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MATHEMATICS COMPETITIONS

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MATHEMATICS COMPETITIONS

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The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the President

It is only a few months before the WFNMC meeting in Riga, a new opportunity to get together and pool our experience and imagination in promoting mathematical challenges for young people in all the continents around the world. In Riga, once again we will have the privilege of taking part in a meeting of a unique group: a group of mathematicians, teachers, problem-posers and solvers, united in a collective and collaborative effort to open new mathematical horizons and perspectives for students, where novel situations can be analyzed, structured and conquered. We expect the participation to be numerous and the outcome fruitful.

WFNMC has been an active participant in the present panorama in which opening windows to mathematical discovery, thought and adventure for those who seem to have particular talent, motivation and interest is highly positive and has given rise to the organization of very fruitful projects and events including math Olympiads that have a life-changing effect on those who are drawn to them and have the opportunity to take part, as well as math clubs, math houses, math camps, and many other original and innovative initiatives.

The WFNMC has an unquestionably vital and pertinent mission, as an inclusive organization designed to support groups in countries and regions, each with its particular perspective and varying degrees of experience and tradition, to move forward in their work of exploring how young students can grow, assimilate and contribute new ideas to ever expanding areas of elementary mathematics, on many different levels of difficulty requiring what is for each student a personally new line of thought. We are all aware of how meeting with more challenging mathematics can transform individual lives, making analytical and critical thought meaningful, and giving form to a context for original ideas and creative solutions.

Yet with time it becomes ever clearer that mathematics education itself is in fact at a crossroads; and it is our belief that important decisions must be made regarding new directions that open up more challenging opportunities in mathematics and mathematical problem solving for ALL students.[1]

We cite three cogent reasons for so doing.

Being true to the nature and evolution of mathematics

As we conceive what school mathematics can and should be, we must bear in mind that starting in the first half of the nineteenth century, mathematics itself took a new direction in its evolution that has not ceased, and this aspect must be kept clearly focused: mathematics is a creation of the human mind and as a creation has no limits. Elementary mathematics and its subset, school mathematics, is a fertile field in which new relationships, results and problems are constantly being explored and analyzed. When we speak of giving all students access to more challenging mathematics in school we mean giving students the opportunity to look at mathematical concepts and ideas from fresh perspectives, solve problems that have elements that make them different from problems already encountered and practiced, develop their mathematical thinking, see mathematics as fun, be as original and creative in the mathematics classroom as they are in all their other activities.

Changes in the way mathematics itself is done: taking seriously the implications of the computer in mathematics and in math education

Reflecting on the profound revolution that the math curriculum must experience given the changes in the demands we face when confronting algorithmizable mathematics, we find that with the calculator and the computer, there is no need to practice algorithms to satiety in order not to make mistakes. It *is* certainly necessary to construct appropriate meaning for mathematical concepts, understand algorithms (for carrying out operations, solving equations, identifying errors, etc.) and know why

[1] This and related issues are treated thoroughly by *ICMI Study 16: Challenging Mathematics in and beyond the Classroom*, edited by Ed Barbeau and Peter Taylor and published by Springer in January, 2009.

they function, but none of this is an end in itself, but rather a means for solving problems that range from the purely imaginative to the most bound to reality in applying mathematics, and can be practiced and perfected in the context of solving problems.

In discussions analyzing the reasons why in many countries there has been a notable decline in the number of students going into careers in mathematics and science, we believe that the true reasons lie within our own profession as mathematics teachers and educators, that is, the lack of relevance of what we are teaching and the way we are teaching it to a student of the twenty-first century who knows full well that a calculator will do her/his arithmetic and a computer her/his algebra or basic calculus instantly and without error. These can only be relevant as tools to much more profound learning experiences which require them but for which they do not suffice, such as problem solving or investigating and discovering new mathematical facts and relationships.

We must provide an answer to the demands of a knowledge-based society

Quoting Alvin Toffler in a recent interview published in *El Tiempo* (Bogotá, March, 2009) we find an opinion that is congruent with many trends and policies throughout the world as well as our own. Says Toffler

> No more education for the masses.
> [It is necessary] to eliminate all the educational systems that prepare young people to work in industrialized models ... *and also* prepare them for yesterday instead of tomorrow.

Not only is a "formation" in mathematics needed in order for a person to get a decent job or to be able to do all that she/he can and should as a professional, it is also necessary to survive and to protect one's personal assets. Today's citizen has to deal with actuarial designs behind money management scams that are getting more and more difficult to understand and evaluate. Many are mathematical models that are irresponsible and our citizens have to be able to understand what they are committing to.

What responsibilities does a society have to prepare its citizens not merely to survive but to take advantage of all of the positive things a knowledge-based society has to offer?

In Riga we hope to discuss the possibility of subscribing a document that makes clear our beliefs regarding the need to provide opportunities for all students to face challenges in mathematics, our commitments, and to map out a plan to put them into action.

Just a few days ago, the gigantic large hadron collider built near Geneva began to show the first experimental collisions, and the Atlas project, twenty years in the making, is now a reality that links some 160 universities and research centers from 40 countries around the globe in an effort that can be described as reproducing the conditions, as those of the Big Bang theory, of the conversion of energy into matter. This monumental project, requiring international collaboration to be viable, can be thought of as a symbol that reminds us of other international scenarios that have the potential to change fundamentally the development of a science.

Although far removed from the special and deliberate costs related to building the experimental resources required by a project such as Atlas, the work of raising mathematical expectations for all young people is enormous, demanding nothing less than a change of mentality, alongside careful selection and structuring of experiences, problems, puzzles, as well as making these widely available. Almost certainly our vehicle will be the Internet, that will prove to be an important ally in reaching young people from every country and region; our task to compete with everything from the latest popular music to the latest video game—a true challenge in itself. But we are confident it will be possible to learn to use this fantastic means of communication to reach teachers and students alike with a wide range of fascinating material, carefully packaged, and to encourage them to think of mathematics in a new and truer light, as an opportunity to have intellectual fun that will make a difference in the way they face and analyze almost every aspect of their lives, the choices they make, the challenges they attempt. There is much work to be done, but we already have the tools and the meeting in Riga will test our will to do it.

María Falk de Losada
President of WFNMC
Bogotá, December 2009

From the Editor

Welcome to *Mathematics Competitions* Vol. 22, No. 2.

Again I would like to thank the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:
The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

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Jaroslav Švrček,
December 2009

WFNMC 6th Congress
First Announcement

Dear colleagues,
With this we announce that the 6th Congress of World Federation of National Mathematics Competitions (WFNMC) will be held in **Riga, Latvia**, **from 25 July, 2010** (arrival) **till 31 July, 2010** (departure). Two previous congresses were held in Cambridge, UK (2006) and Melbourne, Australia (2002).

The work of the Congress will be supervised by Prof. Maria Falk de Losada, the President of WFNMC, and International Program Committee (co-chairs Prof. Alexander Soifer, University of Colorado, and Prof. Agnis Andžāns, University of Latvia).

The congress will be organized at the University of Latvia, in its central building (Rainis boul. 19) and at the campus of Faculty of Physics and Mathematics (Zellu Street 8).

1 Scientific Content

The work of the congress will concentrate around 4 central topics:

1. A bridge between research mathematics / theoretical computer science and competitions.
2. Competition problems and methods of solution.
3. Type and organization of the competitions.
4. Preparation of students and teachers.

We see broad areas where the suggestions of people from computer science could be valuable and thought-provoking, e.g. can we use problems the solutions of which employ databases?, maybe a fifth area "algorithmics" along with algebra, geometry, combinatorics and number theory should be introduced?, etc.

In the area of each topic, short communications, plenary lectures and workshops will be organized.

If the International community will send in enough proposals concentrating around some other topic, it also can be included into the agenda.

Each possible participant of the congress is kindly invited to do at least one of the following.

1. Send in an abstract (up to 1 page, format A4) of a communication (30 minutes together with a discussion) by **1st March, 2010**.
2. Send in a proposal for workshops, indicating topic, short description, organizers, expected number of participants (up to 2 hours) by **1st March, 2010**.
3. Send in a proposal for an exhibition, indicating name, kind and number of items displayed etc by **1st March, 2010**.

   The proposals will be reviewed by the IPC and authors will be informed about the decision by 15th March, 2010.

   In the case of acceptance, the author(s) will be asked to submit a full paper. The length of the paper will depend on the decision of IPC.
4. Send in up to 3 original contest problems with the solutions and analysis why this particular problem seems to the author to be interesting/useful/stimulating etc. by **1st March, 2010**.

   The IPC intends to prepare a congress problem book and publish it, possibly after the congress.

All communication concerning proposals should be sent by e-mail to the address **nms@lu.lv**, indicating "WFNMC Abstract proposal", "WFNMC Problem proposal", etc. in the Subject line.

2 About Riga and Latvia: Practical Information

Latvia and its capital Riga are situated on the east coast of Baltic Sea. Its neighbours are Estonia (north), Lithuania (south), Russia and Belarus (east).

Riga can be reached easily by air from many European cities. Among other possibilities, you can come from Vilnius (Lithuania) by bus or from Helsinki by a boat to Tallin (Estonia) and then by bus to Riga.

In July, the temperature is usually between 18 and 25 degrees Celsius and not rainy (although there can be exceptions). The temperature of the water in Riga seaside (30 minutes by train from the centre of the city) is from 16 to 18 degrees Celsius.

In the summer there are usually a lot of concerts, art exhibitions etc. in Riga. You can enjoy also many museums.

3 Registration

The registration to the congress can be done through the congress web site **http://nms.lu.lv/WFNMC**

The fee is **500 Euro not including accommodation** but including

1. the congress problem/solution book (additional to previous congresses),
2. the abstract book (additional to previous congresses),
3. publishing the proceedings (additional to previous congresses),
4. postal expenses for sending the proceedings and the problem/solution book to the participants,
5. refreshments during the congress sessions,
6. 5 lunches,
7. 5 dinners,
8. 2 excursions on buses (one maybe on a ship),
9. a concert,
10. a free ticket for a week on all buses, trams, and trolleys in Riga.

4 Important!

Immediately after the congress the **6th International Conference on Creativity in Mathematics Education and the Education of Gifted Students** (MCG) will start in Riga (arrival 1st August, departure 7th August). Many of the participants of the congress also might be interested in this conference.

See conference web-page at http://nms.lu.lv/MCG

5 Further Information

The congress web-page is at http://nms.lu.lv/WFNMC

It will be constantly upgraded. Also detailed information about the registration/ payment procedure can be found there.

For special questions, please, write to nms@lu.lv, indicating "WFNMC" in the subject line.

We are looking forward to see all of you in Riga!

Agnis Andžāns,
Chair of the organizing committee

A Short History of the World Federation of National Mathematics Competitions

(In connection with the 25th anniversary of the organization)

Petar S. Kenderov

1 The World of Mathematics Competitions

A mathematics competition for primary school students was held in Bucharest, Romania, as early as 1885[1]. There were 70 participants and eleven prizes, awarded to two girls and nine boys. One cannot completely rule out the possibility that similar competitions were held elsewhere even before 1885. Nevertheless, the Eötvös competition in Hungary (held in 1894) is widely credited as the forerunner of contemporary mathematics (and physics) competitions for secondary school students. The competitors were given four hours to solve three problems (no interaction with other students or teachers was allowed). The problems in the Eötvös competition were designed to check creativity and mathematical thinking, not just acquired technical skills. In particular, the students were asked to provide a proof of a statement. The Eötvös competition model is now widely spread and still dominates a large portion of competition scene.

The year 1894 is notable also for the birth of the famous math journal *KöMal* (an acronym of the Hungarian name of the journal, which translates to "High School Mathematics and Physics Journal"). It was founded by Dániel Arany, a high school teacher in Györ, Hungary. The journal was essential in the preparation of students and teachers for competitions. Readers were asked to send solutions to problems published in the journal. As noted by G.Berzsenyi[2], about 120–150 problems were published in *KöMal* each year. The response was

[1] Berinde, V., Romania—The Native Country of International Mathematical Olympiads. A brief history of Romanian Mathematical Society. CUB PRESS 22, 2004.

[2] Century 2 of *KöMal*, ed. by Vera Oláh (editor), G.Berzsenyi, E. Fried and K. Fried (assoc. editors)) OOK-PRESS, Veszprém, Hungary.

impressive: about 2500–3000 solutions were received yearly. The best solutions and the names of their authors were published in following issues of the journal. This type of year-round competition helped many young people discover and develop their mathematical abilities. Many of them later became world-famous scientists (for more information in this direction, see the Web-pages of *KöMal*[3]).

About the same time, similar development occurred in Romania. The first issue of the monthly *Gazeta Matematică* was published in September 1895. The journal organized a competition for school students, which improved in format over the years and eventually gave birth to the very effective contemporary national system of competitions in Romania. Soon other countries started to organize mathematics competitions. In 1934, a Mathematical Olympiad (with this name) was organized in Leningrad, USSR (now St. Petersburg).

