VOLUME 23 NUMBER 1 2010

MATHEMATICS COMPETITIONS

JOURNAL OF THE

WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

MATHEMATICS COMPETITIONS
JOURNAL OF THE WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

(ISSN 1031 – 7503)
is published biannually by

AMT PUBLISHING
AUSTRALIAN MATHEMATICS TRUST
UNIVERSITY OF CANBERRA ACT 2601
AUSTRALIA

With significant support from the UK Mathematics Trust.

Articles (in English) are welcome.
Please send articles to:

The Editor
Mathematics Competitions
World Federation of National Mathematics Competitions
University of Canberra ACT 2601
Australia
Fax:+61-2-6201-5052

or

Dr Jaroslav Švrček
Dept. of Algebra and Geometry
Faculty of Science
Palacký University
Tr. 17. Listopadu 12
Olomouc
772 02
Czech Republic
Email: svrcek@inf.upol.cz

Mathematics Competitions Vol 23 No 1 2010

TABLE OF CONTENTS

Contents | Page

World Federation of National Mathematics Competitions

Executive

Secretary:	Professor Kiril Bankov Sofia University St. Kliment Ohridski Sofia BULGARIA
Immediate Past President & Chairman, Awards Committee:	Professor Petar S. Kenderov Institute of Mathematics Acad. G. Bonchev Str. bl. 8 1113 Sofia BULGARIA
Treasurer:	Professor Peter Taylor Australian Mathematics Trust University of Canberra ACT 2601 AUSTRALIA

Regional Representatives

Africa:	Professor John Webb Department of Mathematics University of Cape Town Rondebosch 7700 SOUTH AFRICA
Asia:	Mr Pak-Hong Cheung Munsang College (Hong Kong Island) 26 Tai On Street Sai Wan Ho Hong Kong CHINA
Europe:	Professor Nikolay Konstantinov PO Box 68 Moscow 121108 RUSSIA
	Professor Francisco Bellot-Rosado Royal Spanish Mathematical Society Dos De Mayo 16-8#DCHA E-47004 Valladolid SPAIN

North America: Professor Harold Reiter
Department of Mathematics
University of North Carolina at Charlotte
9201 University City Blvd.
Charlotte, NC 28223-0001
USA

Oceania: Professor Derek Holton
Department of Mathematics and Statistics
University of Otago
PO Box 56
Dunedin
NEW ZEALAND

South America: Professor Patricia Fauring
Department of Mathematics
Buenos Aires University
Buenos Aires
ARGENTINA

The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the President

The Journal

In this latest number of the Journal, we are proud to present a series of articles that reflect a rich panorama of events and problems, as well as the creativity shown by the mathematicians and math teachers behind them in setting new challenges for students on many different levels and in many different cultures.

Congress in Riga

The WFNMC Congress in Riga in July will bring many of us together to learn from one another, examine our own work in new light, capture new ideas and invigorate our sources of mathematical energy.

As we prepare for the WFNMC Congress in Riga and look to chart our course of action for the coming years, there are two lines that we see as fundamental to the work of our Federation.

1. Our Federation must continue to be instrumental in bringing the Olympiad experience to students in all corners of the globe, sharing expertise with those setting up new national, transnational and regional events.

One of the ways this can be done is to seek alliances and to approach new groups, such as those responsible for organizing on the international level events for elementary and middle school students such as the World Youth Mathematics Intercity Competition, as well as the organizers of the IMC and similar university competitions, to thus extend the reach of mathematical competitions so that the work of the Federation enriches the mathematical experience of the youngest students and undergraduates alike throughout the world. Our journal reflects this *math competition eclecticism* in a way that our congresses and membership must strive to match.

Not only are we committed to fostering events designed to allow each student who so wishes to pursue his or her optimal level of development in doing mathematics, but moreover, as we have seen in the past and will see again in the days to come, the problems posed in math competitions, especially in the most challenging, often link the work of the Federation and its members to research in mathematics, as well as constituting research in elementary mathematics as former president Petar Kenderov has shown so clearly.

Last year at the 50th IMO in Bremen, former IMO medalists, now widely respected mathematicians, gave lectures on their work that greatly inspired the participants. Our journal is an excellent vehicle for articles written along the same lines and our congresses can be enriched by similar talks telling specific stories of how Olympiad experiences have flowered into beautiful mathematics.

> 2. We envision the possibility of closer contacts and stronger interaction with other international study groups.

We have seen how challenging mathematics can create a context in which students can develop their ability to think mathematically far beyond what is normally expected in the curriculum. This can lead to

- Making important contributions to research relating to the nature of mathematical thinking. Among those who have devoted their research to characterizing mathematical thinking and those who have studied the links between psychology and mathematics education, room for collaborative research can be explored.

- Research relating to how the experience of school mathematics can be brought closer to developing the mathematical thinking of students in ways that will be personally satisfying and fun, and enable the student to leave all options open when making life choices. Collaboration with groups whose research concerns the curriculum could be sought.

Having experienced the varied ways in which different students attack the same problem and the ways in which they bring previous ideas to bear, gives special insight into the nature of mathematical thinking to those who have experience in creating competitions and competition problems and allows them to fully identify the range of flaws introduced by parceling school mathematics into predetermined lines in order facilitate the application of routine algorithms.

- Seminal work in research concerning the generation of new lines of development in teacher education thus creating space for collaborative research with groups devoted to study in this area.

Mathematics competitions and other challenging experiences in mathematics have been shown to be the key to bringing many new young people into the field of mathematics. Moreover, teachers are strongly linked to students' successful participation in mathematics competitions. Tools must be developed to allow many more teachers—most teachers—to be able to open the door to doing school mathematics that, in addition to giving them the basic mathematical structure they need, fascinates and forms students and frees their creativity.

- Contributions that develop ways to nurture students' mathematical creativity and links with groups whose research is concentrated on creativity in mathematics learning.

When the call went out for papers for ICMI Study 16 *Challenging Mathematics in and beyond the Classroom*, many of the potential contributions came from groups studying creativity. It is clear that we must solve the problem of constructing learning situations which allow all students to show the same creativity in the mathematics classroom that they naturally exhibit in their other activities.

- Generating new perspectives with groups who study the relationships between the history and pedagogy of mathematics.

Those who have dedicated their work to creating for competitions new problems in elementary mathematics have an important edge that can be summarized as having a holistic approach to mathematics far richer than permitting, as is common in the curriculum, the compartamentalization of mathematics into disjoint topics, much in the way mathematics can be experienced when bringing to bear the organic and interrelated way it can be seen to have developed historically. This identifies areas of common interest to be studied.

Doing it right

Recently, through the wonderful mailing list maintained by Jerry Becker, came this quote from John Wooden. "If you don't have time to do it right, when will you have time to do it over?"

We believe that providing opportunities for mathematical challenge for all students is doing it right. There remains the burden of proof and we have outlined an agenda.

María Falk de Losada
President of WFNMC
Bogotá, June 2010

From the Editor

Welcome to *Mathematics Competitions* Vol. 23, No. 1.

Again I would like to thank the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:
The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

The Editor, *Mathematics Competitions*
Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA

or to

Dr Jaroslav Švrček
Dept. of Algebra and Geometry
Palacky University of Olomouc
17. listopadu 1192/12
771 46 OLOMOUC
CZECH REPUBLIC

svrcek@inf.upol.cz

Jaroslav Švrček,
June 2010

The members of the Department of Algebra and Geometry with great sorrow announce that their unforgettable colleague, teacher and friend

Prof. Svetoslav Bilchev

passed away.

He was born on 09. 30. 1946 in the town of Ruse. In 1964 he graduated the Second Mathematics High School. Then he took place in the V-th and VI-th international school olympiads in mathematics and won 3 third prizes. In 1969 he graduated the Moscow State University "M. Lomonosov", took his doctoral degree in 1975 and was elected as Associate Professor in 1979. More than 40 years he devoted to the University of Ruse. He specialized abroad many times in Russia, Greece, Norway, England, Italy, Israel. He was a coordinator and took part in many international scientific and educational projects with either the University of Ruse as a contractor, or the UBM or international mathematical organizations. He took part in the preparing of the Bulgarian team for IMO in 1981, the Greek team in 1987 and the English team in 1995. He took part in 5 international congresses on mathematical education, in 4 congresses of the World Federation for mathematical competitions, in the every year conferences of the UBM since 1971, in many international forums in Cyprus, Italy, Denmark

His scientific interest were enormous. They include differential geometry, ordinary and partial differential equations, differential games, mathematical modeling in economics, geometric inequalities, transformations, mathematics education, education of talented pupils and students, mathematical competitions and Olympiads. Many pupils from Ruse and the country will remember Prof. Bilchev as their teacher. He has more than

140 scientific works published in many countries of Europe, America, Asia and Australia.

He won many national and international awards as golden sign of the Society “Georgi Kirkov”, medals “Kiril and Metodii” third and second degree, awards from the Ministry of Education, honorary plates “Ruse” in 1991, 200 and 2003, awards from many international mathematical institutions.

He is associate invited professor of the North University of Baia Mare, Romania.

He is a member of the Union of the Scientists—Ruse, Union of the Bulgarian Mathematicians (UBM), of the AMS, the World Federation for mathematical competitions and many others.

He is a member of the Editorial board of many mathematical journals.

He has taken an active part in creating the Centre of Mathematics, being its head; the Pedagogical Faculty, being its dean in the period 1994-2003. Many times he was elected as a member of the Academic Council of the University of Ruse. He initiated and organized many times Evenings devoted to poetry and Evenings devoted to humour and jokes.

Several mandates and at the moment he is the head of the Department of Algebra and Geometry. He gave much for the professional realization of the colleagues from the Department.

On the 11-th of March 2010 Prof. Bilchev chaired for the last time a session at our Department.

We give our last honours, respect and admiration for everything done by our colleague and friend Professor Svetoslav Bilchev.

Emily Velikova
March, 25th, 2010
Department of Algebra and Geometry
University Russe
BULGARIA

Several Remarks on Possible Components of Advanced Mathematical Thinking

Romualdas Kašuba

Romualdas Kašuba teaches mathematics, communication skills and ethics at Vilnius University, Lithuania. He obtained his Ph.D. at the University of Greifswald in Germany. From 1979 he is a jury member of the Lithuanian Mathematical Olympiad. Since 1996 he is the Deputy Leader of the Lithuanian IMO team, since 1995 also the leader of Lithuanian team at the "Baltic Way" team-contest. In 1999 he initiated the Lithuanian Olympiad for youngsters. He is the author of the book What to do when you do not know what to do?, *Part I and II, Riga 2006-2007 and also of issues* Once upon a time I saw the puzzle, *Part I, II and III, Riga 2007-2008 (all in English). He also represents Lithuania at ICMI.*

It is very difficult and in the same time so very exciting to speak about advanced mathematical thinking. It is a bit similar as if you are going to speak about high society having a feeling of belonging more to the usual society. Nevertheless it is very promising and challenging. It is very similar to the reflections about what are the differences between a mathematical problem and an advanced mathematical problem. The differences sometimes seem to be not so essential. For example, one can say that advanced mathematical thinking sometimes appears to be only a nice combination of relatively simple mathematical elements each of them being not so important or nice or otherwise remarkable. Three concrete samples of advanced attractive but accessible and involving problems first of all for secondary and tertiary level are presented.

1 Eternal span between simple and not so simple

Let us imagine that we have to deal with the following problem, which is appearently not an average problem—but let us first formulate the situation and also what is expected to achieve. We will probably mention later where this problem is taken from and add other important details together with some considerations and even some kind of methaphysical remarks, which are difficult not to make.

