competitive mathematics tasks to support all students in reaching their full potential.

These findings reinforce the importance of motivation and confidence in mathematical achievement. Educators should consider implementing strategies that help students develop positive associations with mathematical problem-solving, which can ultimately lead to higher engagement and success. Additionally, designing low-pressure competition formats that emphasize enjoyment and confidence-building could further promote equitable participation in mathematical problem-solving across genders.

Discussion and Conclusion

The findings of this study contribute to the evolving discourse on gender differences in mathematical competitions, challenging long-standing assumptions. While earlier studies suggested that boys consistently outperformed girls in competitive mathematics settings (Gneezy et al., 2003), this study presents a more nuanced perspective. When competitions are structured to be supportive, equitable, and less anxiety-inducing, female students demonstrate strong performance, particularly in pattern recognition and applied reasoning. These results suggest a potential redefinition of the gender gap, indicating that competition structure and preparation may influence performance disparities more than inherent ability. These findings align with recent research showing that gender disparities in mathematical performance are narrowing due to shifting educational practices and societal attitudes (Hyde et al., 2008; Ceci & Williams, 2020). Large-scale assessments such as PISA and TIMSS indicate that the gender gap in mathematics varies across cultural and educational contexts, with some countries achieving gender parity or even female advantages in mathematical performance (Stoet & Geary, 2013). This suggests that sociocultural and institutional factors play a significant role in shaping mathematical achievement rather than innate cognitive differences. Research on online and in-person mathematical competitions supports this claim, demonstrating that competition formats, engagement levels, and structural changes in assessment design significantly impact gendered outcomes in mathematics (Applebaum et al., 2020).

The performance results presented in Table 1 provide additional insight into these patterns. Female students significantly outperformed male students in pattern recognition (Task 3, $p = 0.039$) and common sense reasoning (Task 7, $p = 0.005$). These findings suggest that structured, pattern-based reasoning tasks align more closely with female students' strengths, supporting previous research indicating that girls excel in structured problem-solving and applied reasoning tasks (Ellison & Swanson, 2018). Additionally, the overall performance difference ($p = 0.046$), with girls scoring higher across all tasks combined, further supports the idea that competition format and question structure can influence gender disparities in performance.

The stronger performance of female students in specific tasks within this study challenges prior findings that positioned boys as superior competitors in mathematics (Leder et al., 2000). Several key factors may explain this shift, including efforts to promote gender equity in STEM education through targeted interventions and inclusive competition formats (Wang & Degol, 2017), reduced stereotype threat due to structured preparation (Spencer et al., 1999), and increased confidence and task perception parity between genders (Schunk & Gunn, 1986). Additionally, problem-solving strategies vary by gender, with female students excelling in structured, pattern-based reasoning tasks (Ellison & Swanson, 2018), suggesting that the nature of competition tasks may favor certain cognitive strengths. Evidence from online mathematical competitions, such as the Virtual Mathematical Marathon, further supports the idea that when female students are provided opportunities to engage deeply with non-standard problems in a structured, low-pressure environment, they perform at equal or even superior levels compared to male students (Freiman & Applebaum, 2011; Applebaum et al., 2013).

Although the results suggest a narrowing gender gap, alternative explanations must be considered. Research by Niederle and Vesterlund (2010) highlights that competitive pressure can distort actual gender differences in mathematical ability. Their findings suggest that female students tend to underperform in high-stakes competitive environments despite possessing the necessary skills, whereas they excel in structured, lower-pressure settings. This aligns with our findings that girls performed well in this structured school-based competition, indicating that competition format plays a crucial role in shaping gender-based performance.