To compete means to compare your abilities with the abilities of others. The broader the base of comparison, the better. This seems to be the motivation for the natural transition from school competitions to town competitions, to national and finally, to international competitions. In 1959 the flagship of mathematics competitions, the International Mathematics Olympiad (IMO), was born. It took place in Romania with participants from seven countries: Bulgaria, Czechoslovakia, German Democratic Republic, Hungary, Poland, Romania, and the Soviet Union (USSR). The second IMO (1960) was organized by Romania as well, but since then it has been hosted by a different country every year (except 1980, when no IMO was held). Over the years, the participation grew dramatically: the 2008 IMO in Spain gathered 537 competitors from 99 countries. Similar was the participation in IMO in Vietnam, 2007: 94 countries with 526 school students. Nowadays this is the most prestigious mathematics competition. Directly or indirectly, it influences all other enrichment activities in mathematics. With its high standards, the IMO prompts the participating countries to constantly improve their educational systems and their methods for selecting and preparing students. This, over the years, yielded a great variety of competitions and mathematical enrichment activities around the world. There are "Inclusive" (open for all) competitions which are intended for

[3]http://www.komal.hu/info/bemutatkozas.e.shtml

students of average abilities, while "exclusive" (by invitation only) events target talented students. There are "multiple-choice" competitions where each problem is supplied with several answers, from which the competitor has to find the correct one. In contrast, "classical style" competitions (like the IMO) require the students to present arguments (proofs) in written form. In "correspondence" competitions, such as those organized by journals *KöMal* and *Gazeta Matematică* or the contemporary "Tournament of Towns", the students do not necessarily meet each other, while in "presence" competitions the participants are working on the solution of problems in the presence of other competitors. There are even mixed-style competitions, with a presence-style first stage and correspondence-style subsequent stages. The majority of competitions are "individual", what counts finally is the score of the individual participant. There are many competitions however where the result of the whole team is what matters. Competitions may differ also by participants' age (for primary school students, for secondary school students, for students in colleges and/or universities) as well as by participants' affiliation: from one school, from several schools or from all the schools in a town, nation wide competitions, international competitions, etc. Nevertheless, there are many other competitions or competition-like events which completely "escape" such "classification attempts" and essentially enrich the variety of measures oriented toward identification, motivation and development of mathematical talent worldwide.

The world of mathematics competitions today embraces millions of students, teachers, research mathematicians, educators, publishers, parents, etc. Hundreds of competitions and competition-like events with national, regional, and international importance are organized every year. A remarkable international cooperation and collaboration gradually emerged in this field. How the system works could be seen from the following story. The Australian Mathematics Competition[4] (AMC) was started in 1978 with the intention to transfer to Australian soil the positive impact of Canadian Mathematics Competition[5]. However, soon the AMC reached half a million participants. It became much larger than the Canadian Mathematics Competition. In turn, the European

[4] http://www.amt.edu.au/amcfact.html
[5] http://cemc.uwaterloo.ca

competition "Kangaurou des Mathématique"[6] (modelled, as the name suggests, after the AMC), which started in 1991, started in 1991 and in 2009 had more than five million students from different countries participating.

It would not be an exaggeration to say that the rise and development of Mathematics Competitions is among the characteristic phenomena of the 20th century. The World Federation of National Mathematics Competitions (WFNMC) was a natural response to the need of international collaboration in this area. It is also a tool to enhance this international collaboration.

2 WFNMC in dates

The World Federation of National Mathematics Competitions was founded in 1984 through the inspiration of Professor Peter O'Halloran (1931–1994) from Australia. The Fifth International Congress on Mathematical Education (ICME 5), held in that year in Adelaide, Australia, had a section on Mathematics Competitions. At one of the sessions of this section, chaired by Peter O'Halloran, the creation of an international organization related to Mathematics Competitions was discussed. The response was very positive. A Committee was elected with the mandate to develop the Federation. Peter O'Halloran became the founding President of the Federation. Here is what Ron Dunkley (one of the Presidents of WFNMC after Peter O'Halloran) wrote in Bulletin No 40[7] of ICMI (the International Commission on Mathematical Instruction) about the first days of the Federation: "*. . . While others assisted in the formation, it was the vision and leadership of Professor Peter O'Halloran of Canberra, Australia, that led directly to the Federation's being*". Professor Peter James Taylor, another Past President of WFNMC, wrote in ICMI Bulletin No 49[8]: "*The founder of WFNMC was Peter O'Halloran, who was President until his death in 1994. He conceived the idea of such an organization in which mathematicians from different countries could compare their experiences and hopefully improve their activities as a result.*" Peter O'Halloran's

[6] http://www.mathkang.org/
[7] http://www.mathunion.org/ICMI/bulletin/40/WFNMC-Report.html
[8] http://www.mathunion.org/ICMI/bulletin/49/Report_WFNMC.html

own reflection on the early days of the Federation is summarized in his "From the President" article in the Journal *Mathematics Competition*[9] : "*In the early 1980s as Executive Director of the Australian Mathematics Competition, I was constantly receiving requests for information and guidance in organizing mathematics competitions from all parts of the world and, in particular, developing countries. I realized that there was a need for an international organization to exchange ideas and information on mathematics competitions as well as to give encouragement to those mathematicians and teachers who are involved with the competitions. Consequently my colleagues in Canberra and my friends at Waterloo and Nebraska launched the Federation at ICME 5 in Adelaide, Australia, in 1984.*"

A significant part of what are now the major activities of the Federation were initiated by Professor O'Halloran himself or during his Presidency. He felt the importance of communications and, in 1985, the publication of *Newsletter of WFNMC* was started. From 1988 it became a journal and got the name *Mathematics Competitions*. During his Presidency the Federation started its own series of Conferences which are conducted every four years (just in the middle between two consecutive ICMEs). The first Conference took place in Waterloo, Canada, from August 16th to August 21st, 1990. It was organized by Ronald Garth Dunkley and his colleagues from the Canadian Mathematics Competition Committee. The next conferences were in:

- Bulgaria (July 23–28 ,1994), organized by Petar S. Kenderov and his colleagues from the Union of Bulgarian Mathematicians and the International Foundation "St. St. Cyril and Methodius";
- Zhong Shan, P. R. China (July 22–27, 1998), with Qiu Zonghu as Chairman, assisted by Pak-Hong Cheung, Andy Liu and Wen-Hsien Sun;
- Melbourne, Australia (August 4–11, 2002), with Peter Taylor as principal organizer assisted by Warren Atkins, Sally Bakker and John Dowsey;
- Cambridge, England (July 22–28, 2006), with Tony Gardiner, Adam McBride, Bill Richardson and Howard Groves as principal organizers.

[9] *Mathematics Competition*, Vol.3 No. 2, December 1990, Page 2

The next one is to take place in Riga, Latvia, in 2010.

The regular meetings of the Federation during ICMEs (Budapest 1988, Quebec 1992, Seville 1996, Tokyo 2000, Copenhagen 2004, Monterrey 2008) also contribute significantly to the well-being of the organization. In particular, this is the time when the so called "Business meetings" of the Federation are conducted at which organizational issues are discussed and Federation officers for the next 4-year period are determined.

During the Presidency of Peter O'Halloran, the Awards of the Federation were established—"David Hilbert Award" (1990) and "Paul Erdös Award" (1991). The awards are intended to recognize people with significant contributions and achievements in developing Mathematical Competitions and Mathematical Enrichment Programs in general.

In 1992 a *World Compendium of Mathematics Competitions* was published containing information about 230 mathematics competitions around the world. Also since 1992 WFNMC has been "under the umbrella" of the Australian Mathematics Trust (AMT). The support of AMT is of greatest importance for the existence of the Federation.

Further important impulse for the activities of WFNMC and a recognition for what it does for mathematics education came in 1994 when, upon the initiative of Peter O'Halloran, the Federation became the forth Affiliated Study Group of ICMI.

At the "Business meeting" of the Federation during the Conference in Bulgaria in 1994 it was decided that Professor Blagovest Sendov from the Bulgarian Academy of Sciences would inherit the Presidency of WFNMC from Peter O'Halloran in 1996. Soon after the conference, Peter O'Halloran realized he had severe health problems. Before his death on 25 September 1994, Peter O'Halloran passed the leadership to Blagovest Sendov. However, the circumstances in Bulgaria required that Sendov enter the political life. He was elected in the National Parliament and became its Chairman. In early 1996 he resigned from the Federation's Presidency. The latter was passed to Ron Dunkley from the University of Waterloo, Canada, who had been Vice-President of the Federation since its inception. Under his leadership a Constitution[10]

[10] http://www.olympiad.org/wfnmccon96.html

of WFNMC was approved during ICME-8 in Seville, Spain. Ron Dunkley was President of WFNMC till 2000. At ICMI-9 in Tokyo, Japan, Professor Peter Taylor was elected President of WFNMC. Under his guidance a "Policy Document: Competitions and Mathematics Education"[11] was adopted on August 10th, 2002, at the Federation's business meeting in Melbourne, Australia. Peter Taylor initiated also an amendment of the Constitution of WFNMC which limited the presidential term to four years. It was adopted at the administrative meeting of WFNMC during ICME-10 in Copenhagen, 2004. At the same meeting Professor Petar S. Kenderov from the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, was elected President of WFNMC. Another amendment of the Constitution was adopted at the administrative meeting during ICME-11 in Monterrey, Mexico. At that meeting María Falk de Losada from Antonio Narino University, Bogota, Colombia, was elected President of the Federation for the term 2008–2012.

3 Goals of WFNMC

The name of the Federation leaves the impression that its major goals are related to competitions only. To some extent, this may have been the case in the earlier stages of development of the Federation when, for example, on page 2 of Vol. 1, No 1, of the journal *Mathematics Competitions* one can find the statement: "*The foundation members of the Federation hope that it will provide a focal point for people interested in, and concerned with, running national mathematics competitions; that it will become a resource centre for exchanging information and ideas on national competitions; and that it will create and cement professional links between mathematicians around the world.*"

In later issues of the *Mathematics Competitions* (again on the beginning pages) one can trace the evolution of the vision for Federation's goals toward improving mathematics education in general. The official viewpoint is now expressed in the preamble of the Federation's Constitution[12]: "*The World Federation of National Mathematics Competitions is a voluntary organization, created through the inspiration of Professor*

[11] http://www.olympiad.org/wfnmcpol02.html
[12] http://www.olympiad.org/wfnmccon04.html

Peter O'Halloran of Australia, that aims to promote excellence in mathematics education and to provide those persons interested in promoting mathematics education through mathematics contests an opportunity of meeting and exchanging information."

Further, in Article 3 of the Constitution we see:

"*The aims of the Federation are:*

(i) *to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;*

(ii) *to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;*

(iii) *to provide opportunities for the exchanging of information in mathematics education through published material, notably through the Journal of the Federation;*

(iv) *to recognize through the WFNMC Awards system, persons who have made notable contributions to mathematics education through mathematical challenge around the world;*

(v) *to organize assistance provided by countries with developed systems for competitions in countries attempting to develop competitions;*

(vi) *to promote mathematics and to encourage young mathematicians.*"

The wider viewpoint on the goals of the Federation is outlined also in the Policy Statement[13] mentioned above: "*The scope of activities of interest to the WFNMC, although centered on competitions for students of all levels (primary, secondary and tertiary), is much broader than the competitions themselves. The WFNMC aims to provide a vehicle for educators to exchange information on a number of activities related to mathematics and mathematics learning. These activities include*

- *Mathematical competitions of various kinds*
- *Mathematical aspects of problem creation and solution, a dynamic branch of mathematics.*
- *Research in mathematics education related or pertaining to competitions or the other types of activities listed here.*

[13] http://www.olympiad.org/wfnmcpol02.html

- *Enrichment courses and activities in mathematics.*
- *Mathematics Clubs or "Circles".*
- *Mathematics Days.*
- *Mathematics Camps, including live-in programs in which students solve open-ended or research-style problems over a period of days.*
- *Publication of Journals for students and teachers containing problem sections, book reviews, review articles on historic and contemporary issues in mathematics.*
- *Support for teachers who desire and/or require extra resources in dealing with talented students.*
- *Support for teachers, schools, regions and countries who desire to develop local, regional and national competitions.*

With qualification, WFNMC also facilitates communication through its Journal and Conferences, in the following areas

- *Topics in informatics parallel to those in mathematics. This applies particularly in that no equivalent body exists for informatics. It takes into account that the disciplines are closely related, that many journals cover both topics, and that in many countries the organisation of competitions in mathematics and informatics, and mathematics and informatics themselves, are closely related.*
- *Recreational mathematics, including mathematical puzzles, particularly as they might inspire the creation of mathematics problems.*

WFNMC is concerned with activities particularly when they have international significance or are significant within their own country."

Further information about the role of competitions for mathematics education, for attracting talent to science, for educational institutions and for the whole society, is contained in a paper presented at one of the sessions of Section 19 ("*Mathematics Education and Popularization of Mathematics*") at the International Congress of Mathematicians in Madrid, 2006[14].

[14] Petar S. Kenderov, Proceedings of the International Congress of Mathematicians, Madrid, Spain, 2006, p.1583–1598

4 The Essence and the Role of WFNMC

Like any other event with positive social impact, each competition or competition-like event generates a group of people dedicated to it. The group consists of team trainers, problem creators, organizers, and other people involved. Taken together, this group maintains and gives the shape of the event. It determines the current status and the future development of the event. This joint obligation (to keep the event floating) serves as a cohesive factor that gradually transforms the group (of sometimes potential rivals) into a vibrant network where collaboration prevails over rivalry. Such networks have a great "value-added" effect. Learning from others becomes a major source for improvement of own work. Unlike electrical networks in physics, where energy is conserved and where nodes with higher potential lose part of their potential to nodes with lower potential, mathematics competition networks tend to increase the potential of all the "nodes" involved and increase the "energy" in the group. Typical examples of such networks are those associated with the following competitions:

- the IMO,
- Le Kangourou Sans Frontières [www.mathkang.org],
- the Australian Mathematics Competition,
- the International Mathematics Tournament of Towns[15],
- the Ibero-American Mathematics Olympiad[16],
- the Asian-Pacific Mathematics Olympiad[17],

The list is far too long to enumerate all networks that deserve to be mentioned here.