> *Find an integer whose binary representation contains 2005 0's, 2005 1's and which is divisible by 2005.*

If we suspect that this is an Olympiad problem then it appears to be more than a right suspicion—yes, this is indeed an Olympiad problem. Now it becomes clear that such a problem, which might be called "all-in-that-world-is-around-2005", is a problem, that was proposed in A.D. 2005. Again, that is indeed the case. Now reading the text of problem it would be not so difficult and even relatively easy to prophetize in what country it might have been proposed. It is obvious that it is a problem from a serious country with a remarkable solving tradition. Keeping the intrigue we will not yet reveal the name of the country where that problem was taken from.

We would like firstly to discuss some interesting circumstances, which are of highest importance. First of all, "I ask you very confidently ain't she nice". And if it is then the banal question "why" would follow. Using some words as slogans we may even boldly ask: is there anything interesting in that problem? Does that problem contain something that might be interesting for an average bright boy? Useful for him? What a use could he get after solving it? In what way could it help him in life? In solving of other problems?

So we have to deal with a problem: show us a number with 2005 zeroes and with 2005 ones and even divisible by 2005. In one sentence just three times the very same number 2005 is mentioned. What is so nice in number 2005? In 2005 it was indeed a remarkable number appearing many times, say, in each daily newspaper or on each bill. It would be less than easy for us to repeat what a problem was proposed - in other words we remember that problem easily and the formulation is short as possible. One thing that makes that problem appear to be attractive—

you come, you see and you remember what is needed. The sort of "veni, vidi, vici". With veni and vidi we are already done but with the vici it might be more complicated than it seems from the first glimpse.

It is also clear that among the circumstances making that problem to belong to the advanced level is the number 2005 itself. Nowadays we would have taken another numbers and so we'll do—the use of actual year numbers is also psychologically understandable—such a number is a not a number we are afraid of because we meet such number many times a day and not necessary even as a data. Otherwise such number might be reduced if we start solving the problem in order to understand what kind of difficulties might appear dealing with such a problem. And the first step could be reducing numbers. Instead of 2005, which is for us the number from the past, which we still remember, but which will never come back, we will take another year number 2009 and only now we will start reducing it. Instead of 2009 the number 209 might be taken, or even 29. Also 9 and even 2 is possible. The reader might now really start smiling or even laughing and he would be right. Because in both those simplest cases the intrigue of problem remains. And regarding simplest cases we are gathering the information we get insight and this is the first feature of advanced mathematical thinking—to get full information in order to be able to produce the skeleton of the future solution. It could be told that in standard mathematical thinking main things are of the highest importance and in advanced mathematical thinking all details are essential. Or expressing the same with other words you can only repeat: "you never (not always) know".

So after simplifying we get two versions—one which is the simplest one and yet even another, which is the over simplest one. From the latter we will start.

> *Find a number whose binary expression contains 2 zeroes and 2 ones and which is also divisible by 2.*

The second will be already more complicated but still very promising as every involving problem ought to be.

> *Find a number whose binary expression contains 9 zeroes and 9 ones and which also divisible by 9.*

Returning to that over simplest problem and doing it we might feel a slight pity because there remains not much or almost nothing from the initial problem, because 2 zeros and 2 ones mean altogether 4 binary digits and this is already comparable with number of fingers—there are only 16 possible candidates 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110 and 1111. First 8 could and will be eliminated as numbers starting with zeroes and so only 8 numbers remain, i. e. 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111. From these 8 exactly three contain 2 zeros and 2 ones:

$$1001, 1010, 1100.$$

The first of them doesn't suit because it is our usual 9, so it is apparently odd and not divisible by 2, but the remaining pair 1010 and 1100 decimal mean 10 and 12 and so both are precious for us, for both are even, so both serve as answers.

Now with 9 zeros and 9 ones divisible by 9 we will not write down all possible cases. Still, our practice, we already gained, will appear of use and importance. So paraphrasing the Hunting of the Snark you can repeat with the Bellman "Do all that you know, and try all that you don't: Not a chance must be wasted to-day!"

So we did some training and now we may try to make some advanced step in our process. Firstly, it would be natural to know the binary appearance of 9—we know it, it was mentioned before and that appearance is 1001. What can we achieve having this? We clearly see that in that binary expression there are two 0's as well as two 1's—this is not bad. Now we could try to make bigger numbers from that 1001 because we need more 0's and more 1's but not losing the divisibility by 9. We might use the modern idea of cloning—in arithmetic we might not feel the slightest fear against it—in mathematics it couldn't cause any harm—in our case cloning preserves divisibility—that is writing clones of 1001 or numbers $10\,011\,001, 100\,110\,011\,001, \ldots$ we can quickly achieve the numbers with 8 or 10 zeroes and ones and even easily write them down. But we need the number with 9 zeroes and 9 ones and that number remains in between these two numbers. So we can modify our behavior: try to get the number divisible by 9 with, say, three 1's and three 0's and then put it together with 1001 also not losing the divisibility by 9. We again start from 9 or binary 1001 adding $9 + 9$, then

$9+9+9$ and acting binary and carefully waiting for the equal odd number of zeroes and ones in these sums. $9+9$ binary means $1001+1001$ or 10010 (3 zeroes, 2 ones—bad!), then $10\,010+1001$ is $11\,011$ (again not the best). We go on counting $11\,011+1001 = 100\,100$ (again two 0's and 1's as it was when started). We still go on getting that $100\,100+1001$ is $101\,101$ (four 1's and two 0's). Going on adding 1001 we will get the following numbers: $110\,110$ (or 54), then even $111\,111$ (no wonder, that is 63), then $1\,001\,000$ (72), followed by $1\,010\,001$ (81) and consisting of four 0's and three 1's and then $1\,011\,010$ (90) consisting, vice versa, of three 0's and four 1's. Now putting these numbers together we will get the "composite" number, consisting of the same odd (but not of 3 as we suspected but just of 7) number of zeroes and ones or $10\,100\,011\,011\,010$ (this is usual $10\,458$) and putting this together with 1001 we get the answer $101\,000\,110\,110\,101\,001$, which is usual $334\,665$.

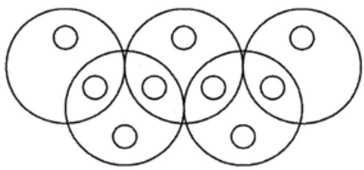
Now it is technologically clear what to do with 2005—first write it binary and then combine adding with cloning and putting together.

So mechanically speaking the advanced mathematical thinking very often is a combination of the simplest elements but not any combination—it might be even formulated that advanced mathematical thinking is an advanced combination of the the simplest elements of mathematical thinking or facts. It is strikingly similar as in the poetry—advanced poetry is an advanced combination of the very same words which we all use many times and every day. Everyone may try. It is worth trying! We can only repeat with the Bellman and with his Father Lewis Carroll:

"Do all that you know, and try all that you don't. Not a chance must be wasted to-day!" Further on we would like to present for the reader some product, which only several days ago was proposed for some mathematical contest and which is supposed to be a creative sample for the secondary level. As it was probably mentioned, problems were not originally invented by the author but taken from various sources. Then they were adopted or, otherwise speaking, in some sense reformulated. Some structuring of problem questions was also proposed together with introducing acting heroes. The arguments for that and even against that (if they appear) would be also understandable. But this is also worth trying and it will never cease and that process of popularizing will never come to its end.

2 Some problems from probably involving samples for youngsters

The author would like to discuss some problems from the sample which he in fall 2007 selected and adopted for some Lithuanian individual contest paying special attention to the formulations of the problems. First problems are more for grades 5 and even 6 (usual ages of solvers 12–13 years), and the rest are more for grades 7 and even 8 (usual ages of solvers 14–15 years). In the age above 10 some advanced problems are possible to realize because young minds are concretely inventive and to the highest degree. There are many mathematicians who believe that human mind is the most inventive at 18.

> *1. Pooh Bear is preparing for the Olympic Games. His coach Piggy (don't mix him with Piglet) proposed him such a problem with clearly Olympic flavor:*
>
> 
>
> *Using each of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 once Pooh Bear is to fill in the nine small circles in the Olympic symbol above so that the sum of the numbers inside each circle is 14.*

That problem by its nature (cf. the solution in [1] , p. 22–23) is indeed not difficult and is of concrete type "try to place the numbers" but in the same time it is clearly advanced in its perfect realization and its clearly related to the last Olympic games. This is believed to make the problem be more attractive for all ages. It is also worth mentioning that it has practically the only solution 861743295—the reversed solution, or 592347168, could be regarded to be the same as the original one.

> *2. Serious-minded Tom starts to write the series of numbers*
>
> $$18, 41, 64, 87, \ldots$$
>
> *in which each successive number exceeds the previous one by 23 units. Jerry, even more serious at that time, claims that if*

Tom never ceases with his writing then sooner or later he will get in his series a number consisting entirely of 9's as well.

(A) Does a number consisting entirely of 9's really appear in Tom's series?
(B) If that will be indeed the case then write down the number and indicate at which place it will stay in Tom's series;
(C) Might it be that there will happen more such numbers consisting entirely of 9's in that series?

Here it is enough to apply the powerful tool commonly used by the many previous generations and which is now completely eliminated by the almighty computer industry, that powerful tool being long division. Making that it will so soon come out that dividing the number 99 999 by 23 we will again obtain the dream rest 18.

3. Once Pooh Bear on his way home found the drawing of some rectangular forest parcel, which was divided into 9 smaller rectangles. Inside some of these rectangles their perimeters were indicated 9 (the perimeter of the rectangular is the length of a thread surrounding it). Inside one smaller rectangle Pooh Bear found the question mark. What is the perimeter of that rectangle?

10	11	12
5		
11		?

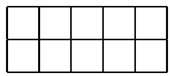
Here the possible advancement may be to wash away all what you do not need or eliminate what is unnecessary. This is a bit similar when you throwing away the unnecessary ballast from your aerostat in order to get him higher. So there is no need to keep the middle row and the middle column—you may wash them away.

10	12
11	?

Now you almost unavoidably are landing by the solution 13 in the form of association

$$10 + ? = 11 + 12.$$

> *4. Robinson Crusoe together with his assistant Man Friday are marking in turn either one or two squares of* 5×2 *board below. Robinson Crusoe starts first. The player who marks the last square wins. Prove that Robinson, who starts, can always win the game independently of what his assistant Friday undertakes.*



Some kind of advance may be measured also when you are applying old ideas using somehow new package. In that case we are playing the old game modulo 3.

> *5. Alice wishes using each of the digits* $2, 3, 4, 5, 6, 7, 8$ *and* 9 *once to fill in the boxes in the equation below to make it correct. Of the three fractions being added, what is the value of the largest one?*

$$\frac{1}{\Box \times \Box} + \frac{\Box}{\Box \times \Box} + \frac{\Box}{\Box \times \Box} = 1$$

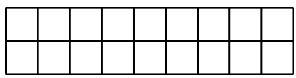
This is the brilliant example of what can be made by skilled hands (cf. the solution in [1] , p. 21). This is a rare example of the problem where the skilled mathematician has no advantage against the smart high-school boy—the author knows some convincable examples. In solving it is important to come to the conclusion that we in no way should allow 5 and 7 to the denominator—so both of them will be placed in the "higher" places. Then we have to understand that they will belong to the different fractions and afterwards we quickly land at the example

$$\frac{1}{3 \cdot 6} + \frac{5}{8 \cdot 9} + \frac{7}{2 \cdot 4} = 1$$

with the largest fraction being 7/8.

It is, as mentioned, exceptionally nice democratical problem in the sense that all who are interested in solving of that problem are practically equal in their chances. As usual in order to get more satisfaction it is worth first try to do it spending for that at least 5–10 minutes.

> *6. Robinson Crusoe and his assistant Man Friday are marking in turn either one or two neighbouring (sharing common side) unmarked squares in the 2 x 9 board below. The player who marks the last square wins. If Robinson starts, show that he can always win the game independently of what Friday is doing.*



Now the trick modulo 3 is of no help because marking 1 or 2 squares you can't reach the state when the number of remaining fields would be, say, 15. Now you can divide it in two symmetrical parts removing 2 middle fields and then preserving the symmetry you'll be successful.