Additionally, the students in this study had significant prior exposure to non-standard problem-solving strategies, which may have mitigated gender-based performance disparities (Buser et al., 2014). The low-stakes nature of this competition also aligns with prior research showing that female students perform better in less pressurized, time-flexible environments (Reuben et al., 2015). However, as this study was conducted at a high-performing school, its findings may not be fully generalizable to broader student populations with different socioeconomic backgrounds and prior exposure to mathematics. Research further indicates that competition environments and assessment formats significantly impact gender performance, with online assessments often reducing anxiety-related performance barriers seen in traditional, time-limited settings (Applebaum et al., 2021).

A key takeaway from this study is the strong correlation between task enjoyment and successful problem-solving across genders. This finding reinforces prior research emphasizing the role of motivation, interest, and self-efficacy in mathematics achievement (Dweck, 2006). According to self-determination theory (Deci & Ryan, 2000), when students feel competent and autonomous, they are more likely to engage with mathematical challenges. Encouraging positive engagement, confidence, and interest in mathematics competitions could be an effective strategy for improving performance across genders. Further research on international competitions such as the Kangaroo Contest has demonstrated that female students excel in environments that emphasize structured, conceptual engagement over speed-based assessments, reinforcing the importance of designing gender-inclusive competition frameworks (Applebaum et al., 2020).

The results from Table 2 further highlight the role of task difficulty perception in performance. Task N6 (Spatial Ability) was identified as the most challenging for most students, with 33 participants selecting it. However, only 11 of those students successfully solved it, indicating a disconnect between perceived and actual difficulty. This aligns with previous studies suggesting that students often overestimate the difficulty of spatial reasoning tasks, potentially discouraging their engagement in problem-solving activities (Applebaum et al., 2021). In contrast, Task N1 (Geometry and Proportion) was the most favored among students, with a 95% success rate among those who selected it as their favorite, reinforcing the idea that enjoyment and familiarity contribute significantly to mathematical performance.

The findings of this study offer practical recommendations for education policymakers and competition organizers. Classroom strategies that emphasize conceptual mastery over competition can help build students' confidence in mathematics (Good et al., 2003). Additionally, reimagining competition formats to incorporate team-based challenges, extended problem-solving durations, and diverse question types can foster greater gender equity in participation and performance (Slavin, 1995). Increasing the visibility of female role models in mathematics and expanding mentorship programs may help counteract stereotypes and encourage greater female participation in competitive mathematics (Lockwood & Kunda, 1997). Research suggests that mentorship programs and culturally responsive curriculum adjustments can significantly increase female engagement in mathematics, particularly when students are provided with role models who demonstrate success in competitive mathematical problem-solving (Applebaum et al., 2021).

This study provides compelling evidence that gender disparities in mathematical competitions may be diminishing when competition environments are structured to be inclusive and supportive. The findings suggest that performance differences are not inherently due to ability but rather are shaped by confidence, preparation, and competition design. By fostering environments that promote engagement and minimize anxiety, educators can help ensure greater gender parity in mathematics competitions and encourage students—regardless of gender—to pursue STEM-related fields. Evidence from mathematical competitions held in both traditional and digital formats indicates that structural adjustments, stereotype threat reduction, and strategic engagement approaches can enhance gender parity in mathematics performance (Freiman & Applebaum, 2011; Applebaum et al., 2013).

Future research should focus on refining competition structures to emphasize collaboration, strategic thinking, and long-term engagement, rather than high-pressure, speed-based assessments. Additionally, continued investment in mentorship programs, gender-inclusive curricula, and stereotype-reduction strategies will ensure sustained female participation in mathematics competitions. Addressing external factors such as stereotype threat, motivation, and competition pressure will be key in ensuring that all students can excel in mathematical problem-solving and STEM-related fields. By continuously improving educational strategies and competition frameworks, we can advance toward a more equitable and inclusive mathematical learning environment for future generations.

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Liga Matemática in Spain

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Nicolás Atanes Santos was born in Burgos, Spain in 2004 although when only one year-old he moved with his parents to Pamplona. During his high school years, he participated in the Mathematical Olympiad while completing the International Baccalaureate Diploma. Since February 2nd 2020, he has been a popularizer of mathematics and in 2022 became a Mathematics student. Since 2023 he is the head of Mathematical Competitions of the National Association of Mathematics Students, overseeing the Liga Matemática.