A good mathematics competition journal also creates such a network, which comprises the editorial board, the editors, the frequent authors, and readers. Famous examples are these journals:

- *Kvant* (Russia)[18],
- *Crux Mathematicorum* (Canada)[19],

[15] http://www.amt.edu.au/imtot.html
[16] http://www.oei.es/oim/index.html
[17] http://www.math.ca/Competitions/APMO/
[18] http://kvant.info/
[19] http://journals.cms.math.ca/CRUX/

- *Mathematics Magazine* (USA)[20],
- *Mathematical Gazette* (UK)[21].

The countless other forms of mathematics enrichment also create networks. All these networks operate autonomously and independently from each other though many of the problems they face are similar in nature. Advancements in one network are not easily transferred to other networks. This is where the role of WFNMC is clearly seen:

- to facilitate communication among the different networks,
- to identify common problems faced by different networks,
- to provide a proper framework for discussion of those problems,
- to help newcomers join one (or more) of the networks.

As a matter of fact, some competition networks are connected to each other because they have common members (people who belong to two or more networks). Such people are of special interest to WFNMC because they, on one hand, know the situation in some networks and, on the other, can directly realize the goals of the Federation in the respective networks. Through them, the role of the Federation becomes feasible. Therefore, the essence of WFNMC is a "Global Network of Networks" which we further refer to as "Competitions Network" though it does not only include competitions.

This global Competitions Network resembles existing networks in other mathematical areas, such as Algebra, Geometry, Analysis, Differential Equations, Numerical Methods, etc. In fact, the Competitions Network covers the classical mathematical area known under the (somewhat misleading) name "Elementary Mathematics". Like other networks, this one operates and lives through its journals, conferences, workshops and e-mail. Periodical regularity of mathematics competition however adds to the strength and vitality of mathematics competitions networks since people meet more often. Unlike other networks which are engaged mainly with research, the Competitions Network also facilitates the dissemination of best practices in curriculum development and in the

[20] http://www.mathematicsmagazine.com/

[21] http://www.m-a.org.uk/resources/periodicals/the_mathematical_gazette/index.html

work with talented youngsters. New problem-solving techniques, new classes of problems, and new ideas about organizing competitions spread quickly around the world. We should not forget also that, through this global network, the Elementary Mathematics (which constitutes an important part of our mathematical heritage) is preserved, kept alive and further developed.

Since WFNMC is an Affiliated Study Group of ICMI which, in turn, is a Commission of the International Mathematical Union, the Competitions Network behind WFNMC is integrated into the global mathematical community.

The WFNMC provides also a framework and a fruitful environment for the discussion of important issues related to mathematics education, to the work with higher ability students and, last but not least, to its own future.

5 Structure and Activities of WFNMC

The structure of the Federation as well as its current activities have evolved as a result of a long and gradual development which, in some aspects reached its steady state. According to the Constitution (as amended in 2008), the **Executive Committee** of the Federation consist of: President, three Vice-Presidents (one of whom is a Senior Vice-President), Secretary, Immediate Past President (chairing the Awards Committee), Publication Officer (Editor of *Mathematics Competitions*) and Treasurer. There are also three Standing Committees: **The Program Committee** (responsible for the development of programs for Federation conferences and chaired by the Senior Vice-President), the **Awards Committee** (receives and assesses nominations for Federation awards, chaired usually by the Past President), the **Committee of Regional Representatives** (responsible for the implementation of Federation programs in the various regions of the world; currently there are regional representatives from Africa, Asia, Europe, North America, Oceania and South America). The names of the people currently occupying the mentioned positions (for the term 2008–2012) as well as a lot of other information related to WFNMC can be seen in the website of the Federation[22].

The major activities through which the Federation achieves its goals are:

- Publication of *Mathematics Competitions* Journal;
- Conducting Conferences and Meetings during ICMEs;
- Presentation of Federation Awards;
- Participation in projects initiated and supported by other organizations.

The Journal. Since its very beginning (as *Newsletter of WFNMC*), *Mathematics Competitions*[23] journal has been playing a special role in the life of the Federation. It publishes materials concerning all aspects of competitions and other related activities: problem-solving, problem creation, pieces of interesting mathematics, know-how on organizing competitions, statistical studies on competition results, gender issues, etc. This way it disseminates new and fruitful ideas coming from different parts of the world.

The journal also records the life of the Federation. It is published by the Australian Mathematics Trust (AMT) on behalf of WFNMC. AMT also delivers the Journal free of charge to people from countries that cannot afford a subscription of the Journal. Warren Atkins was Editor of this Journal from its beginning (1985) till the business meeting of WFNMC during ICME 10 in Copenhagen (2004) where, upon his request, the role of Editor of *Mathematics Competitions* was passed to Jaroslav Švrcek from Palacký University in Olomouc, Czech Republic. Over the years the Editor of the Journal was helped in his work by different persons. Here is an (incomplete) list of names: George Berzsenyi, Heather Sommariva, Richard Bollard, Andrei Storozhev, Gareth Griffith, Bruce Henry.

Conferences and Meetings. An irreplaceable role for the Federation is played by its conferences and the meetings during ICMEs. Both events allow the membership of the Federation to meet every two years. The conferences of the Federation have a special flavor. For instance, there are sessions devoted to problem-solving, problem setting and

[22]http://www.amt.edu.au/wfnmc.html
[23]http://www.amt.edu.au/wfnmc.html

problem improvement where participants work together. Participants are frequently asked to share a favourite problem. Conferences are accompanied by real competitions, sometimes involving the conference participants as well. Among key-note speakers at those Conferences one meets the names of: Paul Erdös, John Conway, Ben Green, Robin Wilson, Kaye Stacey, Anne Street, Jozsef Pelikan, Alexander Soifer, Maria Falk de Losada, André and Jean-Christophe Deledicq, Andy Liu, Simon Singh and many others. In this connection again the special role of the Australian Mathematics Trust should be underlined. Due to its support, persons from non-affluent countries were able to participate in the conferences of the Federation.

Federation's Awards. The Federation has created two international awards – David Hilbert Award and Paul Erdös Award. Both awards are to recognize the contribution of people toward development of mathematics competitions and mathematics enrichment activities in their own countries or internationally. The awards are named after the famous mathematicians David Hilbert and Paul Erdös whose work was a challenge and inspiration for generations of mathematicians. Since 1996 the Hilbert Award has not been awarded. The two awards have been merged and now the Federation has only the Paul Erdös Award. Every two years up to three persons receive this award upon decision of the Federation Executive Committee based on the Awards Committee recommendations. Listed below are the names of the Federation's Awards recipients.

David Hilbert Award:

1991: Arthur Engel (Germany), Edward Barbeau (Canada), Graham Pollard (Australia);

1992: Martin Gardner (USA), Murray Klamkin (Canada), Marcin E. Kuczma (Poland);

1994: Maria Falk de Losada (Colombia), Peter Joseph O'Halloran (Australia);

1996: Andy Liu (Canada).

Paul Erdös Award:

1992: Luis Davidson (Cuba), Nikolay Konstantinov (Russia), John Webb (South Africa);

1994: Ronald Garth Dunkley (Canada), Walter Mientka (USA), Urgengtserengiin Sanjmyatav (Mongolia), Jordan Tabov (Bulgaria), Peter James Taylor (Australia), Qiu Zonghu (P. R. China);

1996: George Berzsenyi (USA), Tony Gardiner (UK), Derek Holton (New Zealand);

1998: Agnis Andzans (Latvia), Wolfgang Engel (Germany), Mark Saul (USA);

2000: Francisco Bellot Rosado (Spain), Istvan Reiman (Hungary), János Surányi (Hungary);

2002: Bogoljub Marinkovic (Yugoslavia), Harold Braun Reiter (USA), Wen-Hsien Sun (Taiwan);

2004: Warren Atkins (Australia), André Deledicq (France), Patricia Fauring (Argentina);

2006: Simon Chua (Philippines), Ali Rejali (Iran), Alexander Soifer (USA);

2008: Shian Leou (Taiwan), Hans-Dietrich Gronau (Germany), Bruce Henry (Australia).

Participation in other organizations' projects

Typical examples are mentioned here in order to illustrate what is meant.

Members of WFNMC are engaged in organizing and conducting various discussion or topic study groups of ICMEs devoted to the role of competitions in mathematics education. The reference[24] is a good example.

The Federation was a key player in the ICMI Study 16 *Challenging Mathematics in and beyond the classroom*[25]. It was finalized in 2009 and

[24] http://www.amt.edu.au/icme10dg16.html
[25] http://www.amt.edu.au/icmis16.html

the results are published in *New ICMI Study Series*, Vol. 12, Barbeau, Edward J.; Taylor, Peter J. (Eds.), 2009, V, 325 p. 5 illus., ISBN: 978-0-387-09602-5. The progress of the work is reflected in the reference[26].

Several members of WFNMC participated in the development of Project *MATHEU*[27], which was carried out with the support of the European Community within the framework of the Socrates Programme. The outcomes of *MATHEU* Project are oriented toward the creation of a challenging environment which students of higher ability in European schools will be identified, motivated and supported.

6 People involved with WFNMC

Here are the names of the persons who had duties (as officers) with WFNMC:

Presidents of WFNMC

Peter Joseph O'Halloran (Australia), Blagovest Sendov (Bulgaria), Ronald Garth Dunkley (Canada), Peter James Taylor (Australia), Petar Stoyanov Kenderov (Bulgaria), María Falk de Losada (Colombia).

Vice-Presidents

Ronald Garth Dunkley (Canada), Walter Mientka (USA), Pierre-Olivier Legrand (French Polynesia), Matti Lehtinen (Finland), Petar Stoyanov Kenderov (Bulgaria), Anthony David Gardiner (UK), Maria Falk de Losada (Colombia), Peter Crippin (Canada), Alexander Soifer (USA), Robert Geretschläger (Austria), Ali Rejali (Iran).

Secretaries of WFNMC

Sally Bakker (Australia), Sandra Britton (Australia), Alexander Soifer (USA), Kiril Bankov (Bulgaria)

[26] http://www.springer.com/education/mathematics+education/book/978-0-387-09602-5

[27] http://www.matheu.org/

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The Nordic Mathematics Competition

Matti Lehtinen

Matti Lehtinen was born in Helsinki, Finland, in 1947. He obtained his Ph.D. at the University of Helsinki in 1975 where he worked until 1987. He then worked at the Finnish National Defence University from which he retired in 2009. He occasionally teaches math history courses at the Universities of Helsinki and Oulu. He has trained Finnish IMO teams since 1973. He is currently editor-in-chief of the Finnish mathematical web and paper journal Solmu.

1 The Nordic Countries

The five Nordic countries, Denmark, Finland, Iceland, Norway and Sweden are a group which shares a number of geographical and cultural properties. The majority of their surface lies between the same latitudes as that of Alaska. All are relatively small in terms of population, ranging from 9 million for Sweden to one third of a million for Iceland. Situated in the periphery of Europe, they assumed Western civilization between the ninth and twelfth centuries. They were united under the same crown from the 14th to the early 16th century. They are now all independent countries with Iceland being the last when it gained independence from Denmark in 1944. Their societies are stable and the level of welfare high. At present, Denmark, Finland and Sweden are members of the European Union and Iceland is applying for membership. The Nordic Council has existed from 1952 as an organization connecting the legislative assemblies of the five countries. The Council has various sub-organsations coordinating the member countries' activities e.g. in culture and education.

Culturally, the countries share a common basis in Lutheran protestantism and Scandinavian law. Linguistically the group is rather homogenous, too. The Icelandic language preserves a more antiquated form of the Scandinavian language which itself belongs to the German group of languages. Norwegian has inherited much from Danish, among others one of the two ways of writing the language. Swedish, besides being the language of Sweden, is also the second official language of Finland, but spoken by a dwindling minority of the population there (now around 5 %). The Finnish language is totally different: it belongs to the Finno-Ugric group of languages and is related only to Estonian, Hungarian, Sami and a number of small languages spoken in Russia. In Finland, government employees are generally required to understand Swedish. So in general, one supposes that communication between the Nordic countries is possible by some variant of "Scandinavian". The Nordic Council does not accept English as a working language but the youth of all five countries usually communicate in English.

2 Mathematics in the Nordic Countries

The Nordic countries have produced a number of important mathematicians. Probably the earliest name to achieve an international reputation is that of Georg Mohr of Denmark, the first mathematician to explain how Euclidean constructions can be performed without the use of a straightedge. Mohr's discovery, made in the early 17th century, unfortunately was forgotten until the early 20th century. The most prestigious mathematician coming from a northern country is undoubtedly Niels Henrik Abel, the Norwegian whose achievements include proving the impossibility of solving the fifth degree equation in the early 19th century. The Swedish mathematician Gösta Mittag-Leffler played a central role in the early stages of the international organization in mathematics around 1900. In more modern times, the list of Field's medalists includes Lars V. Ahlfors of Finland, Atle Selberg of Norway and Lars Hörmander of Sweden.

The level of education in Nordic countries is considered to be high. Basically this means that the education is uniform. On the other hand, the countries do not have a tradition of elite schools with high standards. For instance, the much advertized achievements of Finland in the OECD PISA study do not reflect any excellence in teaching mathematics proper.