3 From the sample of the last Lithuanian Team-contest 2007

Now the author would like to present the problems, which he proposed to the Lithuanian team-contest in mathematics. It was already the 23^{rd} edition of that popular competition in which the teams (usually some 20) of invited regions consisting of 5 persons during 4 hours have to manage with 20 problems. Immediately after the writing is over about 30 skilled coordinators start their job and so on the same evening winners are announced and awards of all 3 contests distributed. None of the problems are originally invented by the author but the process of conversion of problems to some satisfactory unit takes by the author of these lines really quite a long time and cost lot of work and reflection—usually several months. Two goals are there to pursue—and each of them is more important then the other one. Firstly, the prepared sample of problems must give some good preparation material for the International

Baltic Way team contest of countries around the Baltic See, which arouse also from that Lithuanian team contest. Another goal is to find the problems, all of which being advanced, challenging and accessible—each of them probably and naturally in different degree. But all these three components must be included and to be seen in any of that problem. The reader might see to what degree the composer of the sample succeeded in achieving his aims.

> *1. Is it possible to represent the fraction* $3/2011$ *as a sum of three fractions all numerators of which are* 1 *and all denominators are different and odd positive integers?*

This problem is taken from the celebrated Romanian sources with the reference to American Mathematical Society. This problem remained unsolved during the Lithuanian competition. This advanced problem again looks very concrete and even naive. This is very actual also for the (slightly) advanced solving because you may have the difficulties with identifying which problems are worth doing. This is a case of problem which is difficult to solve in the concrete case. It is difficult to realize to what series might that fraction belong. If you chance to identify that it is a special case of the fraction $3/(6n+1)$ your chances to find the suitable representation are much higher and you sooner or later you will land at the representation of the kind

$$\frac{3}{6n+1} = \frac{1}{2n+1} + \frac{1}{(2n+1)(4n+1)} + \frac{1}{(4n+1)(6n+1)}$$

which is hardly imaginable if you stay "with concrete numbers".

> *2. Prove that for any positive integer* N *there exist a multiple of* N *whose decimal digits add up to* N*.*

The problem, which is formulated in one sentence, is always attractive if it only can be solved. It is not necessary to think exceptionally about Fermat's theorem. There are some other statements—the twin's problem, for instance. The solution of especially that problem starts with words: "consider all remainders of the numbers 10^k leave upon division by n. Since there are only finitely many residues, there exists. . . " The whole thing lasts not very long.

3. The five numbers 1, 2, 3, 4, 5 *are written on a blackboard. A student may erase any two numbers a and b and replace them with the numbers ab and a + b. If this operation is performed repeatedly, can the numbers* 21, 27, 64, 180, 540 *ever appear on the board?*

Nice problem which deals with ability to reverse the whole happening. If it was possible then the moment when the 4^{th} number divisible by 3 appears on the blackbord is of special interest. At that moment the 5^{th} number which is not divisible by 3 must necessarily have rest 2 when dividing by 3 and so it will be for ever. But 64 has another rest—so it's impossible.

4. Call a rectangle splittable if it can be divided into two or more square parts such that the side of each square is of integral length and there is a unique square with smallest side length. Find the dimensions of the splittable rectangle with the least possible area.

Beautiful problem for engaged minds—unexpected in formulation and despite of that quite accessible in solution. First move is to prove that the smallest square cannot lie on the side of triangle for otherwise it would have the larger neighbors from both sides and this is appearently bad. Even more bad it would be for the smallest square to be in the corner. It means it lies somewhere really inside. Some addition considerations lead to the answer or splittable rectangle of size 5.

1	1	2	2	3	3	3
1	1	2	2	3	3	3
4	4	4	5	3	3	3
4	4	4	6	6	7	7
4	4	4	6	6	7	7

The same numbers indicate the same square and different numbers belong to the different squares. This problem is overtaken from the Estonian mathematical competition.

5. A regular (5×5)*-array of lights is defective, so that toggling the switch for one light causes each adjacent light in the same* *row and in the same column as well as the light itself to change state, from on to off, or from off to on. Initially all the lights are switched off. After a certain number of toggles, exactly one light is switched on. Find all possible positions of this light.*

This is exceptionally nice problem and it is advanced at least in several aspects. First aspect is that it is difficult to make yourself believe that in such a small 5×5 square some not so easy things may be performed. As in many nice combinatorial type problems you are not so sure how to perform. You would like to perform as if you were dealing immediately with the general case. But to deal immediately with the general case it is not easy. And then you may be forced to perform in such a way that you are looking for at least one case when you will make only one light switch on. This would happen if you continued the investigation in which combination you may get one light switched on. You will have that it is possible to get the central light switched on. Working rather carefully you will get the concrete combination allowing you to have only the central light switched on:

			p	p
		p		
	p	p		p
p				p
p		p	p	

By p we indicate the positions we are touching or pressing.

The question remains what are next possible places where we can have the lamp switched on. You may feel psychologically inconvenient to look for special combinations of pressings. Of course, here you will have the possibility to rotate. Going on with your efforts you may land on some special combination of pressings allowing you to have the following success:

	p		p	
p	p		p	p
	p			
		p	p	p
			p	

Rotating you have three other possibilities.

And now you are forced to look for some theory, which is really not easy—of course, after some practice the things are easier. So step after step you may land at some double filtering. First filter is the special position of labels using even-odd ideas:

1	1	0	1	1
0	0	0	0	0
1	1	0	1	1
0	0	0	0	0
1	1	0	1	1

You may say: for each on-off combination of lights in the array, you define the *position value* to be the sum of the labels of those positions at which the lamps are switched an. It may be checked that pressing any switch always gives another on-off combination of lights *with the same parity.*

If we make the rotation then we get another set of labels with the same property and, of course, with another position number.

1	0	1	0	1
1	0	1	0	1
0	0	0	0	0
1	0	1	0	1
1	0	1	0	1

And now combining these two filters you can state that the realisable positions are these which have 0 in both positions and these are exactly our five states:

0	0	0	0	0
0	X	0	X	0
0	0	X	0	0
0	X	0	X	0
0	0	0	0	0

The fields with X are those where it is possible to have the only switched on state.

> *6. Baron Munchausen acting as a skilled farmer has got four straight fences, with respective lengths* 1*,* 4*,* 7 *and* 8 *metres. What is the maximum area of the quadrilateral Baron Munchausen can enclose?*

Afterwards the author was told that this problem ([1], p. 16) was often employed by some prominent professor in some famous university in order to check quickly whether the given student is inventive. Quickly means in some minutes. The solution uses the fact that there are some two squares whose sum is as powerful as the sum of another two. But before that you must form these two pairs. For that you may need to do some small flip over.

4 Some concluding remarks

The author spend many years teaching students and also preparing them for the competitions of different level starting from the ordinary class and to IMC or IMO. The result of these reflections was also the book "What to do, when you do not know what to do", which appeared in two parts in 2006 and 2007. Selecting of problems is very interesting process which is nowadays essentially globalised. Differently from the political or economical globalisation, which is beloved not by all, the globalisation of problems is only to be welcomed. So a good problem appearing in any country or textbook becomes known all over the world and nobody can say anything against that. There are many even annual textbooks of creative problems, which could be mentioned every time when these matters are concerned. Such are, first of all, Romanian samples, which usually consist only of original problems with very seldom exceptions.

Still preparing samples for these 3 competitions as well as that publication the author was also very much influenced and remains also very much under impression by publication of Simon Chua, Andy Liu and Bin Xiong "World Youth Mathematics Intercity Competition", which he found in Mathematics Competition, a journal of the World Federation of National Mathematics Competitions. Such masterpieces for youngsters even nowadays are difficult to imagine as possible to create.

References

[1] Chua S., Liu A., Xiong B., *World Youth Mathematics Intercity Competition*, Mathematics Competitions, vol. 21, No 1 (2008), p. 8–31.

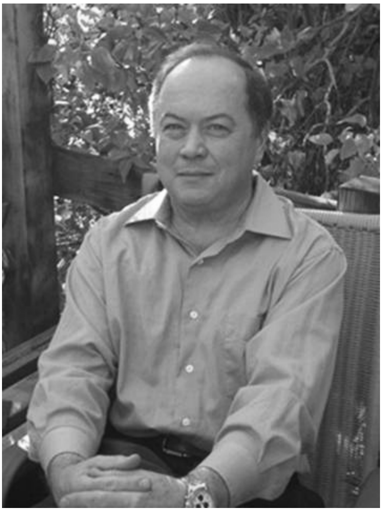
[2] Kašuba R. (2006). *What to do when you don't know what to do?* Mācību grāmata, Rīga.

[3] Kašuba R. (2007). *What to do when you don't know what to do?* Part II, Mācību grāmata, Rīga.

Romualdas Kašuba
Vilnius University
LITHUANIA
E-mail: romualdas.kasuba@maf.vu.lt

Building a Bridge III: from Problems of Mathematical Olympiads to Open Problems of Mathematics

Alexander Soifer



Born and educated in Moscow, Alexander Soifer has for 29 years been a Professor at the University of Colorado, teaching math, and art and film history. He has written six books and some 200 articles, including The Mathematical Coloring Book, *Springer, September 2008. In fact, Springer has contracted Soifer for seven books, five of which are coming out shortly and two are to be written in 2009–10. He founded and for 25 years ran the Colorado Mathematical Olympiad. Soifer has also served on the Soviet Union Math Olympiad (1970–1973) and USA Math Olympiad (1996–2005). He has been Secretary of WFNMC (1996–present), and a recipient of the Paul Erdös Award (2006). Soifer was the founding editor of the research quarterly* Geombinatorics *(1991–present), whose other editors include Ronald L. Graham, Branko Grünbaum, Heiko Harborth, Peter D. Johnson Jr., and Janos Pach. Paul Erdös was an editor and active author of this journal. Soifer's Erdös number is 1.*

1 Introduction

This is part III of the triptych "Building a Bridge". In part I, I showed an example of how a mathematician can keep an eye open for research ideas that could be used in Mathematical Olympiads. Part II provided an example of a walk in the opposite direction, when an Olympiad problem led to research and problems that are still open today, 20 years later.

The illustration problems in parts I and II had one thing in common: problems were created on one of the shores, and then transplanted across the bridge. I have encountered situations, however, when a problem was conceived on the bridge—and then transplanted to both shores: the shore of olympiads and the shore of mathematical research. I will illustrate such an affair here in the context of a problem I created in early 2004 while at Princeton University.

2 Coffee Hours at Princeton

Coffee pays an important role in mathematics, as Paul Erdős famously observed:

> Mathematician is a machine that converts coffee into theorems.

During the years 2002–2004 I was visiting Princeton University with its fabulous mathematics department, a great fixture of which was a daily 3 to 4 PM coffee hour in the commons room, attended by everyone, from students to the *Beautiful Mind* (John F. Nash Jr.). For one such coffee hour, in February 2004, I came thinking—for the hundredth time in my life—about the network of evenly spaced parallel lines cutting a triangle into small congruent triangles. This time I dealt with equilateral triangles, and the crux of the matter was a demonstration that n^2 unit triangles can cover a triangle of side n. I asked myself a question where the continuous clashes with the discrete: what if we were to enlarge the side length of the large triangle from n to $n+\varepsilon$, how many unit triangles will we need to cover it? This comprised a new open problem:

Cover-Up Problem 1. Find the minimum number of unit equilateral triangles required to cover an equilateral triangle of side $n + \varepsilon$.