Background

On February 2nd 2020, I started promoting mathematics because I noticed that sports like football, handball, and chess are disciplines everyone knows, as we hear about them daily, whereas this is not the case with mathematics ([3]). That same year, a regional newspaper defined my popularization of mathematics as a fight to take mathematics over the covers of sports newspapers ([6]).

Mathematical competitions serve as a platform for mathematics outreach, as sometimes newspapers manage to highlight young, talented mathematicians and humanize the mathematical community by featuring local young minds as new protagonists. This type of coverage often leads to a call for others to participate in these competitions or, at the very least, to take an interest in mathematics itself. The issue is that these competitions typically last for a very short period, often just one week a year, while mathematics is a discipline that exists every day. As a result, the impact of this coverage does not always reach the audience at the right time. Mathematics should be more popular and talked about in newspapers, as Erica Klarreich said in April 2023 in [7].

Sport is a competition that makes use of this. More than 1 in 6 people closely follow professional or college sports ([4]). For many, sports offer the excitement they seek. The unpredictability of the games, the thrill of victory, and even the sting of defeat create an emotional rollercoaster that fans find captivating. This excitement drives them to dedicate time and energy to following their teams, especially those with whom you have an emotional bond ([9]). Thanks to this, most people, whether or not they play or watch sport, are aware of sport through the media ([8]). In 2004, mathematician Jordan Ellenberg wrote an article in Slate considering math competitions like the International Mathematical Olympiad and MATHCOUNTS a sport ([2]). The Mathematics community itself may be considered a sport too ([5]).

The format of this mathematics competition that tries to adjust to the sport format and its success was presented during the World Federation of National Mathematics Competitions mini-conference, held in Sydney on 6 July 2024.

Format

Along the same lines, in 2023 we thought about how to organize a mathematics competition with the following objectives: to promote mathematics, maintain its continuity, and unite mathematics students. This competition was conceptualized by me and introduced to the National Association of Mathematics Students in Spain, as it was intended to be a student competition for university students. It was presented during the 2023 National Meeting of Mathematics Students in Badajoz, Spain, with a format designed around these objectives.

First, the competition would feature matches from September to June, excluding rest weeks and final exam periods. The calendar included dates for each round, with each round lasting one week and featuring matches between pairs of teams. Then, we worked on the match format, which was inspired by sports competitions. A classification based on deliberations after each match would not have been exciting, so we allowed between 3 and 6 people per team to play, solving 3 competitive mathematics problems. These problems were designed to be challenging—not immediately solvable—but clever enough to require the broad knowledge of “intuitive” mathematics to be solved. Additionally, we introduced the possibility of providing answers during the match, giving rise to what we call the "Eureka effect"—the excitement of providing a correct answer on the way to winning the match. The first team to solve all 3 problems wins the match, avoiding a 3-3 tie. For example, if a match is at 2-1 and the first team solves the remaining problem, the provisional final score will be 3-1, regardless of whether the second team solves any of the remaining problems in the remaining time.

We also wanted to ensure that the teams—groups of students from the same university—were recognized and encouraged to compete positively. Typically, players from the same team connect with the referee and the opposing team through video calls and virtual meetings, though teams in close proximity often arrange to play the match in person. The referee acts as the corrector, and their role begins when the match starts. During the match, players decide on their strategy, but when a team solves a problem, they must announce it immediately, similar to Archimedes’ famous exclamation. The Eureka effect promoted by the is unique to this competition, creating excitement for both the participants and the spectators.