3 Mathematics competitions in the Nordic Countries

Each country runs its own math competitions. In Sweden, the competition is called The School Mathematics Contest and is run by The Swedish Mathematical Society. The Danish competition is named after Georg Mohr and it is arranged by a group supported by the Danish Mathematical Society and the Danish Association of Mathematics Teachers. In Norway, the competition again gets its name from Abel and is arranged by the Norwegian Mathematical Society. In Finland, the competition is called the High School Mathematics Contest and is run by the Association of Teachers of Mathematics, Physics and Chemistry in concurrence with competitions in physics and chemistry. The Icelandic competition is run by the Icelandic Mathematical Society. The Abel Competition has three rounds, the others two. Their level of difficulty is well below the level of the IMO or national olympiads in the top IMO countries.

4 The Nordic countries and the IMO

The first Nordic country to participate in the International Mathematical Olympiad (IMO) was Finland in 1965. Up until that year, the IMO had been an event arranged between the then Socialist countries only. Since 1976 Finland has been a regular participant. Sweden has participated in all the IMOs since 1967. Norway entered the IMO in 1984 and Iceland in 1985. Denmark was the last Nordic country to enter the IMO, starting in 1991. The Nordic countries have never been top performers at the IMO. Straightforward numerical comparisons between different IMOs are dubious, but a general impression of the performance can be obtained from the relative rank of the country. With this criterion, Sweden has done best: the average relative rank over the years is 43.0 % (100 % means top, 0 % bottom). Norway 37.4 %, Finland 32.6 %, Denmark 31.0 % and Iceland with a tiny population remains at 17.4 %. Only once, in 1982, has a Nordic country been in the top quartile, when Finland achieved the relative rank 75.9 %.

5 The Nordic Mathematics Competition

Over the years, the low performance of the Nordic teams has bothered those responsible for selecting and training the teams. In 1986, during the IMO in Warsaw, the leaders of the Nordic teams decided to seek as a partial remedy the creation of a competition whose level would be half-way between the national competitions and the IMO. It was decided to run the competition with a minimum of organizational work. As is the case with many other competitions, the practices adopted in the beginning persist and they are described below. The principal initiator of the Nordic Competition was Dr. Åke Samuelsson, the longtime leader of the Swedish IMO team and also the first chairman of the International Mathematical Olympiad Advisory Board.

The first Nordic Mathematical Olympiad took place on March 30, 1987, with Åke Samuelson as the main organizer. There were 47 participants from Finland, Iceland, Norway and Sweden. The next competition on April 11, 1988 was smaller, as Finland was absent due to a technical error (the mathematician responsible for the competition in Finland lost the problems in his highly disorganized office). The third competition on April 10, 1989 for the first time had a Danish participant. The fifth competition held on April 10, 1991, the word Olympiad was dropped from the name. Since then, the competition has been known as the Nordic Mathematical Competition or Nordic Mathematical Contest, NMC.

6 How it works

The informal character of the NMC is well described by the fact that the first time its regulations were published was not until 1995 but in practice the competition has been run by tradition. The rules were rewritten in 2009 under the heading “Established practices”. The central facts are the following:

- The competition is basically targeted to prospective IMO participants of the five nations (although high school students who might not qualify to the IMO because of the age limit still can do the NMC).

- Participation is limited to a maximum of 20 contestants for each country. The participants are selected by the national organization responsible for the IMO participation in each country.
- The participants work at their own schools at a preset time.
- The contest has four problems. Working time is four hours. The problems are marked by integers on a scale 0 to 5.
- The main organizing duties are rotated between the organizations responsible for IMO participation; i.e. the main organizer of the competition is changing after a year—cyclic in the order Finland, Denmark, Sweden, Norway and Iceland.
- The main organizer sets the examination paper, in English, on the basis of problem suggestions from the participating countries. The problems cover the main IMO topics algebra, geometry, number theory and combinatorics. The main organizer also produces a preliminary marking scheme.
- Each country has a contact person who arranges the translation of the problems into the local language, takes care of the arrangements with the schools, and makes a preliminary marking of the answers together with adequate translations in case the language of the competitor cannot be read by the main organizer. The languages that need to be translated are Finnish and Icelandic.
- The main organizer coordinates the marking and provides diplomas to the participants. The diplomas for the top students (approximately 20) show their rank, the rest are recognized for their participation. A curiosity is that the diploma is always written in the language of the main organizer. A sample diploma can be seen on page 37.
- Representatives of the participating countries meet at the IMO to discuss the date of the next NMC as well as any other business related to it.

As can be seen, the NMC is done in an extremely cheap way. No travel costs arise and all work is done by volunteers. The negative side of course is that the NMC is not an event. The participants do not meet

Norræna stærðfræðikeppnin 2009
The Nordic Mathematical Contest 2009

Olli Halminen

fær viðurkenningu fyrir
að hafa tekið þátt
í 23. norrænu stærðfræðikeppninni
þann 2. apríl 2009

Fyrir hönd skipulagsnefndarinnar

each other, there is no prize delivering ceremony, and the result can be announced only after the marking process has been completed. The process involves at least two mailings and work done by many persons, and some delays are inevitable.

The setting of the date of the NMC creates complications due to different conditions in each country. These conditions involve dates of national competitions, periods during which schools are extremely unwilling to accept extra duties, vacation times and the changing dates of Easter. The date is usually around April 1st. In most of the Nordic countries the school year ends at the end of May and the aim is to have the diplomas ready by that time so that the participants might get some recognition at their schools.

The NMC is a low-profile event created for a definite and limited purpose. It has been running now for over two decades and has served the purpose reasonably well. It helps choose the IMO teams by facing the students with real competition problems yet on an accessible level. Experience has shown that the adopted difficulty level is good. Usually, a couple of the 70 to 80 participating students score full marks, and the distribution of scores goes all the way to a few zeros.

An aspect of the NMC not to be overlooked is that it strengthens the ties between the relatively few northern mathematicians interested in competitions. As things are now, it is likely that the NMC will continue along its well-established lines.

7 Problems of recent NMCs

The problems of the NMC can be found on various websites, at least in the Nordic languages. A booklet containing the problems of the 20 first NMCs have been published in English in the Latvian Laima series [1]. We present here the problems and solutions of the last three NMCs.

NMC 21, March 29, 2007

1. Find one solution in positive integers to the equation

$$x^2 - 2x - 2007y^2 = 0.$$

2. A triangle, a line and three rectangles, with one side parallel to the given line, are given in such a way that the rectangles completely cover the sides of the triangle. Prove that the rectangles completely cover the interior of the triangle.

3. The number 10^{2007} is written on the blackboard. Anne and Berit play a game where the player in turn makes one of the two operations:

(i) replace a number x on the blackboard by two integer numbers a and b greater than 1 such that $x = ab$;

(ii) erase one or both of two equal numbers on the blackboard.

The player who is not able to make her turn loses the game. Who has a winning strategy if Anne begins?

4. A line through a point A intersects a circle in two points, B and C, in such a way that B lies between A and C. From the point A draw the two tangents to the circle, meeting the circle at points S and T. Let P be the intersection of the lines ST and AC. Show that $AP/PC = 2 \cdot AB/BC$.

NMC 22, March 31, 2008

1. Determine all real numbers A, B and C such that there exists a real function f that satisfies

$$f(x + f(y)) = Ax + By + C$$

for all real x and y.

2. Assume that $n \geq 3$ people with different names sit at a round table. We call any unordered pair of them, say M and N, *dominating*, if

(i) M and N do not sit on adjacent seats, and

(ii) in one (or both) of the arcs connecting M and N along the table edge, all people have names that come alphabetically after the names of M and N.

Determine the minimal number of dominating pairs.

3. Let ABC be a triangle and let D and E be points on BC and CA, respectively, such that AD and BE are angle bisectors of ABC. Let F and G be points on the circumference of ABC such that AF and DE are parallel and FG and BC are parallel. Show that

$$\frac{AG}{BG} = \frac{AC + BC}{AB + CB}.$$

4. The difference between the cubes of two consecutive positive integers is a square n^2, where n is a positive integer. Show that n is the sum of two squares.

NMC 23, April 2, 2009

1. A point P is chosen in an arbitrary triangle. Three lines are drawn through P which are parallel to the sides of the triangle. The lines divide the triangle into three smaller triangles and three parallelograms. Let f be the ratio between the total area of the three smaller triangles and the area of the given triangle. Show that $f \geq \frac{1}{3}$ and determine those points P for which $f = \frac{1}{3}$.

2. On a faded piece of paper it is possible, with some effort, to discern the following:

$$(x^2 + x + a)(x^{15} - \dots) = x^{17} + x^{13} + x^5 - 90x^4 + x - 90.$$

Some parts have been lost, partly the constant term of the first factor of the left side, partly the main part of the other factor. It would be possible to restore the polynomial forming the other factor, but we restrict ourselves to asking the question: What is the value of the constant term a? We assume that all polynomials in the statement above have only integer coefficients.

3. The integers 1, 2, 3, 4 and 5 are written on a blackboard. Two integers a and b are wiped out and are replaced with $a + b$ and ab. Is it possible, by repeating this procedure, to reach a situation where three of the five integers on the blackboard are 2009?

4. There are 32 competitors in a tournament. No two of them are equal in playing strength, and in a one against one match the better one always wins. Show that the gold, silver, and bronze medal winners can be found in 39 matches.

8 Solutions

NMC 21

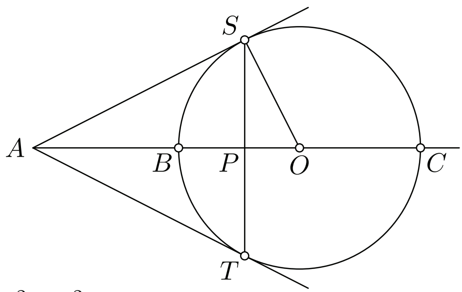
1. Since $2007 = 9 \cdot 223$ and 223 is a prime, 223 must divide either x or $x-2$. Try $x = 225$. Then $x^2 - 2x = 225 \cdot 223 = 15^2 \cdot 223 = 2007 \cdot 5^2$. So $(225, 5)$ is one solution.

2. We show that an arbitrary point P in the interior of the triangle is in one of the rectangles. To this end, draw two lines through P, one parallel and one perpendicular to the given line. Of the four points in which these lines meet the sides of the triangle at least two, say A and B, must be in one of the three rectangles, say R_1. Now if APB is a line, P as an interior point of the segment AB, and P is in R_1. If A and B are on perpendicular lines through P, the segments AP and BP are in R_1, and so P is in R_1.

3. Anne has a winning strategy. Her first move is $10^{2007} \to \left(2^{2007}, 5^{2007}\right)$ and her strategy is that after her move the numbers on the blackboard are $2^{a_1}, \ldots, 2^{a_k}, 5^{a_1}, \ldots, 5^{a_k}$. This is the case after her first move. Assuming Berit makes a move $2^{a_j} \to \left(2^{b_j}, 2^{c_j}\right)$ Anne can answer with $5^{a_j} \to \left(5^{b_j}, 5^{c_j}\right)$. If Berit erases 2^j, then Anne erases 5^j. The same works with 2 and 5 interchanged. So, after every move of Berit, Anne can make a move. Since there is only a finite number of possible situations, Berit must be the first who is unable to move.

4. We first show that for fixed A, B and C, the position of P is independent of the choice of the circle in the problem. Indeed, let Γ_1 and Γ_2 be circles through B and C and let, for $i = 1, 2$, S_i and T_i be the points where tangents from A to Γ_i meet Γ_i and let P_i be the point where S_iT_i meets the line ABC. The power of A with respect to Γ_1 and Γ_2 is $AB \cdot AC$ but also AS_i^2 and AT_i^2. So S_1, S_2, T_1, T_2 all lie on a circle Γ_3 with center A. Denote the point of intersection of S_1T_1 and S_2T_2 by Q. Now the power of Q with respect to Γ_3 is $PS_1 \cdot PT_1 = PS_2 \cdot PT_2$. But this means that P has equal power with respect to Γ_1 and Γ_2. The set of points having equal power with respect to two intersecting circles is the line through the intersection points. So Q on BC or $Q = P$. Any two circles lead to a common P, which means that P is unique.

It follows that we can work with a circle having BC as a diameter. Let O be the center of this circle and let its radius be r. Set $a = AO$, $b = PO$. From similar triangles ASO and SPO we obtain $OS/AO = PO/OS$ or $r^2 = ab$. Then, finally,



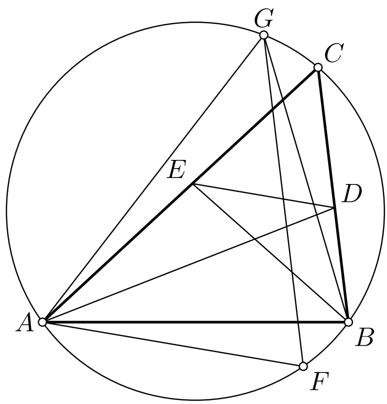
$$\frac{AP}{PC} = \frac{a-b}{b+r} = \frac{a^2-ab}{ab+ar} = \frac{a^2-r^2}{r^2+ar} = \frac{a-r}{r} = \frac{AB}{\frac{BC}{2}} = 2\cdot\frac{AB}{BC}.$$

NMC 22

1. Let f, A, B and C be as in the problem. Let z be a real number. Set $x = z - f(0)$ and $y = 0$. Then $f(z) = f(z - f(0) + f(0)) = A(z - f(0)) + B \cdot 0 + C = Az - Af(0) + C$. So there exist a and b such that $f(z) = az + b$ for all real numbers z. So $Ax + By + C = f(x + f(y)) = a(x + f(y)) + b = ax + a(ay + b) + b = ax + a^2y + (a + 1)b$. So possible triples (A, B, C) are (a, a^2, c), where c is arbitrary and $a \neq -1$ is arbitrary, and $(-1, 1, 0)$.