During the next coffee hour, I posed the problem to a few Princeton colleagues. The problem immediately excited John H. Conway, the John von Neumann Professor of Mathematics. From the commons room he went to the airport, to fly to a conference. On board the airplane, John found a way (Figure 1) to do the job with just n^2+2 unit triangles! (Area considerations alone show the need for at least $n^2 + 1$ of them.) Conway shared his cover-up with me upon his return—at a coffee hour, of course. Now it was my turn to travel to a conference, and have quality time on an airplane. What I found (Figure 2) was a totally different cover-up with the same number, $n^2 + 2$ unit triangles!

Upon my return, at a coffee hour, I shared my cover-up with John Conway. We decided to publish our results together. John suggested setting a new world record in the number of words in a paper, and submitting it to the *American Mathematical Monthly*. On April 28, 2004, at 11:50 AM (computers record the exact time!), I submitted our paper that included just two words, "$n^2 + 2$ can" and our two drawings. I am compelled to reproduce our submission here in its entirety.

Can $n^2 + 1$ unit equilateral triangles cover an equilateral triangle of side $> n$, say $n + \varepsilon$?

John H. Conway & Alexander Soifer

Princeton University, Mathematics, Fine Hall, Princeton, NJ 08544, USA

conway@math.princeton.edu   asoifer@princeton.edu

$n^2 + 2$ can:

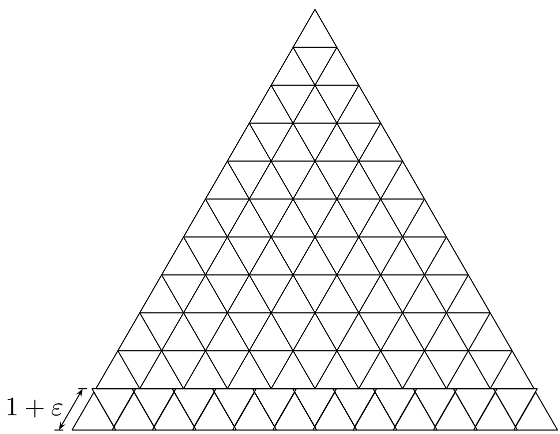


Figure 1

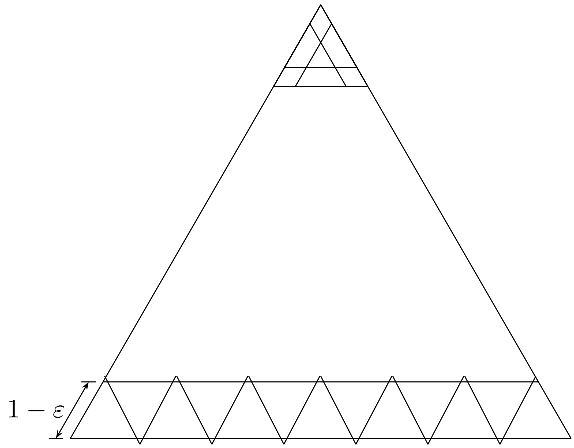


Figure 2

The American Mathematical Monthly was surprised, and did not know what to do about our new world record of a 2-word article. Two days later, on April 30, 2004, the Editorial Assistant Mrs. Margaret Combs acknowledged the receipt of the paper, and continued:

> The Monthly publishes exposition of mathematics at many levels, and it contains articles both long and short. Your article, however, is a bit too short to be a good *Monthly* article… A line or two of explanation would really help.

The same day at the coffee hour I asked John, "What do you think?" His answer was concise, "Do not give up too easily." Accordingly, I replied *The Monthly* the same day:

> I respectfully disagree that a short paper in general—and this paper in particular—merely due to its size must be "a bit too short to be a good *Monthly* article." Is there a connection between quantity and quality?… We have posed a fine (in our opinion) open problem and reported two distinct "behold-style" proofs of our advance on this problem. What else is there to explain?

The Monthly, apparently felt outgunned, for on May 4, 2004, the reply came from *The Monthly*'s top gun, Editor-in-Chief Bruce Palka:

> *The Monthly* publishes two types of papers: "articles," which are substantive expository papers ranging in length from about six to twenty-five pages, and "notes," which are shorter, frequently somewhat more technical pieces (typically in the one-to-five page range). I can send your paper to the notes editor if you wish, but I expect that he'll not be interested in it either because of its length and lack of any substantial accompanying text... The standard way in which we use such short papers these days is as "boxed filler" on pages that would otherwise contain a lot of the blank space that publishers abhor... If you'd allow us to use your paper in that way, I'd be happy to publish it.

John Conway and I accepted the "filler", and in the January 2005 issue our paper [12] was published. *The Monthly*, however, invented the title without any consultation with the authors, and added our title to the body of the article!

We also ran our little article in *Geombinatorics*, where we additionally observed that the "equilaterality" is essential, for otherwise $n^2 + 1$ triangles, similar to the large triangle T and with the ratio of sizes 1: $n+\varepsilon$ can cover T (Figure 3).

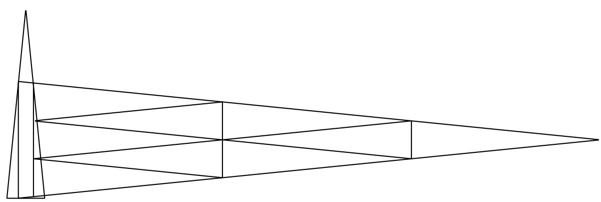


Figure 3

3 Cover-Up at Colorado Mathematical Olympiad

The XXI Colorado Mathematical Olympiad took place shortly after, on April 16, 2004. Needless to say, I wanted to include the Cover-Up problem in some form, preferably as a story.

To Have a Cake (Problem 4, Colorado Mathematical Olympiad)

a) We need to protect from the rain a cake that is in the shape of an equilateral triangle of side 2.1. All we have are identical tiles in the shape of an equilateral triangle of side 1. Find the smallest number of tiles needed.

b) Suppose the cake is in the shape of an equilateral triangle of side 3.1. Will 11 tiles be enough to protect it from the rain?

Solution. 4 a). Mark 6 points in the equilateral triangle of side 2.1: vertices and midpoints (Fig. 4). A tile can cover at most one such point, therefore we need at least 6 tiles. On the other hand, 6 tiles can do

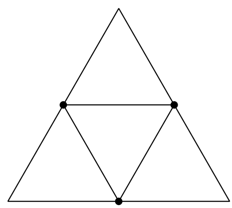


Figure 4

the job. Let me show two different ways, corresponding the general case coverings by Conway and I presented in the previous section. We can first cover the corners (Fig. 5 left), and then use 3 more triangles to cover the remaining hexagon (Fig. 5 right).

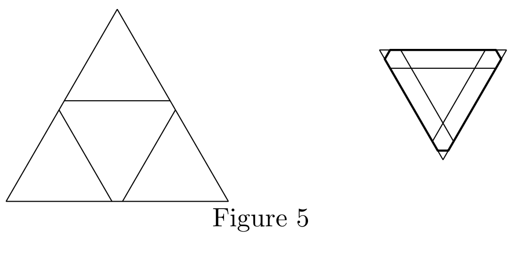


Figure 5

Alternatively, we can cover the top corner (Fig. 6), and then use 5 triangles to cover the remaining trapezoid.

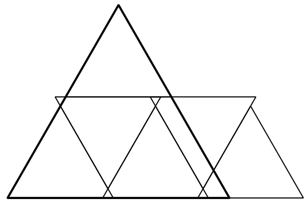


Figure 6

b). We can push the first covering method from a) by covering a cake of side up to 2.25 with 6 tiles (see Fig. 7, where $x = 0.25$).

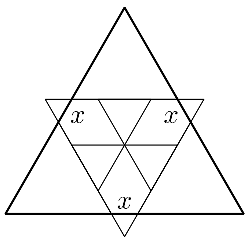


Figure 7

Let us use this to cover a cake of side, say, 2.2 and put this covering in the top corner, and then take care of the remaining trapezoid with 5 tiles (Fig. 8).

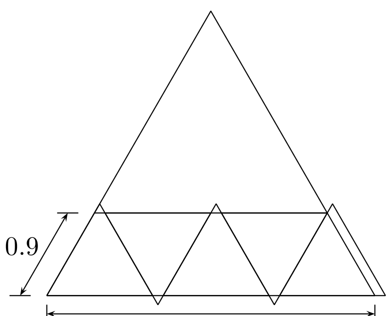


Figure 8

Or we can use 4 tiles in the top corner, and then use 7 tiles for a larger trapezoid (Fig. 9).

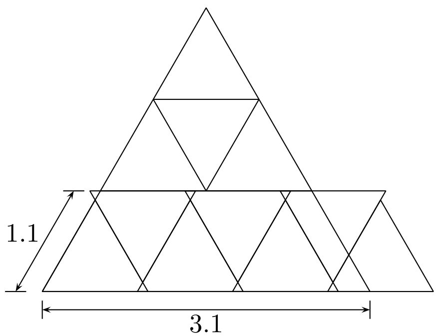


Figure 9

4 Across the Bridge—Back to Research

As I was going across our bridge back to the shore of research, the Columbia University brilliant undergraduate student Mitya Karabash joined me for further explorations of this problem. First of all, we observed the following result, which is better than its simple proof:

Non-Equilateral Cover-Up 2. [14]. For *every* non-equilateral triangle T, n^2+1 triangles similar to T and with the ratio of sizes $1:(n+\varepsilon)$, can cover T.

Proof. An appropriate affine transformation maps the equilateral triangle and its covering depicted in Figure 2 into T. This transformation produces a covering of T with n^2+2 triangles similar to T. We can now cover the top triangle with 2 tiling triangles instead of 3 as shown in Figure 10, thus reducing the total number of covering triangles to n^2+1.

Mitya and I then generalized the problem from covering a triangle to covering much more complex figures we named *trigons*.

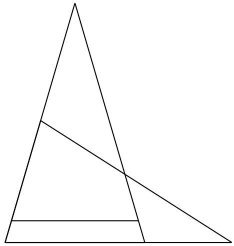


Figure 10

n-Trigon T_n is the union of n triangles from the standard triangulation of the plane such that a triangular rook can find a path between any two triangles of T_n, i.e., the union of n edge-connected triangles. If the triangulation is equilateral, then we say that the n-trigon is *equilateral*. You can see an example of an equilateral 9-trigon in Figure 11.

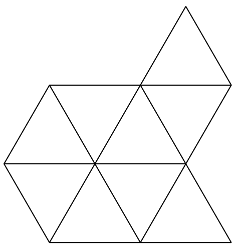


Figure 11

In our cover-up games, we assume that the ratio of the corresponding sides of the triangles forming the trigon and the titling triangles is $(1 + \varepsilon)/1$. We proved an important result:

Karabash-Soifer's Trigon Theorem 3. [14]. An n-trigon T_n can be covered with

1. $n + 2$ triangles if the trigon is equilateral;
2. $n + 1$ triangles if the trigon is non-equilateral.

In spite of all the progress, however, one "little" question remains open. I will formulate it here as a conjecture (because John Conway and I thought we knew the answer—we just had no idea how to prove it):

Cover-Up Conjecture 4 (Conway-Soifer, 2004). Equilateral triangle of side $n+\varepsilon$ cannot be covered by n^2+1 unit equilateral triangles.

Right after the Cover-Up Problem 1, I created the *Cover-Up Squared Problem*. Naturally, a square of side n can be covered by n^2 unit squares. When, however, I let the side length increase merely to $n+\varepsilon$, I found a new open problem:

Cover-Up Squared Problem 5. [13]. Find the smallest number $P(n)$ of unit squares that can cover a square of side length $n+\varepsilon$.

I devised a covering approach illustrated in Figure 12.

