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MATHEMATICS COMPETITIONS

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WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

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The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the President

It is my pleasure as the recently elected President of the WFNMC to address the readers of the WFNMC journal.

Analyzing the achievements of the Federation under the guidance of former presidents Petar Kenderov, Peter Taylor, Ron Dunkley, Blagovest Sendov and Peter O'Halloran in its 24 years, it is clearly my duty to follow up on the many successful programs and initiatives of the WFNMC under their leadership. And it is also natural to look for new activities, new directions that will enrich the Federation, its members and all those who have benefited from this fine journal, from the Federation's biannual meetings and conferences, from the awards program, among others.

It is our aim to encourage new persons to join the Federation, to look to attract new groups working in mathematics education, and especially in the field of providing mathematical challenges for students, organizing competitions and developing students' mathematical creativity, as a means of enriching and continually renewing the mission and accomplishments of the WFNMC.

In July of this year, two important events of the Federation took place. Every four years ICMI organizes the International Congress of Mathematics Education (ICME) and in 2008 ICME-11 was held in Monterrey, México.

Since the WFNMC is an Affiliated Study Group of ICMI, its biannual meeting and Erdös awards presentations were held within the program of ICME-11.

Erdös Awardees

An Erdös prize for 2006 was awarded to Ali Rejali by Michelle Artigue, president of ICMI and Petar Kendarov, then president of WFNMC.

The Awards Committee announced 2008 awards for Bruce Henry of Australia, Leou Shan of Taiwan and Dieter Gronau of Germany.

Bruce Henry was presented with his award by Michelle Artigue, president of ICMI and Petar Kendarov, then president of WFNMC.

Peter Taylor was commissioned to present the Erdös award to Dieter Gronau at the final Jury meeting at the IMO in Madrid, and to present the award to Professor Leou of Taiwan when meeting with him in the coming weeks.

A report on the progress of ICMI Study 16 "Challenging Mathematics in and Beyond the Classroom" was given by Peter Taylor. The work is completed and the study volume will be published later in 2008.

At this meeting it was fascinating to have Michel Spira speak about the Brazilian Olympiad for Public Schools, sponsored by the President of Brazil, Luis Ignacio da Silva. More than 18 million students took part in the Olympiad in 2008.

New officers

The second meeting of WFNMC at ICME-11 was devoted to reports from each of the members of the past Executive Committee. Then elections were held and the results are as follows:

THE EXECUTIVE

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Vice President:	Ali Rejali (Iran)
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WFNMC Miniconference

Prior to IMCE-11, the WFNMC held a miniconference on 5 July 2008, which gave the opportunity to hear speakers address many of the issues of interest to the Federation and its members. A closing dinner was held in the Museum of the History of Mexico, among Aztec and Mayan cultural images.

Miniconference programme

The miniconference programme featured the following programme:

Petar Kenderov (Bulgaria) Is Elementary Mathematics Elementary?
Andy Edwards (Australia) Reach Problems.
Leo Schneider (USA) New York State Mathematics League Contest Problems.
Alexander Soifer (USA) Building a Bridge.
Frank De Clerck (Belgium) On the impact of the Flemish Mathematical Olympiad on the curriculum of mathematics in high school.
Josef Molnár, *Jaroslav Švrček* (Czech Republic) About new methods of youth creativity Competitions.
Romualdas Kašuba (Lithuania) Venid y Ved.
Andy Liu (Canada) Math Fair. Workshop on problem creation. Non-competitive Problems.

Biannual conference in Latvia

The next biannual conference of WFNMC will take place in Riga, Latvia, in 2010 under the guidance of Agnis Andžāns and other members of the organizing committee, a group active in many of the previous biannual conferences. The conference will run from Monday to Friday, with a day devoted to excursions. The format will be problem workshops each morning and two plenary lectures each afternoon. All participants will be invited to contribute with a proposed talk. Speakers will be chosen by a local program committee.

Maria Falk de Losada
President of WFNMC
December 2008

From the Editor

Welcome to *Mathematics Competitions* Vol. 21, No. 2.

Again I would like to thank the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:
The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

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Jaroslav Švrček,
December 2008

The Erdös Award:

Call for Nominations

The Awards Committee of the WFNMC calls for nominations for the Erdös Award.

As described in the formal nomination procedures (see www.amt.edu.au/wfnmc.html), nominations should be sent to the Chair of the Committee at the address below by 31 May 2009 for consideration and presentation in 2010. Such a nomination must include a description of the nominee's achievements together with the names & addresses of (preferably) four persons who can act as referees.

Committee Chair

Petar S. Kenderov
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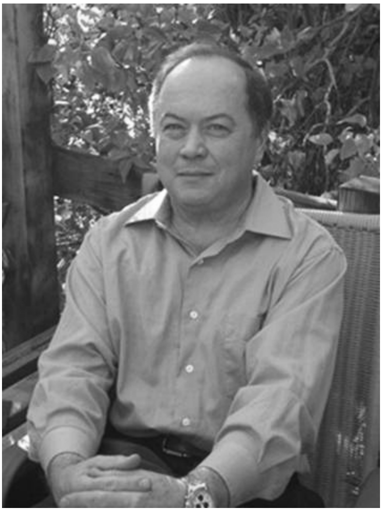
Building a Bridge I: from Problems of Mathematical Olympiads to Open Problems of Mathematics