2. We show by induction that the number of dominating pairs is at least $n - 3$. If $n = 3$, there are no adjacent pairs. Assume that for all aggregates of n people there are at least $n - 3$ dominating pairs. Assume that $n + 1$ people sit around the table. Assume Z is the person last in the alphabetical order. When Z goes away, the two persons who sat beside Z no longer form a dominating pair. All other dominating pairs are still dominating, since there were also other people than Z sitting between them. Since n people create at least $n - 3$ dominating pairs, the number of dominating pairs with $n + 1$ people around the table is at least $(n + 1) - 3$. On the other hand, if the people withdraw from the table one by one in alphabetical order, the number of dominating pairs always reduces by one until only three people and no dominating pairs remain. So the number of dominating pairs must be exactly $n - 3$.

3. Since $FG \| BC$, $\angle FGB = \angle GBC$. This implies $\angle GAC = \angle BAF$ and $\angle GAB = \angle CAF = \angle CED$ (because $ED \| AF$). As $ED \| AF$ and



$FG \| BD$, $\angle AFG = \angle ADC$. So ABG and EDC are similar. Using the bisector theorem we have

$$DC = \frac{AC}{AC + AB} \cdot BC$$

and

$$EC = \frac{BC}{AB + BC} \cdot AC.$$

The claim follows from the similarity proved above

$$\frac{AG}{BG} = \frac{EC}{DC} = \frac{AC + AB}{AB + BC}.$$

– This proof, evident when looking at the figure, presupposes $\angle AGB = \angle ACB$ which means that G and C are on the same side of AB. That this indeed always is the case, can be proved.

4. We assume $(m+1)^3 - m^3 = n^2$. Then n is odd and $4(3m^2+3m+1) = (2n)^2$, $3((2m)^2+2\cdot 2m+1) = (2n)^2-1$ and $3(2m+1)^2 = (2n-1)(2n+1)$. The consecutive odd numbers $2n-1$ and $2n+1$ have no common factors. So one of them is a square and the other is 3 times a square. If $2n+1 = (2t+1)^2$, we would have $2n = 4t^2+4t$, and n would be even. So $2n-1 = (2t+1)^2$ or $n = 2t^2+2t+1 = t^2+(t+1)^2$.

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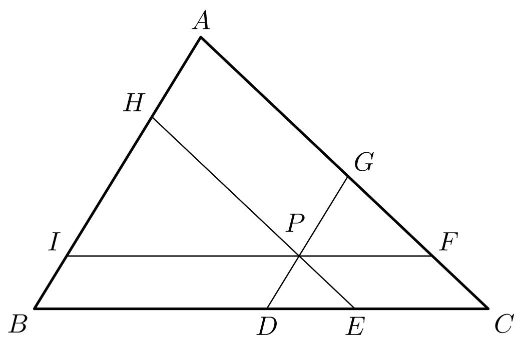
1. Let ABC be the triangle. The lines through P meet the sides of the triangle at D and E, F and G, H and I, respectively. The triangles ABC, DEP, PFG and IPH are similar and $BD = IP$, $EC = PF$. If $BC = a$, $IP = a_1$, $DE = a_2$ and $PF = a_3$, then $a_1 + a_2 + a_3 = a$. The areas of the triangles are ka^2, ka_1^2, ka_2^2 and ka_3^2, for some k. So

$$f = \frac{ka_1^2 + ka_2^2 + ka_3^2}{ka^2} = \frac{a_1^2 + a_2^2 + a_3^2}{(a_1 + a_2 + a_3)^2}.$$

Since the arithmetic mean is less or equal to the quadratic mean,

$$\frac{(a_1 + a_2 + a_3)^2}{9} \leq \frac{a_1^2 + a_2^2 + a_3^2}{3}.$$

The means are equal if and only if $a_1 = a_2 = a_3$ This is equivalent to $f \geq \frac{1}{3}$.



In case of equality, $a_1 = a_2 = a_3$. The three small triangles are congruent. Then also $CF = FG = GA$ and $AH = HI = IB$. Since AIF and ABC are similar and P is the midpoint of IF, the extension of AP bisects BC. So P is on the median of ABC from A. Likewise, P is on the other medians. So P is the intersection of the medians of ABC.

2. Let the first factor of the polynomial of the left be $P(x)$, the other $Q(x)$ and denote the right hand side by $R(x)$. Then $P(0) = P(-1) = a$, $R(0) = -90$ and $R(-1) = -180-4 = -184$. We observe that $90 = 2{\cdot}3^2{\cdot}5$

and $184 = 2^3 \cdot 23$. Since a is a factor of 90 and 184, $a = \pm 1$ or $a = \pm 2$. If $a = 1$, then $P(1) = 3$. But $R(1) = 4 - 180 = -176$ and $R(1)$ is not a multiple of 3. So $a \neq 1$. If $a = -2$, then $P(1) = 0$, but $R(1) = -176$. So $a \neq -2$. One easily notices that $R(x) = (x^4+1)(x^{13}+x-90)$. If $a = -1$, then $P(2) = 5$, but $2^4 + 1 = 17$ and $2^{13} + 2 - 90 = 8 \cdot 1024 + 2 - 90$, and we see that the number is not a multiple of 5. So $a \neq 1$. The only possibility is $a = 2$. [One can show that $Q(x) = (x^4 + 1)(x^{11} - x^{10} - x^9 + 3x^8 - x^7 - 5x^6 + 7x^5 + 3x^4 - 17x^3 + 11x^2 + 23x - 45$.]

3. The operation either diminishes the number of odd numbers on the blackboard or leaves it unchanged. Also, the operation increases both numbers or keeps one of them unchanged (in case one of a and b is one). To reach three 2009s, the operation can never be applied to two odd numbers. Assume that the desired outcome has been obtained. The first 2009 to appear on the blackboard must have been $a+b$. Then either $ab > 2009$ or $ab = 2008$. In the latter case, number one has been wiped out and 2008 cannot be used to produce another 2009. In both cases, there are only three numbers from which new 2009s can be produced. Let c and d be two of them such that $c + d = 2009$. Then again either $cd > 2009$ or $cd = 2008$, and one is wiped out and cd is not available. We now have five numbers, four of which are such that they cannot be used to produce a 2009. There is no way to produce another 2009, in contradiction to what was supposed.

4. The gold medalist is found in five rounds, $16 + 8 + 4 + 2 + 1 = 31$ matches. The silver medalist lost to the winner in one of the rounds. The five losers to the winner can settle the best among them in four matches in a "winner continues, loser drops out" tournament arranged so that the losers to the winner in the first two rounds first meet each other, the winner meets the one who lost to the gold medalist in round three etc. The bronze medalist has only lost matches to the winner or the silver medalist. If the silver medalist lost to the gold winner in round k, $k > 1$, he had won $k - 1$ matches on the first rounds and $5 - k$ or $6 - k$ matches in the silver rounds. So there are at most $6 - k + k - 1 = 5$ candidates for bronze. Four matches are again required to settle the medal. The total number of matches needed is $31 + 4 + 4 = 39$.

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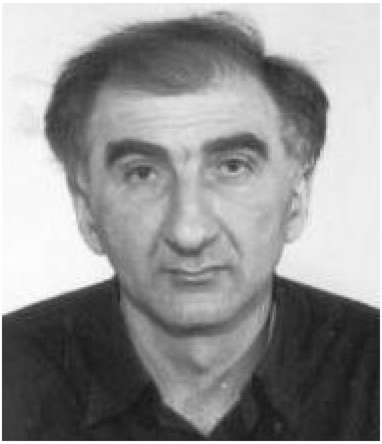
Old inequalities, new proofs

Nairi M. Sedrakyan



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In this paper we have chosen well known inequalities, to which we will present new proofs:

The first example is an inequality, which was proposed in the USA National Mathematical Olympiad in 1980.

Example 1. (USA 1980)

Prove the inequality

$$\frac{a}{b+c+1}+\frac{b}{a+c+1}+\frac{c}{a+b+1}+(1-a)(1-b)(1-c)\leq 1,$$

where $0\leq a,b,c\leq 1$.

Proof. First we will prove the following lemma.

Lemma

If $0 \leq x, y \leq 1$, then

$$\frac{1}{x+y+1} \leq 1 - \frac{x+y}{2} + \frac{xy}{3}.$$

Indeed, we have $(1-x)(1-y) \geq 0$, hence $x+y-1 \leq xy$, then

$$\frac{1}{x+y+1} - \left(1 - \frac{x+y}{2}\right) = \frac{x+y}{2(x+y+1)}(x+y-1) \leq \frac{xy}{3},$$

which proves the lemma.

Now, using the lemma we have

$$\frac{a}{b+c+1} + \frac{b}{a+c+1} + \frac{c}{a+b+1} \leq$$
$$\leq a - \frac{(b+c)a}{2} + \frac{bca}{3} + b - \frac{(a+c)b}{2} + \frac{acb}{3} + c - \frac{(a+b)c}{2} + \frac{abc}{3} =$$
$$= 1 - (1-a)(1-b)(1-c).$$

Thus

$$\frac{a}{b+c+1} + \frac{b}{a+c+1} + \frac{c}{a+b+1} + (1-a)(1-b)(1-c) \leq 1.$$

Example 2 (China TST[1], 2004)

Let a, b, c, d be positive real numbers such that $abcd = 1$. Prove that

$$\frac{1}{(1+a)^2} + \frac{1}{(1+b)^2} + \frac{1}{(1+c)^2} + \frac{1}{(1+d)^2} \geq 1.$$

Proof. Let

$$\frac{1}{(1+a)^2} + \frac{1}{(1+b)^2} + \frac{1}{(1+c)^2} + \frac{1}{(1+d)^2} \geq R^2, \qquad (R > 0),$$

[1] TST- Selective test for International Mathematical Olympiad

then we have to prove, that $R \geq 1$.

Suppose $R < 1$. We can write

$$\frac{1}{(1+a)^2} + \frac{1}{(1+b)^2} = R^2\cos^2\alpha, \qquad \frac{1}{(1+c)^2} + \frac{1}{(1+d)^2} = R^2\sin^2\alpha,$$

where $0 < \alpha < \frac{\pi}{2}$. Hence $\frac{1}{1+a} = R\cos\alpha\cos\beta$, $\frac{1}{1+b} = R\cos\alpha\sin\beta$, $\frac{1}{1+c} = R\sin\alpha\cos\gamma$, $\frac{1}{1+d} = R\sin\alpha\sin\gamma$, where β, $\gamma \in \left(0; \frac{\pi}{2}\right)$.

We have

$$\begin{aligned} 1 = abcd = \frac{1 - R\cos\alpha\cos\beta}{R\cos\alpha\cos\beta} \cdot \frac{1 - R\cos\alpha\sin\beta}{R\cos\alpha\sin\beta} \cdot \\ \cdot \frac{1 - R\sin\alpha\cos\gamma}{R\sin\alpha\cos\gamma} \cdot \frac{1 - R\sin\alpha\sin\gamma}{R\sin\alpha\sin\gamma} > \\ > \frac{1-\cos\alpha\cos\beta}{\cos\alpha\cos\beta} \cdot \frac{1-\cos\alpha\sin\beta}{\cos\alpha\sin\beta} \cdot \frac{1-\sin\alpha\cos\gamma}{\sin\alpha\cos\gamma} \cdot \frac{1-\sin\alpha\sin\gamma}{\sin\alpha\sin\gamma} = \\ > \frac{1-\cos\alpha\cos\beta}{\sin\alpha\sin\beta} \cdot \frac{1-\cos\alpha\sin\beta}{\sin\alpha\cos\beta} \cdot \frac{1-\sin\alpha\cos\gamma}{\cos\alpha\cos\gamma} \cdot \frac{1-\sin\alpha\sin\gamma}{\cos\alpha\sin\gamma} \geq \\ \geq 1 \cdot 1 \cdot 1 \cdot 1 = 1, \end{aligned}$$

Which is impossible. Hence $R \geq 1$.

Next example has a geometrical origin and is related to the problem of inscription in a square of a triangle with a largest inscribed circle.

Example 3

Prove the inequality

$$\sqrt{1+x^2} + \sqrt{1+y^2} + \sqrt{(1-x)^2 + (1-y)^2} \geq (1+\sqrt{5})\,(1-xy)\,,$$

where $x, y \in [0; 1]$.

Proof. We will prove, that

$$\sqrt{1+x^2} \cdot \sqrt{1+y^2} + \sqrt{(1-x)^2 + (1-y)^2} \geq \sqrt{5}\,(1-xy) \qquad (1)$$

If $xy \geq \frac{1}{2}$, then

$$\sqrt{1+x^2}\cdot\sqrt{1+y^2}+\sqrt{(1-x)^2+(1-y)^2}\geq$$
$$\geq 1+xy\geq\frac{3}{2}>\frac{\sqrt{5}}{2}\geq\sqrt{5}\,(1-xy)\,.$$

If $xy < \frac{1}{2}$, then

$$\sqrt{1+x^2}\cdot\sqrt{1+y^2}+\sqrt{(1-x)^2+(1-y)^2}=$$
$$=\sqrt{(1-xy)^2+(x+y)^2}+\sqrt{\left(\sqrt{1-2xy}\right)^2+(1-x-y)^2}\geq$$
$$\geq\sqrt{\left(1-xy+\sqrt{1-2xy}\right)^2+1^2}=$$
$$=\sqrt{(1-xy)^2+2\,(1-xy)\,\sqrt{1-2xy}+1-2xy+1}\geq$$
$$\geq\sqrt{(1-xy)^2+2\,(1-xy)\,(1-2xy)+1-2xy+1}=\sqrt{5}(1-xy).$$

Here we used the Minkowski inequality

$$\sqrt{a_1^2+b_1^2}+\sqrt{a_2^2+b_2^2}\geq\sqrt{(a_1+a_2)^2+(b_1+b_2)^2}.$$

Thus (1) is proved.