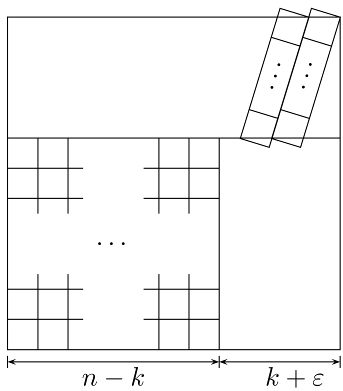


Figure 12

My results were followed by the joint ones by Mitya Karabash and me. The best Mitya and I were able to do in the cover-up squared, was to match Paul Erdős and Ronald L. Graham's dual result [11] on packing squares in a square:

Karabash-Soifer's Theorem 14. [15]. $P(n)=n^2+O(n^{7/11})$.

Let me explain the "big O" notation. We write $f(n)=O(g(n))$ if asymptotically the function $f(n)$ does not grow faster than a constant multiple of $g(n)$.

The Cover-Up Squared Problem remains open, both in search for the asymptotically lowest possible solution and for exact values for small n. Mitya and I have conjectured:

Cover-Up Square Conjecture 14. [15]. $P(n) = n^2 + \Theta(n^{1/2})$.

We write $f(n) = \Theta(g(n))$ if asymptotically $f(n)$ and $g(n)$ are of the same order, i.e., $f(n) = O(g(n))$ and $g(n) = O(f(n))$.

I hope to have achieved my goal of establishing the bridge between the problems of Mathematical Olympiads and research problems of mathematicians. Paul Erdős told me in March 1989 that "Olympiads by themselves are not very important, but they bring a new enthusiasm for mathematics, and in this regard they are important". I agree. Moreover, building this bridge eliminates any separation between Olympiads and "real" mathematics.

I hope that *Springer*'s latest decision to publish my 7 books [2]–[8] would allow for a further contribution to the Bridge between Olympiads and Mathematics.

References

[1] Gal'perin G. A., Tolpygo, A. K., *Moscow Mathematical Olympiads*, edited and with introduction by A. N. Kolmogorov, Prosvetshenie, Moscow, 1986 (Russian).

[2] Soifer A., *Mathematics as Problem Solving*, New expanded edition, Springer, New York, 2008.

[3] Soifer A., *How Does One Cut a Triangle?* New expanded edition, Springer, New York, 2008.

[4] Boltyanski V., Soifer A., *Geometric Etudes in Combinatorial Mathematics*, Center for Excellence in Mathematical Education, Colorado Springs, CO, 1991. New expanded edition Springer, New York, due 2009.

[5] Soifer A., *Colorado Mathematical Olympiad: The First Ten Years and Further Explorations*, Center for Excellence in Mathematical Education, Colorado Springs, 1994.

[6] Soifer A., *Colorado Mathematical Olympiad: The First Twenty Years and Further Explorations*, Springer, New York, due 2009.

[7] Soifer A., *Mathematical Coloring Book: Mathematics of Coloring and the Colorful Life of Its Creators*, Springer, New York, 2008.

[8] Erdős P., Soifer A., *Problems of PGOM Erdős*, Springer, New York, due 2010.

[9] Soifer A., Building a Bridge I: From Problems of Mathematical Olympiads to Open Problems of Mathematics, *Mathematics Competitions*, Australia, Vol. 21, No. 2, 2008, pp. 11–19.

[10] Soifer, A., Building a Bridge II: From Problems of Mathematical Olympiads to Open Problems of Mathematics, *Mathematics Competitions*, Australia, Vol. 22, No. 1, 2009, pp. 59–66.

[11] Erdős, P., and Graham, R. L., On Packing Squares with Equal Squares, *J. Combin. Theory, Ser. A*, **19**, 1975, 119–123.

[12] Conway, J. H., and Soifer, A., Covering a Triangle with Triangles, *Amer. Math. Monthly*, **112(1)**. 2005, 78.

[13] Soifer, A., Cover-Up Squared, *Geombinatorics*, XIV(4), 2005, 221–226.

[14] Karabash, D., and Soifer, A., On Covering of Trigons, *Geombinatorics*, XV(1), 2005, 13–17.

[15] Karabash, D., and Soifer, A., Note on Covering Square with Equal Squares, *Geombinatorics*, XVIII(1), October 2008, 13–17.

Alexander Soifer
University of Colorado at Colorado Springs
P. O. Box 7150, Colorado Springs, CO 80933
USA
E-mail: asoifer@uccs.edu
http://www.uccs.edu/˜asoifer/

A Quarter a Century of Discovering & Inspiring Young Gifted Mathematicians: All the Best from Colorado Mathematical Olympiad

Alexander Soifer

1 The Event

There are many types of mathematical competitions throughout the world. Some expect participants to merely state "good" answers, others are multiple-choice competitions. Some competitions are oral, and completed in a matter of a couple of hours. Others go on for a week or weeks. Over the past 100+ years throughout the world, the word "Olympiad" came to mean the particular type of competition where complete essays-solutions are expected for every problem and an adequate time is offered for solving them.

The First Colorado Springs Mathematical Olympiad took place 25 years ago, on April 27, 1984. The *Colorado Mathematical Olympiad* is the largest essay-type in-person mathematical competition in the United States, with 600 to 1,000 participants competing annually for prizes, such as: gold, silver and bronze medals, college scholarships, computers, computer software, calculators, books, memorabilia, etc. All prizes are made possible by our various sponsors, such as CASIO, Wolfram Research, Texas Instruments, City of Colorado Springs, University of Colorado, School Districts 11, 20, 3 and others, and various high and middle schools from all over the State of Colorado, etc.

Participants are offered 5 problems and given 4 hours to solve them. Problems vary a great deal in difficulty, and we arrange them from the easiest problem to the most difficult problem. This is an individual competition, and we expect all work to be shown. We specifically warn the Olympians right on the problem sheet that "we will give no credit for answers submitted without supporting work. Conversely, a minor error that leads to an incorrect answer will not substantially reduce your credit."

Our Olympiad is genuinely and principally an essay-type competition. We generously reward originality and creativity. Our goal is to allow every student the opportunity to participate in the Olympiad. We offer only individual competition and put no restrictions on the number of participants from a school.

We try to bring the Olympiad problems closer to "real" mathematics: many problems require construction of examples (rather than just analytical reasoning). Sometimes we mimic a "real" mathematical research by offering a series of problems, increasing in difficulty, and leading to generalizations and deeper results. These problems demonstrate ways in which mathematical research works.

We use the least standard, the most interesting, decisively unknown to participants problems. We do not have a large enough organization to offer the competition separately for every grade. Thus, we offer the same problems to everyone who comes, from middle school students to high school seniors. This approach has made finding acceptable problems much more difficult, but has positively improved the quality of the problems. We create problems that require for their solutions a minimal amount of knowledge and a great deal of creativity, originality, and analytical thinking.

Often the problems we select stem from mathematical research we do. We "simply" notice a fragment of the research, which utilizes a beautiful idea. We then translate thus found mathematical gem into the language of an engaging story—and a new Olympiad problem is ready!

I will show you here a selection of our problems: All the Best from the Quarter a Century of the Colorado Mathematical Olympiad. (Veterans of competitions, of course, noticed my homage to Australian Mathematics Competitions in the subtitle of my paper.) Well, maybe not all the best—as my volume is limited—but some of the best! The problems will illustrate how we identify and inspire gifted young mathematicians, and why these particular problems aid us in this task.

We have reached the Quarter a Century mark—the XXV Colorado

Mathematical Olympiad has taken place on April 18, 2008, with Award Presentation Ceremonies (a 4-hour program) a week later. *Springer* has contracted with this author 7 books, related to the enrichment activities: see them in bibliography [2] –[8] , and on *Springer's web site*, including the book http://www.springer.com/math/algebra/book/978-0-387-75471-0 specifically covering the 20 years of the Olympiad and 20 essays on building a bridge from Olympiad problems to open problems of mathematics.

2 The Problems

Limitations of space (in the article) and time (in the talk at the ICME-11, Monterrey, Mexico), allow me to give just 3 examples from the past few years. Since we offer problems of increasing difficulty, I will offer you here one easy, one medium, and one hard problem—in this order.

2006.2. A horse! A horse! My kingdom for a horse!
The Good, the Bad, and the Ugly divide a pile of gold and a horse. The pile consists of 2006 gold coins, and they draw in turn 1, 2 or 3 coins from the pile. The Good gets the first turn, the Bad draws second, and the Ugly takes last. The Ugly does not trust the Bad and never draws the same number of coins as the Bad has drawn immediately before him. The one who takes the last coin is left behind, while the two others cross the prairie together on horseback. Who can guarantee himself the ride out on the horse regardless of how the others draw coins? How can he do this?

Solution. The Good can guarantee himself a horse ride. Observe that $2006 = 6k + 2$ for some integer k. The Good starts by taking 1 coin. Since the Ugly will not draw the same as the Bad, the number of coins drawn by the Bad and the Ugly combined on any round add up to 3, 4, or 5. Thus, by always drawing the difference between 6 and the sum of the draws by the Bad and the Ugly, the Good will eventually bring the number of coins to 1 after his turn. Thus, the Bad will pick up the last coin, and the Good and Ugly will ride off together.

2007.4. Looking for the Positive
A number is placed in each angle of a regular 2007-gon so that that the sum of any 10 consecutive numbers is positive. Prove that one can

choose an angle with the number a in it, such that when we label all 2007 numbers clockwise $a = a_1, a_2, \ldots, a_{2007}$, each sum a_1, $a_1 + a_2$, $\ldots$, $a_1 + a_2 + \cdots + a_{2007}$ will be positive.

Proof in two parts.

1). Let us prove that the sum S of all 2007 given numbers is positive. By permuting the summands cyclically, we get:

$$\begin{array}{c} a_1 + a_2 + \cdots + a_{2007} = S \\ a_2 + a_3 + \cdots + a_1 = S \\ \cdots \\ a_{10} + a_{11} + \cdots + a_9 = S \end{array}$$

By adding together these 10 equalities, we get a positive sum in the left-hand side (as the sum of the 2007 columns, each column positive due to being the sum of 10 consecutive numbers). On the right we get $10S$, which is thus positive, and so is S.

2). Consider the following array of $2007 \times 2 = 4014$ sums, where we add one summand at a time moving clockwise:

$$\begin{aligned} s_{1,1} &= a_1 \\ s_{1,2} &= a_1 + a_2 \\ &\cdots \\ s_{1,2007} &= a_1 + a_2 + \cdots + a_{2007} \\ s_{1,2008} &= a_1 + a_2 + \cdots + a_{2007} + a_1 \\ s_{1,2009} &= a_1 + a_2 + \cdots + a_{2007} + a_1 + a_2 \\ &\cdots \\ s_{1,4014} &= a_1 + a_2 + \cdots + a_{2007} + a_1 + a_2 + \cdots + a_{2007} \end{aligned}$$

Choose $1 \leq k \leq 2007$ such that the sum $s_{1,k}$ is the lowest among the first 2007 sums; if the same lowest sum appears more than once, we choose $s_{1,k}$ with the largest k. This sum is in fact the lowest among all 4014 sums because the latter 2007 sums are equal the earlier 2007 sums plus S, where S is the sum of all given numbers ($S > 0$ as shown in part 1 of the proof). We are done because

$$
\begin{aligned}
0 &< s_{1,k+1} - s_{1,k} = a_{k+1} \\
0 &< s_{1,k+2} - s_{1,k} = a_{k+1} + a_{k+2} \\
&\dots \\
0 &< s_{1,k+2007} - s_{1,k} = a_{k+1} + a_{k+2} + \dots + a_{2007} + a_1 + \dots + a_k
\end{aligned}
$$

Chess 7×7[1]

a) Each member of two 7-member chess teams is to play once against each member of the opposing team. Prove that as soon as 22 games have been played, we can choose 4 players and seat them at a round table so that each pair of neighbors has already played.

b) Prove that 22 is best possible, i.e., after 21 games the result cannot be guaranteed.