This format challenges participants to apply their mathematical knowledge and logical thinking under timed conditions, mirroring the intensity and excitement of athletic competitions. Once a solution is announced, the referee starts the correction process, determining the accuracy of the solution instantly and definitively. There are no partial scores, either the answer is correct or incorrect. Both teams must be informed of the answer, without any reason or explanation given during the match. Sometimes, it may take more or less time to correct a solution, or a post-match correction may be necessary, needing no more than approximately 24 hours for a final deliberation. However, both teams are typically informed throughout the match, almost immediately. So how are the responses evaluated? When a referee examines a solution, they must pay attention to different types of mathematical problems:

Proof-type problems

For this type of problem, the referee must ensure that the solution presented by the players leads to a valid conclusion, without errors that could alter the result. An error in a calculation that still leads to a correct conclusion does not invalidate the result by itself, but a non-trivial assumption may be the reason for considering the solution incorrect.

Number problems

For this type, the solution is a number. Therefore, the referee must first verify that the number provided as an answer is correct. This includes numbers that depend on one or more parameters. While correct reasoning is necessary for the answer to be valid, a numerical result that differs from the correct one invalidates the answer, even if the reasoning is flawless. Players are allowed to submit multiple answers before time runs out, and the answer will be validated once both the correct answer and valid reasoning are provided. As this usually involves calculations, there are no significant differences in how it is solved — perhaps in calculation strategies or insightful ideas that shorten the process — but it only matters that the steps are properly justified, and there is a clear connection between the arguments to make them valid.

In general, decisions are made by the referee based on what the team submits, and each solution is reviewed independently. The goal is for players to discuss possible strategies, plan attacks, and even defenses, solving easier problems first or dedicating more time and players to more challenging ones. Once a problem has been addressed and submitted as the team's solution, both teams receive a match report where they can review the solutions submitted by each team and contest the referee's decisions. Some examples of mathematical problems used are:

> The Fibonacci sequence starts with two 1's and then each number is the sum of the previous two. Find the remainder that would be obtained if the 2024th term of the sequence is divided by 5.

GalUALs — DerUVAda, Final Match

Solution. Let's write the first 20 residues of the Fibonacci sequence:

$$1,1,2,3,0,3,3,1,4,0,4,4,3,2,0,2,2,4,1,0\dots$$

Calculating the 2024 Fibonacci terms would end up being difficult. But notice that the residues, like the Fibonacci sequence, are obtained by adding the last two residues modulo 5. Therefore, looking at the last two terms written above, you can see that this sequence will begin again. So starting from term 20, the same residues are repeated consecutively every 20 terms. Let's look for a multiple of 20 that is close to 2024:

$$\left\lfloor \frac{2024}{20} \right\rfloor = 101,\ 101 \times 20 = 2020.$$

So the remainder of the 2024th term is the same remainder as that of the 4th term, that is, the remainder when dividing by 5 of the 2024th term of the sequence gives a remainder of 3.

Other problems from the first edition include:

> Find all the right triangles with an integer leg and hypotenuse, knowing that the other leg is equal to $\sqrt{2024}$.

Poblema FC — Equipo Nebrija, Week 14

5 positive integers (not necessarily different) are written on the board and all possible sums of pairs of these numbers are calculated. The only results obtained are 31, 38 and 45 (some of them, several times). What are the 5 numbers?

Gatois — Los transfinitos de Ali-cantor, Week 14

We write the numbers from one to twenty consecutively forming the 31-digit number

$$N = 1234567891011121314151617181920.$$

Can we rearrange the digits of N to obtain a perfect square?

Las Bolas Compactas — Epsiloneta, Week 2

Prove that there are sequences of n consecutive natural numbers such that the n numbers are composite numbers.

Una Hora Menos — Dragones de Cantoblanco, Week 2

In the infinite sequence of digits $(1,2,3,4,5,\dots)$, what is 2023rd the digit?

Matemágicos — GaUB,Week 7

What is the maximum number of 1012-element subsets of $1,2,\dots,2024$ that you can choose so that that the intersection of any three subsets has at most one element?

Funtor de Breogán — GaUB, Week 5

Find the number of distinct 8×8 chess boards with two squares removed such that 31 dominoes of size 2×1 cannot be placed.