Hence

$$\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{(1-x)^2+(1-y)^2}=$$
$$=\sqrt{1+x^2}\sqrt{1+y^2}+\sqrt{(1-x)^2+(1-y)^2}-$$
$$-\left(\sqrt{1+x^2}-1\right)\left(\sqrt{1+y^2}-1\right)+1\geq$$
$$\geq\sqrt{5}\,(1-xy)+1-\frac{x^2}{\sqrt{1+x^2}+1}\cdot\frac{y^2}{\sqrt{1+y^2}+1}\geq$$
$$\geq\sqrt{5}\,(1-xy)+1-\frac{x^2}{x}\cdot\frac{y^2}{y}=(\sqrt{5}+1)\,(1-xy)\,.$$

Hence

$$\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{(1-x)^2+(1-y)^2}\geq(1+\sqrt{5})(1-xy).$$

Item a) in the next example was proposed in the 47th IMO in Slovenia.

Example 4. (IMO-2006)

Find the least possible value of the constant M, such that the inequality

$$\left|ab\left(a^2-b^2\right)+bc\left(b^2-c^2\right)+ca\left(c^2-a^2\right)\right|\leq M\left(a^2+b^2+c^2\right)^2$$

holds for any a) real, b) positive numbers $a,\ b,\ c$.

Proof. Note, that

$$\begin{gathered}ab\left(a^2-b^2\right)+bc\left(b^2-c^2\right)+ca\left(c^2-a^2\right)=\\ =ab\,(a-b)\,(a+b)+c^3\,(a-b)+c\left(a^3-b^3\right)=\\ =(a-b)\left((b-c)\,a^2+ab\,(b-c)-c\,(b-c)\,(b+c)\right)=\\ =(a-b)\,(b-c)\,(a-c)\,(a+b+c)\,.\end{gathered}$$

Thus we have to prove the inequality

$$|a-b|\cdot|b-c|\cdot|a-c|\cdot|a+b+c|\leq M\left(a^2+b^2+c^2\right)^2. \qquad (2)$$

For $a=b=c$ the inequality (2) holds for any positive M. Without loss of generality we can assume, that $a\geq b\geq c$ and $(a-c)^2+(b-c)^2=1$, hence there exists $\alpha\in\left[0,\frac{\pi}{4}\right]$, such, that $a=c+\cos\alpha$, $b=c+\sin\alpha$.

Now note, that

$$\begin{gathered}|a-b|\cdot|b-c|\cdot|a-c|\cdot|a+b+c|=\\ =(\cos\alpha-\sin\alpha)\cos\alpha\,\sin\alpha\,|3c+\cos\alpha+\sin\alpha|\,.\end{gathered} \qquad (3)$$

a) we have

$$\begin{gathered}(\cos\alpha-\sin\alpha)\cos\alpha\sin\alpha\,|3c+\cos\alpha+\sin\alpha|=\\ =\frac{1}{2}\sqrt{(1-\sin 2\alpha)\sin 2\alpha\,\sin 2\alpha\,(3d+1+\sin 2\alpha)},\end{gathered}$$

where $d = 3c^2 + 2c(\cos\alpha + \sin\alpha)$. Then $a^2 + b^2 + c^2 = d + 1$.

Now we can write

$$(\cos\alpha - \sin\alpha)\cos\alpha\sin\alpha\,|3c + \cos\alpha + \sin\alpha| =$$
$$= \frac{1}{2\sqrt{\lambda\mu}}\sqrt{(\lambda - \lambda\sin 2\alpha)\sin 2\alpha\ \sin 2\alpha\,((3d+1)\,\mu + \mu\sin 2\alpha)}$$

and choose λ and μ so, that $\lambda = \mu + 2$ and $3\mu = \lambda + \mu$, i.e. $\mu = 2$, $\lambda = 4$. Then

$$(\cos\alpha - \sin\alpha)\cos\alpha\ \sin\alpha\,|3c + \cos\alpha + \sin\alpha| =$$
$$= \frac{1}{4\sqrt{2}}\left(\sqrt[4]{(4 - 4\sin 2\alpha)\sin 2\alpha\ \sin 2\alpha\,(6d + 2 + 2\sin 2\alpha)}\right)^2 \le$$
$$\le \frac{1}{4\sqrt{2}}\left(\frac{6d+6}{4}\right)^2 = \frac{9\sqrt{2}}{32}(d+1)^2.$$

Thus

$$\left|ab\left(a^2 - b^2\right) + bc\left(b^2 - c^2\right) + ca\left(c^2 - a^2\right)\right| \le \frac{9\sqrt{2}}{32}\left(a^2 + b^2 + c^2\right)^2,$$

and the equality holds when $\sin 2\alpha = \frac{4}{5}$, $d = -\frac{7}{15}$. In that case we can choose

$$c = \frac{\sqrt{2} - 3}{3\sqrt{5}}, \quad a = \frac{\sqrt{2} - 3}{3\sqrt{5}} + \cos\left(\frac{1}{2}\arcsin\frac{4}{5}\right),$$
$$b = \frac{\sqrt{2} - 3}{3\sqrt{5}} + \sin\left(\frac{1}{2}\arcsin\frac{4}{5}\right).$$

Hence the least possible value of M is $\frac{9\sqrt{2}}{32}$.

b)

$$(\cos\alpha - \sin\alpha)\cos\alpha\ \sin\alpha\,(3c + \cos\alpha + \sin\alpha) =$$
$$= \frac{1}{4}\sin 4\alpha\left(\frac{3c}{\cos\alpha + \sin\alpha} + 1\right) \le \frac{1}{4}\left(\frac{3c}{\cos\alpha + \sin\alpha} + 1\right) \le$$
$$\le \frac{1}{4}(3c + 1) \le \frac{1}{4}(2c+1)^2 \le \frac{1}{4}\left(3c^2 + 2c(\cos\alpha + \sin\alpha) + 1\right)^2 =$$
$$= \frac{1}{4}\left(a^2 + b^2 + c^2\right)^2.$$

For $a = c + \cos\frac{\pi}{8}$, $b = c + \sin\frac{\pi}{8}$, $c > 0$ we obtain

$$\frac{1}{4}\left(\frac{3c}{\cos\frac{\pi}{8} + \sin\frac{\pi}{8}} + 1\right) \leq M\Big(3c^2 + 2c\left(\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\right) + 1\Big)^2 \qquad (4)$$

Hence for $c \to 0$ we find from (4), that $M \geq \frac{1}{4}$.

Thus in this case the least possible value of M is $\frac{1}{4}$.

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The 50th International Mathematical Olympiad 10–22 July 2009 Bremen, Germany

The 50th International Mathematical Olympiad (IMO) was held July 10–22 in Bremen, Germany. It was also 50 years since the first IMO was held back in the year 1959. Those who are keenly observant will note that this does not seem to add up correctly. However, this is due to the IMO not being held in year 1980. Thus being the 50th anniversary of the IMO, the host country Germany was keen to make it an extra special occasion and it certainly turned out that way.

This IMO was the largest to be held so far. A record number of 565 contesting high school students from a record 104 countries participated. Benin, Mauritania, Syria and Zimbabwe all participated for the first time.

The IMO, like the Olympic Games, has an Opening Ceremony to welcome all the contestants. It opened in a rather unexpected way with "The Breakmathix". A combination of Beat-box and Breakdance. Very entertaining, especially for high school students. There was more than the usual outside interest in the IMO this year with a number of members of the German government and media showing interest. Both the opening and closing ceremonies were reported on German evening television news. The audience was addressed by the following speakers. Dr. Angela Merkel, Chancellor of Germany, who herself had participated in the German Mathematical Olympiads in the past with good results. Andreas Storm, Parliamentary State Secretary, German Federal Ministry of Education and Research who would officially open the 2009 IMO. Renate Jürgens-Pieper, Senator for Education and Science, Freie Hansestadt Bremen who emphasized the relevance and importance of mathematics in today's world. Walter Rasch, Senator a.D., Chairman, Bildung and Begabung e.V., the main sponsor of the 2009 IMO. Ingo Kramer, Board of Governors, Jacobs University Bremen. Jacobs University supported the IMO by providing the venue for most of the IMO's functions to be carried out including accommodation for all the contestants and leaders.

Dr. József Pelikán, Chairman of the IMO Advisory Board, who emphasized that problem-solving is both challenging and fun especially in the land of Gauss, Riemann and Hilbert. Following this there was some mathematical entertainment. Prof. Dr. Albrecht Beutelspacher, University of Gießen, Director of the Mathematikum, demonstrated "Calculating without a Calculator", in particular long multiplication. Then it came time to officially open the IMO with the parade of the participating teams. In the past this was carried out in alphabetical order, but this year it was done in the order of participation in the IMO. Thus it started with Romania and Bulgaria, both of which are the only two countries to have participated in all 50 IMOs. Some countries with long IMO participation such as the USSR no longer exist. The countries that used to make up the USSR were reckoned as having started IMO participation when they competed separately in their own right. Some teams added a little more spice to their parade, such as the Italian team which did a Mission Impossible skit and the British team who threw frisbees into the audience. Of unusual note was the team from the United Arab Emirates whose 5 members were all girls. Finally there was some more Breakmathix. Thus concluded the Opening Ceremony. The students would begin their own "Mission Impossible" the very next day.

The IMO competition consists of two exam papers held on consecutive days. Each paper is of $4\frac{1}{2}$ hours duration and consists of three very challenging mathematics questions. They are from a variety of mathematical areas and require originality, perseverance and good problem solving skills to negotiate a complete solution. This year the dates of the competition were Wednesday 15th and Thursday 16th of July. To qualify for the IMO, contestants must not have formally enrolled at a university and be less than 20 years of age at the time of writing the second exam paper. Each country has its own internal selection procedures and may send a team of up to six contestants along with a team Leader and Deputy Leader. The Leaders and Deputy Leaders are not contestants but fulfill other roles at the IMO.

Most Leaders arrived in Bremen on July 10th. Their first main task was to set the IMO paper. Already for a number of months prior, countries had been submitting proposed questions for the exam papers. The local Problem Selection Committee had considered these proposals and composed a shortlist of 30 problems considered highly suitable for the IMO exams. Over the next few days the *Jury* of team Leaders discussed the merits of the problems. Through a voting procedure they eventually chose the six problems for the exams. The problems selected for the exam are described as follows.

1. An easy number-theory problem proposed by Australia. The problem was essentially to find all solutions to a set of simultaneous congruences.
2. A very nice medium-level geometry question proposed by Russia. This problem needed good geometric observations and technique combining power of a point and similar triangles ideas.
3. An intriguing, difficult algebra problem with a hint of combinatorial thinking proposed by the United States of America. This problem was concerned with the question of when an increasing sequence of positive integers is in fact an arithmetic sequence. The result to be proved is in fact quite surprising.
4. An apparently easy geometry problem proposed by Belgium. A little unfortunate in that it has a trap due to possible diagram dependance if one chooses an approach by Euclidean methods. A standard computational solution by trigonometry in this instance is safer.
5. A novel medium-level functional equation proposed by France. The question involves a set of functional inequalities on the positive integers.
6. A very difficult combinatorics problem. The question was originally proposed by Russia but generalized by Christian Reiher, a member of the German Problems Selection Committee. (Christian is the most decorated IMO participant ever with 4 gold and 1 bronze to his name.) The generalized version was used for the exam. The question is concerned with the existence of a permutation of a sequence such that the set of partial sums of the permuted sequence is disjoint from a predetermined set.

After the exams, the Leaders and their Deputies spend about two days assessing the work of the students from their own countries. They are guided by marking schemes discussed earlier. A local team of markers called *Coordinators* also assess the papers. They are also guided by the marking schemes but may allow some flexibility if, for example, a Leader brings something to their attention in a contestant's exam script which is not covered by the marking scheme. There are always limitations in this process, but nonetheless the overall consistency and fairness by the Coordinators was very good. Only one dispute made it to the Jury room.

The outcome was not quite as expected. Question 4 was meant to be one of the two easiest problems but ended up being substantially harder than question 2 which was meant to be a medium-level problem. (Question 4 had an average mark of 2.9, with 100 complete solutions, whereas question 2 had an average mark of 3.71, with 214 complete solutions.) As expected question 1 was the easiest on this IMO. Its average mark was 4.8, with 324 complete solutions. Question 6 was the most difficult question on the paper. The average mark was just 0.2, with just 3 complete solutions, all coming from the top three contestants. There were 282 (=49.9 %) medals awarded. The distributions being 135 (=23.9 %) bronze, 98 (=17.3 %) silver and 49 (=8.7 %) gold. There were two students: Makoto Soejima (Japan) and Dongyi Wei (China) who achieved the excellent feat of a perfect score. Most gold medalists essentially solved at least five questions, most silver medalists solved three or four questions and most bronze medalists solved two or three questions. Of those who did not get a medal, a further 96 contestants received an honourable mention for solving at least one question perfectly.