This problem occurred to me while I was reading a wonderful unpublished 1989 manuscript of a monograph *Aspects of Ramsey Theory* by Hans Jürgen Prömel and Bernd Voigt. I found a mistake in a lemma, and constructed a counterexample for this lemma's statement. Finding this counterexample is exactly problem 5 b) here. Problem 5 a) is a corrected particular case of that lemma, translated, of course, into a language of a nice "real" story. I found 3 wonderful solutions of part a) of the problem. Here is one of them.

Solution of a). This solution harnesses the power of combinatorics.

In the selection and editing process, Dr. Col. Bob Ewell suggested to use a 7×7 table to record the games played. We number the players in both teams. For each player of the first team we allocate a row of the table, and for each player of the second team a column. We place a checker in the table in location (i, j) if the player i of the first team played the player j of the second team (Fig 1). If the required four players are found, this would manifests itself in the table as a rectangle formed by four checkers (a checkered rectangle)! The problem thus translates into the new language as follows:

[1] First appeared in [9]—see three beautiful solutions of part a).

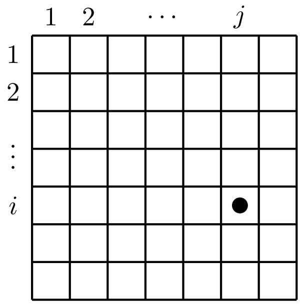


Figure 1

A 7 × 7 table with 22 checkers must contain a checkered rectangle.

Assume that a table has 22 checkers but does not contain a checkered rectangle. Since 22 checkers are contained in 7 rows, by Pigeonhole Principle, there is a row with at least 4 checkers in it. Observe that interchanging rows or columns does not affect the property of the table to have or have not a checkered rectangle. By interchanging rows we make the row with at least 4 checkers first. By interchanging columns we make all checkers to appear consecutively from the left of the first column. We consider two cases.

1. Top column contains exactly 4 checkers (Figure 2). Draw a bold

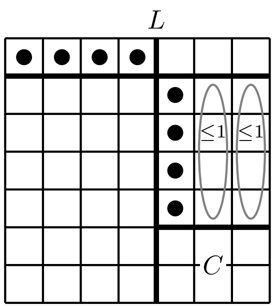


Figure 2

vertical line L after the first 4 columns. To the left from L, top row contains 4 checkers, and all other rows contain at most 1 checker each, for otherwise we would have a checkered rectangle (that includes the top row). Therefore, to the left from L we have at most $4 + 6 = 10$ checkers. This leaves at least 12 checkers to the right of L, thus at least one of the three columns to the right of L contains at least 4 checkers; by interchanging columns and rows we put them in the position shown in Figure 5.3. Then each of the two right columns contains at most 1 checker total in the rows 2 through 5, for otherwise we would have a checkered rectangle. We thus have at most $4 + 1 + 1 = 6$ checkers to the right of L in rows 2 through 5 combined. Therefore, in the lower right 2×3 part C of the table we have at least $22 - 10 - 6 = 6$ checkers—thus C is completely filled with checkers and we get a checkered rectangle in C in contradiction with our assumption.

2. Top column contains at least 5 checkers (Figure 3). Draw a bold

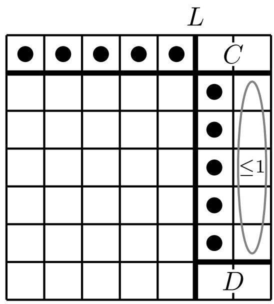


Figure 3

vertical line L after the first 5 columns. To the left from L, top row contains 5 checkers, and all other rows contain at most 1 checker each, for otherwise we would have a checkered rectangle (that includes the top row). Therefore, to the left from L we have at most $5 + 6 = 11$ checkers. This leaves at least 11 checkers to the right of L, thus at least one of the two columns to the right of L contains at least 6 checkers; by interchanging columns and rows we put 5 of these 6 checkers in the position shown in Figure 3. Then the last column contains at most 1 checker total in the rows 2 through 6, for otherwise we would have a checkered rectangle. We thus have at most $5 + 1 = 6$ checkers to the right of L in rows 2 through 6 combined. Therefore, the upper right 1×2 part C of the table plus the lower right 1×2 part D of the table have together have at least $22 - 11 - 6 = 5$ checkers—but they only have 4 cells, and we thus get a contradiction.

Solution of b). Glue a cylinder (!) out of the board 7×7, and put 21 checkers on all squares of the 1^{st}, 2^{nd}, and 4^{th} diagonals (Fig. 4 shows the cylinder with one checkered diagonal; Fig. 5 shows, in a flat representation, the cylinder with all three cylinder diagonals). Assume

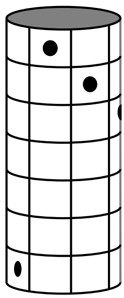


Figure 4

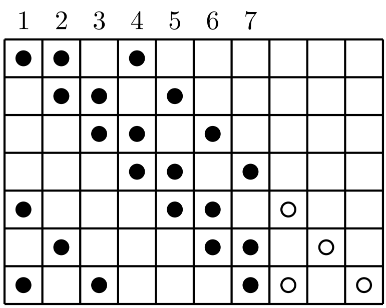


Figure 5

that 4 checkers form a rectangle on our 7×7 board. Since these four checkers lie on 3 diagonals, by Pigeonhole Principle two checkers lie on the same (checkers-covered) diagonal D of the cylinder. But this means that *on the cylinder* our 4 checkers form a square! Two other (opposite) checkers a and b thus must be symmetric to each other with respect to D, which implies that the diagonals of the cylinder that contain a and b must be symmetric with respect to D—but no 2 checker-covered diagonals in our checker placement are symmetric with respect to D. (To see it, observe Fig. 6 which shows the top rim of the cylinder with bold dots for checkered diagonals: square distances between the checkered diagonals, clockwise, are 1, 2, and 4) This contradiction implies that there are no checkered rectangles in our placement. Done!

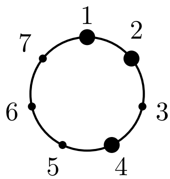


Figure 6

Remark on Problem b). Obviously, any solution of problem b) can be presented in a form of 21 checkers on a 7×7 board (left 7×7 part with 21 black checkers in Fig. 5). It is less obvious, that the solution is *unique*: by a series of interchanges of rows and columns, any solution of this problem can be brought to precisely the one I presented! Of course, such interchanges mean merely renumbering of players of the same team. The uniqueness of the solution of problem b) is precisely another way of stating the uniqueness of the projective plane[1] of order 2, so called "Fano Plane"[2] denoted by $PG(2, 2)$. The Fano plane is an abstract construction, with symmetry between points and lines: it has 7 points and 7 lines (think of rows and columns of our 7×7 board as lines and points respectively!), with 3 points on every line and 3 lines through every point (Fig. 7). Observe that if in our 7×7 board we

[1]A finite projective plane of order n is defined as a set of points with the properties that:

1. Any two points determine a line,
2. Any two lines determine a point,
3. Every point has $n + 1$ lines through it,
4. Every line contains $n + 1$ points.

[2]Named after Gino Fano (1871–1952), the Italian geometer who pioneered the study of finite geometries.

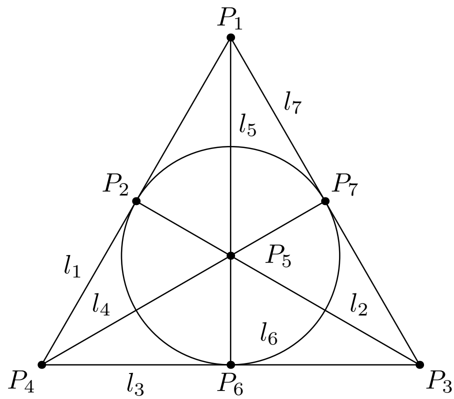


Figure 7

replace checkers by 1 and the rest of the squares by zeroes, we would get the incidence matrix of the Fano Plane.

3 Interest Earned on the Capital

What has our investment capital been to date? In its first Quarter a Century, Colorado Mathematical Olympiad (CMO) has entertained some 17,000 students. They have written over 85,000 essays, and were awarded over $240,000 in prizes. The Olympiad has been a unique joint effort of school districts, schools, institutions of higher education, business community and local and State governments.

In March 1989 I was hosting Paul Erdös for the first time. We did mathematics. We also talked about life, refugees, anti-Semitism, and, of course, Colorado Mathematical Olympiad, for which he contributed a problem. Paul said, "The Olympiad would not be important by itself, but it creates a new enthusiasm for mathematics, and this is very important." This enthusiasm "paid interest":

The winner of the 1st 1984 CMO Russell Shaffer, achieved a perfect score at MIT, then on the National Science Foundation Scholarship earned his Ph.D. degree in Theoretical Computer Science from Princeton University, and is now a computer science researcher.

The winner of the 2nd 1985 CMO, Richard Wolniewitz, earned his Ph.D. degree in Computer Science from the University of Colorado at Boulder. He owns his own software engineering company.

The winner of the 3rd, 4th, and 5th CMO's (1986–1988) David Hunter, earned his first degree from Princeton University, and a Ph.D. in Statistics. He is a Professor at Pennsylvania State University.

The co-winner of the 5th 1988 CMO, Gideon Jaffe, earned his first degree in Mathematics and Drama from Harvard University, and his Ph.D. degree in Philosophy from the University of California at Berkeley. He is a Professor of Philosophy at Berkeley.

Matt Kahle was a "C–" student in geometry in Air Academy High School in Colorado Springs. This did not stop him from winning the 8th and the 9th CMO's (1990–1991). He earned his first two degrees in Mathematics from Colorado State University (he was not admitted to my University for having a "C–" grade average, in spite of my assurances that one day we would be proud of such an alumnus). Matt has earned his Ph. D. in Mathematics from the University of Washington, Seattle, and is now a Post-Doctoral Fellow at Stanford University.

These are just a few examples of the "interest" we gained on the "investment" of the enthusiasm for mathematics among the middle and high school students. Now, after 25 years of its existence, Colorado Mathematical Olympiad has become a part of life in the State of Colorado, an event eagerly awaited by students, their parents and their schools. The road to this success has not always been covered by roses—indeed, sometimes we have encountered thorns as well. Read more about the history of CMO and its problems in [1] , and more in the forthcoming in 2009 book [2] , and start your own Olympiads, big and small, for just your class, or for your school, neighborhood, city, region, country. Good luck!

References

[1] Soifer, A., *Colorado Mathematical Olympiad: The First **Ten** Years and Further Explorations*, Center for Excellence in Mathematical Education, Colorado Springs, 1994.

[2] Soifer, A., *Colorado Mathematical Olympiad: The First* ***Twenty*** *Years and Further Explorations*, Springer, New York, to appear in 2009.

[3] Soifer, A., *Mathematics as Problem Solving*, Center for Excellence in Mathematical Education, Colorado Springs, CO, 1987. New expanded edition to appear in Springer, New York, early 2009.

[4] Soifer, A., *How Does One Cut a Triangle?* Center for Excellence in Mathematical Education, Colorado Springs, CO, 1990. New expanded edition to appear in Springer, New York, early 2009.

[5] Boltyanski, V., and Soifer, A. *Geometric Etudes in Combinatorial Mathematics*, Center for Excellence in Mathematical Education, Colorado Springs, CO, 1991. New expanded edition to appear in Springer, New York, 2009.

[6] Soifer, A., *Mathematical Coloring Book: Mathematics of Coloring and the Colorful Life of Its Creators*, Springer, New York, 2008.

[7] Erdös, P., and Soifer, A., *Problems of pgom Erdös*, Springer, New York (to appear in 2010).

[8] Soifer, A., *Life and Fate: In Search of Van der Waerden*, Springer, New York (to appear in 2009–2010).

[9] Soifer, A., A Journey from Ramsey Theory to Mathematical Olympiad to Finite Projective Planes, *Mathematics Compe* 17 (2), 2004, 8–16.