DerUVAda — Gatois, Week 9

Prove that the product of four consecutive positive integers plus one is a perfect square.

URracas del Ebro — LUGRange, Week 9

Impact

Almost 500 students participated in the Liga Matemática 2023-2024, the very first edition, and another 500 are participating in the Liga Matemática 2024-2025. Since 2023, all students who represented Spain in the International Mathematical Olympiad have become Liga Matemática players. Additionally, nearly every Spanish student competing in the International Mathematics Competition for University Students also participates in Liga. The competition has received support from the mathematical community. Several former mathletes from the International Mathematical Olympiad, such as Álvaro Gamboa, Martín Padrón, Álvaro Acítores, Leonardo Costa, Jorge Casanova, among others, are also players ([1]).

The competition operates on an academic year basis, running from September to June. The diverse representation of universities highlights its role in promoting collaboration and camaraderie among mathematics students from different academic institutions. Teams compete not only for victory but also for the pride of representing their university on a national stage. The 31 teams that made up the Liga Matemática 2023-2024, together with their delegates, are:

1. Delta Chancla (University of Oviedo, Álvaro Lerones),
2. El márgen de Papel (University of Málaga, Pepe Molina),
3. Poblema FC (University of Cantabria, Pablo Asiain Monreal),
4. Dragones de Cantoblanco (Autonomous University of Madrid, Nicolás Rey),
5. EHUler (University of the Basque Country, Nerea Álvarez),
6. Equipo Nebrija (University of Nebrija, David Martín de Diego),
7. Badahoes (University of Extremadura, Paula Sáenz de Tejada),
8. Hamiltogatos (University of Seville, Alejandro Costa),
9. GalUALs (University of Almería, Javier Canton),
10. Ni sí ninot (University of Valencia, Marina Benito),
11. GaUS (University of Seville, Luis Gonzalo),
12. LUGRange (University of Granada, Javier Carvajal Noguera),
13. URracas del Ebro (University of La Rioja, Víctor Alcalde),
14. Funtor de Breogán (University of Santiago de Compostela, Javier Polo),
15. SUMUtorios (University of Murcia, Juan Agustín Lorca García),
16. Eulerianos (Polytechnic University of Valencia, Adrián Toledo),
17. MatUCAs (University of Cádiz, José María García Pérez),
18. Epsiloneta (University of the Balearic Islands, Frank Hammond),
19. ¿Nombre?, ¿Qué nombre? (Polytechnic University of Madrid, Noé Rico),
20. Matemágicos (Public University of Navarra, María Virto),
21. UPCerdős (Polytechnic University of Catalonia, Leonardo Costa),
22. Los transfinitos de Ali-Cantor (University of Alicante, Javier Aldeguer),
23. DerUVAda (University of Valladolid, Javier Gómez de Tejada),
24. Una Hora Menos (University of La Laguna, Pablo Becerra),
25. Las Bolas Compactas (National University of Distance Education, Daniel García Iglesias),
26. Parábolas mis dos hipérbolas (University of Salamanca, Andreu Simó Vidal),
27. Proposición indecente (University of Zaragoza, Marcos Escartín Ferrer),
28. GaUB (University of Barcelona, Robin Ath),
29. Complutense Universidad de Madrid (Complutense University of Madrid, Alberto García),
30. Los hijos de Gauss (Complutense University of Madrid, Jorge del Saz),
31. Gatois (Autonomous University of Barcelona, Oriol Bosquet

Since the second edition, delegates can be players, although they must hold the position for an entire edition. The team's names are intended to be definitive —and most have done so— except for the team from the Autonomous University of Barcelona, which in the second edition changed its name to GalUAB, and some teams, such as that of the University of Cádiz, University of Salamanca and University Nebrija, among others, did not continue in the second edition, while others signed up, and the teams were forced to be limited to being from a single University, and each University to have a single team.

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