A day was set aside for the 50th anniversary IMO celebrations. The centrepiece was the presence of six former IMO superstars: Terence Tao, Béla Bollobás, Timothy Gowers, Stanislav Smirnov, Jean-Christophe Yoccoz and László Lovász who gave mathematical lectures to the audience. They were also on hand during intermissions to interact with the students, many of whom lined up to get autographs and photos with their favourite personality.

The awards were presented at the Closing Ceremony. An 11 year-old contestant from Peru was given an extended applause for his bronze medal. There was more entertainment including a physics show and the performance of the final movement of Beethoven's first symphony by the Kammerphilharmonie of Bremen. Near the conclusion József Pelikán once again encouraged the contestants to keep pursuing their mathematical endeavours with the following illustration: "The transition from IMO mathematics to research mathematics is like first seeing animals at the zoo and then going on to meet them wild in the jungle. Contest problems behave like tame, even domestic animals but one can also grow to understand and love the untamed ones."

The 2009 IMO was supported and organised by Bildung and Begabung in cooperation with Jacobs University in Bremen.

The 2010 IMO is scheduled to be held in Astana, Kazakhstan.

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1 IMO Paper

Wednesday, July 15, 2009
Language: English

First Day

1. Let n be a positive integer and let $a_1, \ldots, a_k$ $(k \geq 2)$ be distinct integers in the set $\{1, \ldots, n\}$ such that n divides $a_i(a_{i+1} - 1)$ for $i = 1, \ldots, k - 1$. Prove that n does not divide $a_k(a_1 - 1)$.

2. Let ABC be a triangle with circumcentre O. The points P and Q are interior points of sides CA and AB, respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ, respectively, and let Γ be the circle passing through K, L and M. Suppose that the line PQ is tangent to the circle Γ. Prove that $OP = OQ$.

3. Suppose that $s_1, s_2, s_3, \ldots$ is a strictly increasing sequence of positive integers such that the subsequences

$$s_{s_1}, s_{s_2}, s_{s_3}, \ldots \qquad \text{and} \qquad s_{s_1+1}, s_{s_2+1}, s_{s_3+1}, \ldots$$

are both arithmetic progressions. Prove that the sequence s_1, s_2, $s_3, \ldots$ is itself an arithmetic progression.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points

Thursday, July 16, 2009
Language: English

Second Day

4. Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E, respectively. Let K be the incentre of triangle ADC. Suppose that $\angle BEK = 45°$. Find all possible values of $\angle CAB$.

5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b, there exists a non-degenerate triangle with sides of lengths

$$a, \ f(b) \text{ and } f(b + f(a) - 1).$$

(A triangle is *non-degenerate* if its vertices are not collinear.)

6. Let $a_1, a_2, \ldots a_n$ be distinct positive integers and let M be a set of $n - 1$ positive integers not containing $s = a_1 + a_2 + \cdots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths $a_1, a_2, \ldots, a_n$ in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points

2 Results

Some Country Scores			**Some Country Scores**		
Rank	Country	Score	Rank	Country	Score
1	China	221	17	Brazil	160
2	Japan	212	18	Canada	158
3	Russia	203	19	Bulgaria	157
4	South Korea	188	19	Hungary	157
5	North Korea	183	19	U.K.	157
6	U.S.A.	182	22	Serbia	153
7	Thailand	181	23	Australia	151
8	Turkey	177	24	Peru	144
9	Germany	171	25	Georgia	140
10	Belarus	167	25	Poland	140
11	Italy	165	27	Kazakhstan	136
11	Taiwan	165	28	India	130
13	Romania	163	29	Hong Kong	122
14	Ukraine	162	30	Singapore	116
15	Iran	161	31	France	112
15	Vietnam	161	32	Croatia	110

The medal cuts were set at 32 for gold, 24 for silver and 14 for bronze.

Distribution of Awards at the 2009 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Albania	24	0	0	0	0
Algeria (4 members)	2	0	0	0	0
Argentina	93	0	1	1	2
Armenia	59	0	0	2	1
Australia	151	2	1	2	1
Austria	66	0	0	2	2
Azerbaijan	91	0	1	2	2
Bangladesh	67	0	0	2	3
Belarus	167	1	4	1	0
Belgium	89	0	1	2	1
Benin (2 members)	3	0	0	0	0
Bolivia (3 members)	9	0	0	0	0
Bosnia & Herzegovina	63	0	0	1	3
Brazil	160	1	3	2	0
Bulgaria	157	1	3	2	0
Cambodia	14	0	0	0	0
Canada	158	1	3	2	0
Chile (4 members)	41	0	1	0	0
China	221	6	0	0	0
Colombia	88	0	1	2	2
Costa Rica (4 members)	34	0	0	1	1
Croatia	110	0	1	4	1
Cuba (1 member)	21	0	0	1	0
Cyprus	45	0	1	0	2
Czech Republic	87	0	1	2	3
Denmark	68	0	1	1	1
Ecuador	26	0	0	0	1
El Salvador (3 members)	13	0	0	0	0
Estonia	40	0	0	0	3
Finland	49	0	0	0	4
France	112	0	1	3	2
Georgia	140	0	3	2	1
Germany	171	1	4	1	0
Greece	86	0	0	3	3
Guatemala (4 members)	14	0	0	0	1

Distribution of Awards at the 2009 IMO

Country	Total	Gold	Silver	Bronze	H.M.
Honduras (3 members)	24	0	0	1	0
Hong Kong	122	1	2	2	0
Hungary	157	1	2	3	0
Iceland	26	0	0	0	1
India	130	0	3	2	1
Indonesia	84	0	0	4	1
Iran	161	1	4	1	0
Ireland	20	0	0	0	0
Israel	80	0	0	3	2
Italy	165	2	2	2	0
Japan	212	5	0	1	0
Kazakhstan	136	0	3	3	0
Kuwait (4 members)	3	0	0	0	0
Kyrgyzstan	33	0	0	0	3
Latvia	61	0	0	1	3
Liechtenstein (2 members)	21	0	0	1	0
Lithuania	77	0	1	1	3
Luxembourg	65	0	0	3	1
Macau	49	0	0	1	2
Macedonia (FYR)	91	0	1	3	1
Malaysia (2 members)	31	0	1	0	0
Mauritania	8	0	0	0	0
Mexico	74	0	0	3	1
Moldova	74	0	0	4	0
Mongolia	72	0	0	3	1
Montenegro (4 members)	23	0	0	0	1
Morocco	32	0	0	0	0
Netherlands	79	0	1	1	2
New Zealand	53	0	0	1	3
Nigeria	17	0	0	0	0
North Korea	183	3	2	1	0
Norway	60	0	0	2	2
Pakistan (5 members)	21	0	0	1	0
Panama (1 member)	12	0	0	0	1
Paraguay (4 members)	14	0	0	0	0

Distribution of Awards at the 2009 IMO

Country	Total	Gold	Silver	Bronze	H.M.
Peru	144	0	4	2	0
Philippines (4 members)	26	0	0	1	0
Poland	140	0	2	4	0
Portugal	99	0	1	3	2
Puerto Rico	23	0	0	0	1
Romania	163	2	2	2	0
Russia	203	5	1	0	0
Serbia	153	1	3	1	0
Singapore	116	0	2	3	1
Slovakia	73	0	0	2	3
Slovenia	58	0	0	1	3
South Africa	84	0	0	2	4
South Korea	188	3	3	0	0
Spain	71	0	0	4	0
Sri Lanka	74	0	0	2	3
Sweden	70	0	0	2	4
Switzerland	79	0	0	3	2
Syria (5 members)	7	0	0	0	0
Taiwan	165	1	5	0	0
Tajikistan	82	0	1	2	0
Thailand	181	1	5	0	0
Trinidad & Tobago	28	0	0	0	2
Tunisia (5 members)	27	0	0	1	0
Turkey	177	2	4	0	0
Turkmenistan	97	0	1	3	0
Ukraine	162	3	1	2	0
United Arab Emirates	3	0	0	0	0
United Kingdom	157	1	3	2	0
United States of America	182	2	4	0	0
Uruguay	21	0	0	0	1
Uzbekistan	85	0	1	2	1
Venezuela (2 members)	13	0	0	0	0
Vietnam	161	2	2	2	0
Zimbabwe (2 members)	5	0	0	0	0
Total (565 contestants)		**49**	**98**	**135**	**96**

The 3rd Middle European Mathematical Olympiad
24–29 September 2009
Poznań, Poland

The 3rd Middle European Mathematical Olympiad (MEMO) was held on September 24–29 in Poznań, Poland. Organizers from the Polish Mathematical Society in cooperation with the Adam Mickiewicz University in Poznań invited teams from 10 Central-European countries (Austria, Croatia, the Czech Republic, Germany, Hungary, Lithuania, Poland, Slovakia, Slovenia and Switzerland). Each national team consisted of up to six representatives. One of the aims of the MEMO is to give young mathematically-gifted students a chance to get acquainted with the atmosphere of an international mathematical competition. So the contestants could not have competed at the IMO 2009, but their age and year of study at secondary school had to give them a chance to qualify for the 2010 IMO. Each country has its own internal selection procedures and may send a team along with the team leader and deputy leader.

In general, the MEMO competition consists of two exam papers which are sat over consecutive days. On the first day competitors individually solve four problems from the areas of algebra, combinatorics, geometry and number theory. The contestants are given the problems in their mother tongue and are given five hours to solve them. On the second day the teams work together to solve eight problems from the same areas as mentioned above, with the time limit of five hours again.

Most of the teams arrived in Poznań on September 24. The next day, the *Jury* consisting of the team leaders terminated their preparation of the MEMO paper. In June 2009 countries had submitted proposed questions for the exam. The Problem Selection Committee composed a shortlist of problems to be taken into account and sent it to the leaders. On their final meeting the day before the competition started, the jury selected the final 12 problems for the competition and translated them to the mother language of the contestants.

While the leaders were preparing the final version of the problems, the contestants along with the deputy leaders devote their time to the history and sightseeing of the Poznań's region. The organizers prepared a trip for them to the first Polish capital the historical town of Gniezno, a very nice archeological site near Biskupin and a park of miniatures in Podbiedziska.

On Saturday and Sunday, September 26–27, contestants sat the papers. The leaders, deputy leaders and coordinators prepared an assessment scheme according to which the problems were evaluated. After the exam, leaders and deputy leaders assessed the work of the students from their own countries. Final evaluation was done after coordination with the local coordinators.

In the individual competition, 7 (12 %) students were awarded a gold medal (Hungary 3, Poland 2, Germany and Slovakia). The absolute winner, Bertalan Bodor from Hungary, was the only contestant who fully solved Problem 4 of the individual competition, which turned to be the most difficult. 13 students (22 %) were awarded a silver medal and 19 students (32 %) a bronze medal. In the team competition, the Polish team won followed by Hungary and Germany. More details can be found on the web-site of the competition[1].

Finally we present the complete set of problems.

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[1] http://www.memo2009.wmi.amu.edu.pl

1 MEMO Problems

Individual Competition

1. Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that

$$f(xf(y)) + f(f(x) + f(y)) = yf(x) + f(x + f(y))$$

for all $x, y \in \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers.

2. Suppose that we have $n \geq 3$ distinct colours. Let $f(n)$ be the greatest integer with the property that every side and every diagonal of a convex polygon with $f(n)$ vertices can be coloured with one of n colours in the following way:

- at least two distinct colours are used, and
- any three vertices of the polygon determine either three segments of the same colour or of three different colours.

Show that $f(n) \leq (n-1)^2$ with equality for infinitely many values of n.

3. Let $ABCD$ be a convex quadrilateral such that AB and CD are not parallel and $AB = CD$. The midpoints of the diagonals AC and BD are E and F. The line EF meets segments AB and CD at G and H, respectively. Show that $\angle AGH = \angle DHG$.

4. Determine all integers $k \geq 2$ such that for all pairs (m, n) of different positive integers not greater than k, the number $n^{n-1} - m^{m-1}$ is **not** divisible by k.

Team Competition

1. Let x, y, z be real numbers satisfying $x^2+y^2+z^2+9 = 4(x+y+z)$. Prove that

$$x^4 + y^4 + z^4 + 16(x^2 + y^2 + z^2) \geq 8(x^3 + y^3 + z^3) + 27$$

and determine when equality holds.

2. Let a, b, c be real numbers such that for every two of the equations

$$x^2 + ax + b = 0, \qquad x^2 + bx + c = 0, \qquad x^2 + cx + a = 0$$

there is exactly one real number satisfying both of them. Determine all the possible values of $a^2 + b^2 + c^2$.

3. The numbers $0, 1, 2, \ldots, n$ ($n \geq 2$) are written on a blackboard. In each step we erase an integer which is the arithmetic mean of two different numbers which are still left on the blackboard. We make such steps until no further integer can be erased. Let $g(n)$ be the smallest possible number of integers left on the blackboard at the end. Find $g(n)$ for every n.

4. We colour every square of a 2009×2009 board with one of n colours (we do not have to use every colour). A colour is called *connected* if either there is only one square of that colour or any two squares of the colour can be reached from one another by a sequence of moves of a chess queen without intermediate stops at squares having another colour (a chess queen moves horizontally, vertically or diagonally). Find the maximum n, such that for every colouring of the board at least one colour present at the board is connected.

5. Let $ABCD$ be a parallelogram with $\angle BAD = 60^\circ$ and denote by E the intersection of its diagonals. The circumcircle of the triangle ACD meets the line BA at $K \neq A$, the line BD at $P \neq D$ and the line BC at $L \neq C$. The line EP intersects the circumcircle of the triangle CEL at points E and M. Prove that the triangles KLM and CAP are congruent.