Alexander Soifer
University of Colorado at Colorado Springs
P. O. Box 7150, Colorado Springs, CO 80933
USA
E-mail: asoifer@uccs.edu
http://www.uccs.edu/˜asoifer/

Tournament of Towns (Fall, 2009)

Andy Liu

Andy Liu is a professor of mathematics at the University of Alberta in Canada. His research interests span discrete mathematics, geometry, mathematics education and mathematics recreations. He edits the Problem Corner of the MAA's magazine Math Horizons. *He was the Chair of the Problem Committee in the 1995 IMO in Canada. His contribution to the 1994 IMO in Hong Kong was a major reason for him being awarded a David Hilbert International Award by the World Federation of National Mathematics Competitions.*

1. Junior A6.
On an infinite chessboard are placed 2009 $n \times n$ cardboard pieces such that each of them covers exactly n^2 cells of the infinite chessboard. The cardboard pieces may overlap. Prove that the number of cells of the infinite chessboard which are covered by an odd number of cardboard pieces is at least n^2.

2. Junior A7/Senior A6.
Olga and Max visited a certain Archipelago with 2009 islands. Some pairs of islands were connected by boats which run both ways. Olga chose the first island on which they land. Then Max chose the next island which they could visit. Thereafter, the two took turns choosing an accessible island which they had not yet visited. When they arrived at an island which was connected only to islands they had already visited, whoever's turn to choose next would be the loser. Prove that Olga could always win, regardless of the way Max played and regardless of the way the islands were connected.

3. Senior A7.
At the entrance to a cave is a rotating round table. On top of the table are n identical barrels, evenly spaced along its circumference. Inside each barrel is a herring either with its head up or its head down. In a move, Ali Baba chooses from 1 to n of the barrels and turns them upside down. Then the table spins around. When it stops, it is impossible to tell which barrels have been turned over. The cave will open if the heads of the herrings in all n barrels are all up or are all down. Determine all values of n for which Ali Baba can open the cave in a finite number of moves.

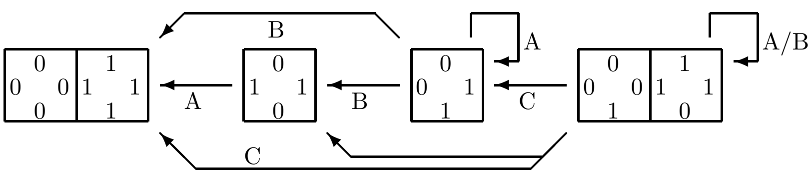
Solution to Problem 1. Olga Ivrii, Toronto.
Partition the infinite chessboard into $n \times n$ subboards by horizontal and vertical lines n units apart. Within each subboard, assign the coordinates (i, j) to the square at the i-th row and the j-th column, where $1 \le i, j \le n$. Whenever an $n \times n$ cardboard is placed on the infinite chessboard, it covers n^2 squares all with different coordinates. The total number of times squares with coordinates (1,1) is covered is 2009. Since 2009 is odd, at least one of the squares with coordinates (1,1) is covered by an odd number of cardboards. The same goes for the other $n^2 - 1$ coordinates. Hence the total number of squares which are covered an odd number of times is at least n^2.

Solution to Problem 2. Central Jury.
We construct a graph, with the vertices representing the islands and the edges representing connecting routes. The graph may have one or more connected components. Since the total number of vertices is odd, there must be a connected component with an odd number of vertices. Olga chooses from this component the largest set of independent edges, that is, edges no two of which have a common endpoint. She will colour these edges red. Since the number of vertices is odd, there is at least one vertex which is not incident with a red vertex. Olga will start the tour there. Suppose Max has a move. It must take the tour to a vertex incident with a red edge. Otherwise, Olga could have colour one more edge red. Olga simply continue the tour by following that red edge. If Max continues to go to vertices incident with red edges, Olga will always

have a ready response. Suppose somehow Max manages to get to a vertex not incident with a red edge. Consider the tour so far. Both the starting and the finishing vertices are not incident with red edges. In between, the edges are alternately red and uncoloured. If Olga interchanges the red and uncoloured edges on this tour, she could have obtained a larger independent set of edges. This contradiction shows that Max could never get to a vertex not incident with red edges, so that Olga always wins if she follows the above strategy.

Solution to Problem 3. Hsin-Po Wang, Taipei.
The task is guaranteed to succeed if and only if n is a power of 2. Suppose n is not a power of 2. Then it has an odd prime factor p. Choose p evenly spaced barrels and make sure that the herrings inside are not all pointing the same way. Ignore all other barrels. At any point, let the herrings in r barrels are pointing up while the herrings in the other s barrels are pointing down. Since $r + s = p$ is odd, $r \neq s$. We may assume that $r > s$. In order for Ali Baba to succeed, he must turn over all r barrels of the first kind or all s barrels of the second kind. A pagan god who is having fun with Ali Baba can spin the table so that if Ali Baba plans to turn over r barrels, the herring in at least one of them is pointing down; and if Ali Baba plans to turn over s barrels, the herring in all of them are pointing up. This way, Ali Baba will never be able to open the cave.

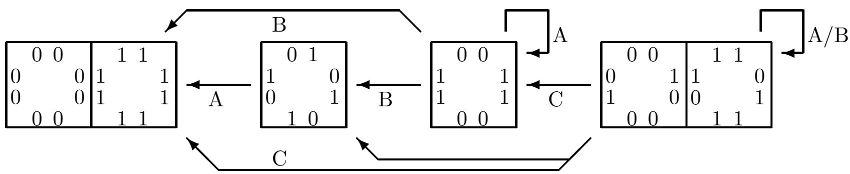
If $n = 2^k$ for some non-negative integer k, we will prove by induction on k that Ali Baba can open the cave. The case $k = 0$ is trivial as the cave opens automatically. The case $k = 1$ is easy. If the cave is not already open, turning one barrel over will do. For $k = 2$, let 0 or 1 indicate whether the herring is heads up or heads down.



The diagram above represents the four possible states the table may be in, as well as the transition between states by the following operations. Operation **A:** Turn over any two opposite barrels. Operation **B:** Turn over any two adjacent barrels. Operation **C:** Turn over any one barrel. By performing the sequence **ABACABA**, the cave will open. The first state is called an *absorbing* state, in that once there, no further transition takes place as the cave will open immediately.

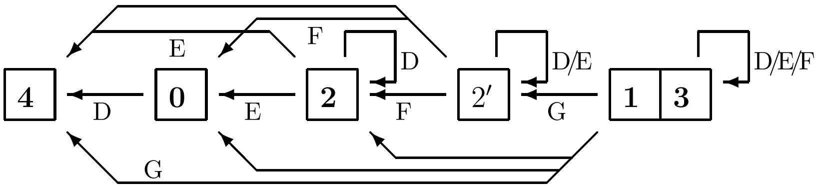
The second state becomes the first state upon the first operation A. The third state remains in place during the first operation A, but will become either the first state or the second state upon the first operation B. In the latter case, it will become the first state upon the second operation A. The fourth state remains in place during the first three operations, but will become any of the other three states upon the operation C. It will become the first state at the latest after three more operations.

The success of the case $k = 2$ paves the way for the case $k = 3$. The process is typical of the general inductive argument so that we give a detailed analysis. The idea is to treat each pair of diametrically opposite barrels as a single entity.



The above diagram, which is essentially copied from that for $k = 2$, is part of a much bigger state-transition diagram for $k = 3$. Here, all the states have the property that opposite pairs of barrels are all matching, that is, both are 0 or both are 1. The operations are modified from those in the case $k = 2$ as follows.Operation **A:** Turn over every other pair of opposite barrels; in other words, turn over every other barrel. Operation **B:** Turn over any two adjacent pairs of opposite barrels. Operation **C:** Turn over any pair of opposite barrels.

By performing the sequence **ABACABA**, the cave will open. These states together form an expanded absorbing state in the overall diagram below.



Here, the box marked m contains all states with m matching opposite pairs, where $0 \le m \le 4$. The box marked 4 is the expanded absorbing state mentioned above. The states with 2 matching pairs are classified according to whether these matching are alternating or adjacent. The former states are contained in the box marked 2 while the latter states are contained in the box marked $2'$. We have four new operations. Operation **D:** Turn over any 4 adjacent barrels. Operation **E:** Turn over any 2 barrels separated by one other barrel. Operation **F:** Turn over any two adjacent barrels. Operation **G:** Turn over any barrel.

Let **X** denote the sequence ABACABA. Then the sequence for the case $k = 3$ is

XDXEXDXFXDXEXDXGXDXEXDXFXDXEXDX.

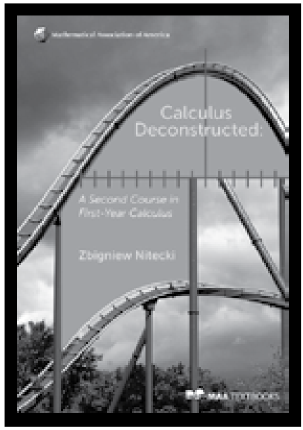
We keep repeating X to clear any state that has entered the box marked 4, to prevent them from returning to another box. Whatever the state the table is in, the cave will open by the end of this sequence.

The general procedure is now clear. We treat each opposite pair as a single entity, thereby reducing to the preceding case. Then we moving progressively all states into the expanded absorbing state. Thus the task is possible whenever n is a power of 2.

Andy Liu
University of Alberta
CANADA

Calculus Deconstructed:
A Second Course in First-Year Calculus

Can be used for the first semester of the (freshman) Honors Calculus course for students with high school calculus background. Also suitable for a "bridge" course using basic analysis.



Zbigniew H. Nitecki

A thorough and mathematically rigorous exposition of single-variable calculus for readers with some previous exposure to calculus techniques but not to methods of proof, this book is appropriate for a beginning Honors Calculus course assuming high school calculus or a "bridge course" using basic analysis to motivate and illustrate mathematical rigor. It can serve as a combination textbook and reference book for individual self-study. Standard topics and techniques in single-variable calculus are presented in the context of a coherent logical structure, building on familiar properties of real numbers and teaching methods of proof by example along the way. Numerous examples reinforce both practical and theoretical understanding, and extensive historical notes explore the arguments of the originators of the subject.

No previous experience with mathematical proof is assumed: rhetorical strategies and techniques of proof (reductio ad absurdum, induction, contrapositives, etc.) are introduced by example along the way. Between the text and the exercises, proofs are available for all the basic results of calculus for functions of one real variable.

650 pp., Hardbound 2009 | *ISBN: 978-0-88385-756-4* | *Catalog Code: CDE*
List: $72.50 | *MAA Member: $57.95*

Also available with a Solutions Manual:

The solutions manual contains solutions for all of the problems contained in the textbook.

325 pp., 2009
ISBN: 978-0-88385-758-8

THE MATHEMATICAL ASSOCIATION OF AMERICA ®

Order your copy today!
1.800.331.1622 ● www.maa.org

Subscriptions

Journal of the World Federation of National Mathematics Competitions

2 Issues Annually

Current subscribers will receive a subscription notice after the publication of the second issue each year.

For new subscribers, information can be obtained from:

Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA
Tel: +61 2 6201 5137
Fax:+61 2 6201 5052
Email: publications@amt.edu.au

or from our web site:
www.amt.edu.au

Useful Problem Solving Books from AMT Publications

These books are a valuable resource for the school library shelf, for students wanting to improve their understanding and competence in mathematics, and for the teacher who is looking for relevant, interesting and challenging questions and enrichment material.

To attain an appropriate level of achievement in mathematics, students require talent in combination with commitment and self-discipline. The following books have been published by the AMT to provide a guide for mathematically dedicated students and teachers.

NEW

Australian Mathematics Competition Primary Problems & Solutions Book 1 2004–2008
W Atkins & PJ Taylor
This book consists of questions and full solutions from past AMC papers and is designed for use with students in Middle and Upper Primary. The questions are arranged in papers of 10 and are presented ready to be photocopied for classroom use.

NEW

Challenge! 1999—2006 Book 2
JB Henry & PJ Taylor
This is the second book of the series and contains the problems and full solutions to all Junior and Intermediate problems set in the Mathematics Challenge for Young Australians, exactly as they were proposed at the time. They are highly recommended as a resource book for classes from Years 7 to 10 and also for students who wish to develop their problem-solving skills. Most of the problems are graded within to allow students to access an easier idea before developing through a few levels.

Bundles of Past AMC Papers

Past Australian Mathematics Competition papers are packaged into bundles of ten identical papers in each of the Junior, Intermediate and Senior divisions of the Competition. Schools find these sets extremely valuable in setting their students miscellaneous exercises.

AMC Solutions and Statistics
Edited by PJ Taylor

This book provides, each year, a record of the AMC questions and solutions, and details of medallists and prize winners. It also provides a unique source of information for teachers and students alike, with items such as levels of Australian response rates and analyses including discriminatory powers and difficulty factors.

Australian Mathematics Competition Book 1 1978–1984
Edited by W Atkins, J Edwards, D King, PJ O'Halloran & PJ Taylor

This 258-page book consists of over 500 questions, solutions and statistics from the AMC papers of 1978-84. The questions are grouped by topic and ranked in order of difficulty. The book is a powerful tool for motivating and challenging students of all levels. A must for every mathematics teacher and every school library.

Australian Mathematics Competition Book 2 1985–1991
Edited by PJ O'Halloran, G Pollard & PJ Taylor

Over 250 pages of challenging questions and solutions from the Australian Mathematics Competition papers from 1985-1991.

Australian Mathematics Competition Book 3 1992–1998
W Atkins, JE Munro & PJ Taylor

More challenging questions and solutions from the Australian Mathematics Competition papers from 1992-1998.

Australian Mathematics Competition Book 3 CD
Programmed by E Storozhev

This CD contains the same problems and solutions as in the corresponding book. The problems can be accessed in topics as in the book and in this mode is ideal to help students practice particular skills. In another mode students can simulate writing one of the actual papers and determine the score that they would have gained. The CD runs on Windows platform only.

Australian Mathematics Competition Book 4 1999–2005
W Atkins & PJ Taylor

More challenging questions and solutions from the Australian Mathematics Competition papers from 1999-2005.

Problem Solving via the AMC
Edited by Warren Atkins

This 210-page book consists of a development of techniques for solving approximately 150 problems that have been set in the Australian Mathematics Competition. These problems have been selected from topics such as Geometry, Motion, Diophantine Equations and Counting Techniques.

Methods of Problem Solving, Book 1
Edited by JB Tabov & PJ Taylor

This book introduces the student aspiring to Olympiad competition to particular mathematical problem solving techniques. The book contains formal treatments of methods which may be familiar or introduce the student to new, sometimes powerful techniques.

Methods of Problem Solving, Book 2
JB Tabov & PJ Taylor

After the success of Book 1, the authors have written Book 2 with the same format but five new topics. These are the Pigeon-Hole Principle, Discrete Optimisation, Homothety, the AM-GM Inequality and the Extremal Element Principle.

Mathematical Toolchest
Edited by AW Plank & N Williams

This 120-page book is intended for talented or interested secondary school students, who are keen to develop their mathematical knowledge and to acquire new skills. Most of the topics are enrichment material outside the normal school syllabus, and are accessible to enthusiastic year 10 students.

International Mathematics — Tournament of Towns (1980–1984)
Edited by PJ Taylor

The International Mathematics Tournament of the Towns is a problem-solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. This 115-page book contains problems and solutions from past papers for 1980-1984.

International Mathematics — Tournament of Towns (1984–1989)
Edited by PJ Taylor

More challenging questions and solutions from the International Mathematics Tournament of the Towns competitions. This 180-page book contains problems and solutions from 1984-1989.

International Mathematics — Tournament of Towns (1989–1993)
Edited by PJ Taylor

This 200-page book contains problems and solutions from the 1989-1993 Tournaments.

International Mathematics — Tournament of Towns (1993–1997)
Edited by PJ Taylor

This 180-page book contains problems and solutions from the 1993-1997 Tournaments.

International Mathematics — Tournament of Towns (1997–2002)
Edited by AM Storozhev

This 214-page book contains problems and solutions from the 1997-2002 Tournaments.

Challenge! 1991 – 1998
Edited by JB Henry, J Dowsey, AR Edwards, L Mottershead, A Nakos, G Vardaro & PJ Taylor

This book is a major reprint of the original Challenge! (1991-1995) published in 1997. It contains the problems and full solutions to all Junior and Intermediate problems set in the Mathematics Challenge for Young Australians, exactly as they were proposed at the time. It is expanded to cover the years up to 1998, has more advanced typography and makes use of colour. It is highly recommended as a resource book for classes from Years 7 to 10 and also for students who wish to develop their problem-solving skills. Most of the problems are graded within to allow students to access an easier idea before developing through a few levels.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

Australian Mathematical Olympiads 1979 – 1995
H Lausch & PJ Taylor

This book is a complete collection of all Australian Mathematical Olympiad papers from the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads Book 1 1981–1993
A Liu

This book contains the papers and solutions of two contests, the Chinese National High School Competition and the Chinese Mathematical Olympiad. China has an outstanding record in the IMO and this book contains the problems that were used in identifying the team candidates and selecting the Chinese team. The problems are meticulously constructed, many with distinctive flavour. They come in all levels of difficulty, from the relatively basic to the most challenging.

Asian Pacific Mathematics Olympiads 1989-2000
H Lausch & C Bosch-Giral

With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981–1995
ME Kuczma & E Windischbacher

Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
Edited by T Andreescu & Z Feng

This book contains one hundred and one highly rated problems used in training and testing the USA International Mathematical Olympiad team. The problems are carefully graded, ranging from quite accessible towards quite challenging. The problems have been well developed and are highly recommended to any student aspiring to participate at National or International Mathematical Olympiads.

Hungary Israel Mathematics Competition
S Gueron

The Hungary Israel Mathematics Competition commenced in 1990 when diplomatic relations between the two countries were in their infancy. This 181-page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions. The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Chinese Mathematics Competitions and Olympiads Book 2 1993–2001
A Liu
This book is a continuation of the earlier volume and covers the years 1993 to 2001.

Bulgarian Mathematics Competition 1992–2001
BJ Lazarov, JB Tabov, PJ Taylor & A Storozhev

The Bulgarian Mathematics Competition has become one of the most difficult and interesting competitions in the world. It is unique in structure combining mathematics and informatics problems in a multi-choice format. This book covers the first ten years of the competition complete with answers and solutions. Students of average ability and with an interest in the subject should be able to access this book and find a challenge.

Mathematical Contests – Australian Scene
Edited by PJ Brown, A Di Pasquale & K McAvaney

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

Mathematics Competitions
Edited by J Švrcek

This bi-annual journal is published by AMT Publishing on behalf of the World Federation of National Mathematics Competitions. It contains articles of interest to academics and teachers around the world who run mathematics competitions, including articles on actual competitions, results from competitions, and mathematical and historical articles which may be of interest to those associated with competitions.

Problems to Solve in Middle School Mathematics

B Henry, L Mottershead, A Edwards, J McIntosh, A Nakos, K Sims, A Thomas & G Vardaro

This collection of problems is designed for use with students in years 5 to 8. Each of the 65 problems is presented ready to be photocopied for classroom use. With each problem there are teacher's notes and fully worked solutions. Some problems have extension problems presented with the teacher's notes. The problems are arranged in topics (Number, Counting, Space and Number, Space, Measurement, Time, Logic) and are roughly in order of difficulty within each topic. There is a chart suggesting which problem-solving strategies could be used with each problem.

Teaching and Assessing Working Mathematically Book 1 & Book 2

Elena Stoyanova

These books present ready-to-use materials that challenge students understanding of mathematics. In exercises and short assessments, working mathematically processes are linked with curriculum content and problem solving strategies. The books contain complete solutions and are suitable for mathematically able students in Years 3 to 4 (Book 1) and Years 5 to 8 (Book 2).

A Mathematical Olympiad Primer

G Smith

This accessible text will enable enthusiastic students to enter the world of secondary school mathematics competitions with confidence. This is an ideal book for senior high school students who aspire to advance from school mathematics to solving olympiad-style problems. The author is the leader of the British IMO team.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Innovation, Industry, Science and Research) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

Newton Enrichment Student Notes

JB Henry

Recommended for mathematics students of about Year 5 and 6 as extension material. Topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

Dirichlet Enrichment Student Notes

JB Henry

This series has chapters on some problem solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory. It is designed for students in Years 6 or 7.

Euler Enrichment Student Notes

MW Evans & JB Henry

Recommended for mathematics students of about Year 7 as extension material. Topics include elementary number theory and geometry, counting, pigeonhole principle.

Gauss Enrichment Student Notes
MW Evans, JB Henry & AM Storozhev

Recommended for mathematics students of about Year 8 as extension material. Topics include Pythagoras theorem, Diophantine equations, counting, congruences.

Noether Enrichment Student Notes
AM Storozhev

Recommended for mathematics students of about Year 9 as extension material. Topics include number theory, sequences, inequalities, circle geometry.

Pólya Enrichment Student Notes
G Ball, K Hamann & AM Storozhev

Recommended for mathematics students of about Year 10 as extension material. Topics include polynomials, algebra, inequalities and geometry.

T-SHIRTS

T-shirts of the following six mathematicians are made of 100% cotton and are designed and printed in Australia. They come in white, Medium (Turing only) and XL.

Leonhard Euler T-shirt

The Leonhard Euler t-shirts depict a brightly coloured cartoon representation of Euler's famous Seven Bridges of Königsberg question.

Carl Friedrich Gauss T-shirt

The Carl Friedrich Gauss t-shirts celebrate Gauss' discovery of the construction of a 17-gon by straight edge and compass, depicted by a brightly coloured cartoon.

Emmy Noether T-shirt

The Emmy Noether t-shirts show a schematic representation of her work on algebraic structures in the form of a brightly coloured cartoon.

George Pólya T-shirt

George Pólya was one of the most significant mathematicians of the 20th century, both as a researcher, where he made many significant discoveries, and as a teacher and inspiration to others. This t-shirt features one of Pólya's most famous theorems, the Necklace Theorem, which he discovered while working on mathematical aspects of chemical structure.

Peter Gustav Lejeune Dirichlet T-shirt

Dirichlet formulated the Pigeonhole Principle, often known as Dirichlet's Principle, which states: "If there are p pigeons placed in h holes and $p>h$ then there must be at least one pigeonhole containing at least 2 pigeons." The t-shirt has a bright cartoon representation of this principle.

Alan Mathison Turing T-shirt

The Alan Mathison Turing t-shirt depicts a colourful design representing Turing's computing machines which were the first computers.

ORDERING

All the above publications are available from AMT Publishing and can be purchased on-line at:

www.amt.edu.au/amtpub.html or contact the following:

AMT Publishing

Australian Mathematics Trust

University of Canberra ACT 2601

Australia

Tel: +61 2 6201 5137

Fax: +61 2 6201 5052

Email: mail@amt.edu.au

The Australian Mathematics Trust

The Trust, of which the University of Canberra is Trustee, is a non-profit organisation whose mission is to enable students to achieve their full intellectual potential in mathematics. Its strengths are based upon:

- a network of dedicated mathematicians and teachers who work in a voluntary capacity supporting the activities of the Trust;
- the quality, freshness and variety of its questions in the Australian Mathematics Competition, the Mathematics Challenge for Young Australians, and other Trust contests;
- the production of valued, accessible mathematics materials;
- dedication to the concept of solidarity in education;
- credibility and acceptance by educationalists and the community in general whether locally, nationally or internationally; and
- a close association with the Australian Academy of Science and professional bodies.