6. Suppose that $ABCD$ is a cyclic quadrilateral and $CD = DA$. Points E and F belong to the segments AB and BC respectively, and $\angle ADC = 2\angle EDF$. Segments DK and DM are height and median of the triangle DEF, respectively. L is the point symmetric to K with respect to M. Prove that the lines DM and BL are parallel.

7. Find all pairs (m, n) of integers which satisfy the equation

$$(m+n)^4 = m^2n^2 + m^2 + n^2 + 6mn.$$

8. Find all non-negative integer solutions of the equation

$$2^x + 2009 = 3^y 5^z.$$

Tournament of Towns

Andy Liu

Andy Liu is a professor of mathematics at the University of Alberta in Canada. His research interests span discrete mathematics, geometry, mathematics education and mathematics recreations. He edits the Problem Corner of the MAA's magazine Math Horizons. *He was the Chair of the Problem Committee in the 1995 IMO in Canada. His contribution to the 1994 IMO in Hong Kong was a major reason for him being awarded a David Hilbert International Award by the World Federation of National Mathematics Competitions. He has trained students in all six continents.*

Here are my favourite problems from the Spring 2009 round of the Tournament.

1. In a convex 2009-gon, all diagonals are drawn. A line intersects the 2009-gon but does not pass through any of its vertices. Prove that the line intersects an even number of diagonals.

2. When a positive integer is increased by 10%, the result is another positive integer whose digit-sum has decreased by 10%. Is this possible?

3. (a) Prove that there exists a polygon which can be dissected into two congruent parts by a line segment which cuts one side of the original polygon in half and another side in the ratio 1:2.

(b) Can such a polygon be convex?

4. A castle is surrounded by a circular wall with 9 towers. Some knights stand on guard on these towers. After every hour, each knight moves to a neighbouring tower. A knight always moves in the same direction, whether clockwise or counter-clockwise. At some hour, there are at least two knights on each tower. At another hour, there are exactly 5 towers each of which has exactly one knight on it. Prove that at some other hour, there is a tower with no knights on it.

5. In triangle ABC, $AB = AC$ and $\angle CAB = 120°$. D and E are points on BC, with D closer to B, such that $\angle DAE = 60°$. F and G are points on AB and AC respectively such that $\angle FDB = \angle ADE$ and $\angle GEC = \angle AED$. Prove that the area of triangle ADE is equal to the sum of the areas of triangles FBD and GCE.

6. A positive integer n is given. Two players take turns marking points on a circle. The first player uses the red colour while the second player uses the blue colour. When n points of each colour have been marked, the game is over, and the circle has been divided into $2n$ arcs. The winner is the player who has the longest arc both endpoints of which are of this player's colour. Which player can always win, regardless of any action of the opponent?

7. At step 1, the computer has the number 6 in a memory cell. In step n, it computes the greatest common divisor d of n and the number m currently in that cell, and replaces m with $m+d$. Prove that if $d > 1$, then d must be prime.

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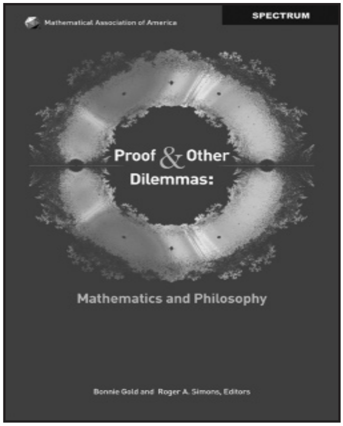

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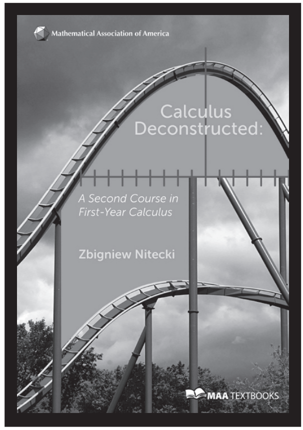

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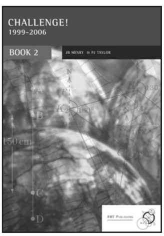

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Edited by W Atkins, J Edwards, D King, PJ O'Halloran & PJ Taylor

This 258-page book consists of over 500 questions, solutions and statistics from the AMC papers of 1978-84. The questions are grouped by topic and ranked in order of difficulty. The book is a powerful tool for motivating and challenging students of all levels. A must for every mathematics teacher and every school library.

Australian Mathematics Competition Book 2 1985–1991
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This CD contains the same problems and solutions as in the corresponding book. The problems can be accessed in topics as in the book and in this mode is ideal to help students practice particular skills. In another mode students can simulate writing one of the actual papers and determine the score that they would have gained. The CD runs on Windows platform only.

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Problem Solving via the AMC
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This 210-page book consists of a development of techniques for solving approximately 150 problems that have been set in the Australian Mathematics Competition. These problems have been selected from topics such as Geometry, Motion, Diophantine Equations and Counting Techniques.

Methods of Problem Solving, Book 1
Edited by JB Tabov & PJ Taylor

This book introduces the student aspiring to Olympiad competition to particular mathematical problem solving techniques. The book contains formal treatments of methods which may be familiar or introduce the student to new, sometimes powerful techniques.

Methods of Problem Solving, Book 2
JB Tabov & PJ Taylor

After the success of Book 1, the authors have written Book 2 with the same format but five new topics. These are the Pigeon-Hole Principle, Discrete Optimisation, Homothety, the AM-GM Inequality and the Extremal Element Principle.

Mathematical Toolchest
Edited by AW Plank & N Williams

This 120-page book is intended for talented or interested secondary school students, who are keen to develop their mathematical knowledge and to acquire new skills. Most of the topics are enrichment material outside the normal school syllabus, and are accessible to enthusiastic year 10 students.

International Mathematics — Tournament of Towns (1980–1984)
Edited by PJ Taylor

The International Mathematics Tournament of the Towns is a problem-solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. This 115-page book contains problems and solutions from past papers for 1980-1984.

International Mathematics — Tournament of Towns (1984–1989)
Edited by PJ Taylor

More challenging questions and solutions from the International Mathematics Tournament of the Towns competitions. This 180-page book contains problems and solutions from 1984-1989.

International Mathematics — Tournament of Towns (1989–1993)
Edited by PJ Taylor

This 200-page book contains problems and solutions from the 1989-1993 Tournaments.

International Mathematics — Tournament of Towns (1993–1997)
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This 180-page book contains problems and solutions from the 1993-1997 Tournaments.

International Mathematics — Tournament of Towns (1997–2002)
Edited by AM Storozhev

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Challenge! 1991 – 1998
Edited by JB Henry, J Dowsey, AR Edwards, L Mottershead, A Nakos, G Vardaro & PJ Taylor

This book is a major reprint of the original Challenge! (1991-1995) published in 1997. It contains the problems and full solutions to all Junior and Intermediate problems set in the Mathematics Challenge for Young Australians, exactly as they were proposed at the time. It is expanded to cover the years up to 1998, has more advanced typography and makes use of colour. It is highly recommended as a resource book for classes from Years 7 to 10 and also for students who wish to develop their problem-solving skills. Most of the problems are graded within to allow students to access an easier idea before developing through a few levels.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

Australian Mathematical Olympiads 1979 – 1995
H Lausch & PJ Taylor

This book is a complete collection of all Australian Mathematical Olympiad papers from the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads Book 1 1981–1993
A Liu

This book contains the papers and solutions of two contests, the Chinese National High School Competition and the Chinese Mathematical Olympiad. China has an outstanding record in the IMO and this book contains the problems that were used in identifying the team candidates and selecting the Chinese team. The problems are meticulously constructed, many with distinctive flavour. They come in all levels of difficulty, from the relatively basic to the most challenging.

Asian Pacific Mathematics Olympiads 1989-2000
H Lausch & C Bosch-Giral

With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981–1995
ME Kuczma & E Windischbacher

Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
Edited by T Andreescu & Z Feng

This book contains one hundred and one highly rated problems used in training and testing the USA International Mathematical Olympiad team. The problems are carefully graded, ranging from quite accessible towards quite challenging. The problems have been well developed and are highly recommended to any student aspiring to participate at National or International Mathematical Olympiads.

Hungary Israel Mathematics Competition
S Gueron

The Hungary Israel Mathematics Competition commenced in 1990 when diplomatic relations between the two countries were in their infancy. This 181-page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions. The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Chinese Mathematics Competitions and Olympiads Book 2 1993–2001
A Liu
This book is a continuation of the earlier volume and covers the years 1993 to 2001.

Bulgarian Mathematics Competition 1992–2001
BJ Lazarov, JB Tabov, PJ Taylor & A Storozhev

The Bulgarian Mathematics Competition has become one of the most difficult and interesting competitions in the world. It is unique in structure combining mathematics and informatics problems in a multi-choice format. This book covers the first ten years of the competition complete with answers and solutions. Students of average ability and with an interest in the subject should be able to access this book and find a challenge.

Mathematical Contests – Australian Scene
Edited by PJ Brown, A Di Pasquale & K McAvaney

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

Mathematics Competitions
Edited by J Švrcek

This bi-annual journal is published by AMT Publishing on behalf of the World Federation of National Mathematics Competitions. It contains articles of interest to academics and teachers around the world who run mathematics competitions, including articles on actual competitions, results from competitions, and mathematical and historical articles which may be of interest to those associated with competitions.

Problems to Solve in Middle School Mathematics
B Henry, L Mottershead, A Edwards, J McIntosh, A Nakos, K Sims, A Thomas & G Vardaro

This collection of problems is designed for use with students in years 5 to 8. Each of the 65 problems is presented ready to be photocopied for classroom use. With each problem there are teacher's notes and fully worked solutions. Some problems have extension problems presented with the teacher's notes. The problems are arranged in topics (Number, Counting, Space and Number, Space, Measurement, Time, Logic) and are roughly in order of difficulty within each topic. There is a chart suggesting which problem-solving strategies could be used with each problem.

Teaching and Assessing Working Mathematically Book 1 & Book 2
Elena Stoyanova

These books present ready-to-use materials that challenge students understanding of mathematics. In exercises and short assessments, working mathematically processes are linked with curriculum content and problem solving strategies. The books contain complete solutions and are suitable for mathematically able students in Years 3 to 4 (Book 1) and Years 5 to 8 (Book 2).

A Mathematical Olympiad Primer
G Smith

This accessible text will enable enthusiastic students to enter the world of secondary school mathematics competitions with confidence. This is an ideal book for senior high school students who aspire to advance from school mathematics to solving olympiad-style problems. The author is the leader of the British IMO team.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Innovation, Industry, Science and Research) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

Newton Enrichment Student Notes
JB Henry

Recommended for mathematics students of about Year 5 and 6 as extension material. Topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

Dirichlet Enrichment Student Notes
JB Henry

This series has chapters on some problem solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory. It is designed for students in Years 6 or 7.

Euler Enrichment Student Notes
MW Evans & JB Henry

Recommended for mathematics students of about Year 7 as extension material. Topics include elementary number theory and geometry, counting, pigeonhole principle.

Gauss Enrichment Student Notes
MW Evans, JB Henry & AM Storozhev

Recommended for mathematics students of about Year 8 as extension material. Topics include Pythagoras theorem, Diophantine equations, counting, congruences.

Noether Enrichment Student Notes
AM Storozhev

Recommended for mathematics students of about Year 9 as extension material. Topics include number theory, sequences, inequalities, circle geometry.

Pólya Enrichment Student Notes
G Ball, K Hamann & AM Storozhev

Recommended for mathematics students of about Year 10 as extension material. Topics include polynomials, algebra, inequalities and geometry.

T-SHIRTS

T-shirts of the following six mathematicians are made of 100% cotton and are designed and printed in Australia. They come in white, Medium (Turing only) and XL.

Leonhard Euler T-shirt

The Leonhard Euler t-shirts depict a brightly coloured cartoon representation of Euler's famous Seven Bridges of Königsberg question.

Carl Friedrich Gauss T-shirt

The Carl Friedrich Gauss t-shirts celebrate Gauss' discovery of the construction of a 17-gon by straight edge and compass, depicted by a brightly coloured cartoon.

Emmy Noether T-shirt

The Emmy Noether t-shirts show a schematic representation of her work on algebraic structures in the form of a brightly coloured cartoon.

George Pólya T-shirt

George Pólya was one of the most significant mathematicians of the 20th century, both as a researcher, where he made many significant discoveries, and as a teacher and inspiration to others. This t-shirt features one of Pólya's most famous theorems, the Necklace Theorem, which he discovered while working on mathematical aspects of chemical structure.

Peter Gustav Lejeune Dirichlet T-shirt

Dirichlet formulated the Pigeonhole Principle, often known as Dirichlet's Principle, which states: "If there are p pigeons placed in h holes and $p>h$ then there must be at least one pigeonhole containing at least 2 pigeons." The t-shirt has a bright cartoon representation of this principle.

Alan Mathison Turing T-shirt

The Alan Mathison Turing t-shirt depicts a colourful design representing Turing's computing machines which were the first computers.

ORDERING

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- a network of dedicated mathematicians and teachers who work in a voluntary capacity supporting the activities of the Trust;
- the quality, freshness and variety of its questions in the Australian Mathematics Competition, the Mathematics Challenge for Young Australians, and other Trust contests;
- the production of valued, accessible mathematics materials;
- dedication to the concept of solidarity in education;
- credibility and acceptance by educationalists and the community in general whether locally, nationally or internationally; and
- a close association with the Australian Academy of Science and professional bodies.