Alexander Soifer



Born and educated in Moscow, Alexander Soifer has for 29 years been a Professor at the University of Colorado, teaching math, and art and film history. He has written six books and some 200 articles, including The Mathematical Coloring Book, Springer, *September 2008. In fact,* Springer *has contracted with Soifer seven books, five of which are coming out shortly and two are to be written in 2009–10. He founded and for 25 years ran the Colorado Mathematical Olympiad. Soifer has also served on the Soviet Union Math Olympiad (1970–1973) and USA Math Olympiad (1996–2005). He has been Secretary of WFNMC (1996–present), and a recipient of the Paul Erdös Award (2006). Soifer was the founding editor of the research quarterly* Geombinatorics *(1991–present), whose other editors include Ronald L. Graham, Branko Grünbaum, Heiko Harborth, Peter D. Johnson Jr., and Janos Pach. Paul Erdös was an editor and active author of this journal. Soifer's Erdös number is 1.*

1 Introduction: Envisioning the Bridge

The famous Russian mathematician Boris N. Delone once said, as Andrei N. Kolmogorov recalls in his introduction to [1], that "a major scientific discovery differs from a good Olympiad problem only by the fact that a solution of the Olympiad problem requires 5 hours whereas obtaining a serious scientific result requires 5,000 hours."

My books [2]–[8] are bricks in building a bridge from problems of Mathematical Olympiads to problems of "real" mathematics. In them, I try to show that problems of competitions and research problems of mathematics stem from the same root, made of the same fabric, have no natural boundaries to separate them. I am grateful to Springer for its service as a mason of this bridge: Springer has recently contracted seven of my books.

It is, therefore, natural to give "real" problems to young high school Olympians (maybe not at Mathematical Olympiads, as they do not last 5,000 hours). In the mid 1960's, as a high school student, I attended the Award Presentation Ceremony of the Moscow Mathematical Olympiad where the Chairman of the Olympiad's Jury, the famed mathematician Andrej Nikolaevich Kolmogorov, paid us young Olympians, an elegant compliment. "Perhaps, the only way to find a proof of Fermat's Last Theorem is to offer it at the Moscow Mathematical Olympiad," he said.

We ought to stop discriminating against young high school mathematicians based on their tender age. However, we can offer true research problems only to a small percent of high school mathematicians. What can we do for others who are not yet ready for research? Here is one way. While reading or creating research mathematics, I have caught myself many times thinking how beautiful, Olympiad-like, certain ideas were. Consequently, some of these striking ideas gave birth to problems I created for the Olympiads. I first notice a fragment of the research, which utilizes a nice, better yet surprising idea. I then translate thus found mathematical gem into the language of secondary mathematics, and try to present it in a form of an engaging story—and a new Olympiad problem is ready!

Sometimes I imitate a "real" mathematical train of thought by offering at a Mathematical Olympiad a series of problems, increasing in difficulty, and leading to generalizations and deeper results. It is also important to realize that "real" mathematics cannot be reduced to just analytical reasoning, for about of 50 % mathematics is about construction of counterexamples. I try to reflect this dichotomy in our Olympiad problems, many of which require not only analytical proofs but also construction of examples.

The bridge we are building can be walked in the opposite direction as well: it is worthwhile for professionals to take a deeper look at problems of Mathematical Olympiads. Those problems just might inspire exciting generalizations and new directions for mathematical research. We will talk about this direction in the next installment of this triptych of papers.

In this first part of the triptych of papers, I will illustrate these ideas in the context of a problem I have recently created for the Colorado Mathematical Olympiad.

2 An Olympiad Problem

My 1996 sabbatical leave was spent in several European countries, a good part of it at Charles University in the old part of Prague. While there, I attended a research number-theoretic talk by the young talented professor Martin Klazar on integral sequences. I enjoyed the talk, and took notes. When in early 2005 I came across these notes for the first time since taking them, I found a note to myself on the margin (yes, the margin again!): "use these ideas at the Olympiad!" Indeed, I put Martin's research ideas into the foundation of part (b) of the hardest problem of the 22^{nd} Colorado Mathematical Olympiad in 2005.

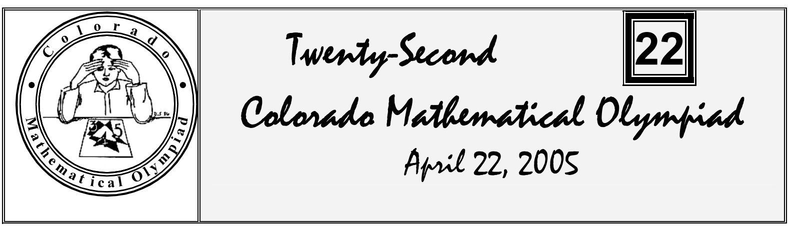


Figure 1: Logo of the 22^{nd} Colorado Mathematical Olympiad

Let us look at this problem and its solution.

2005.5. Love and Death

(a) The DNA of bacterium *bacillus anthracis* (causing anthrax) is a sequence, each term of which is one of 2005 genes. How long can the DNA be if no consecutive terms may be the same gene, and no two distinct genes can reappear in the same order? That is, if distinct genes α, β occur in that order (with or without any number of genes in between), the order $\alpha, \ldots, \beta$ cannot occur again.

(b) The DNA of bacterium *bacillus amoris* (causing love) is a sequence, each term of which is one of 2005 genes. No three consecutive terms may include the same gene twice, and no three distinct genes can reappear in the same order. That is, if distinct genes α, β, and γ occur in that order (with or without any number of genes in between), the order $\alpha, \ldots, \beta, \ldots, \gamma$ cannot occur again. Prove that this DNA is at most 12,032 long.

First Solution to 5(a). Let us prove that in a DNA satisfying the two given conditions, there is a gene that occurs only once. Indeed, let us assume that each gene appears at least twice and for each gene select the first two appearances from the left and call them a *pair*. The first gene from the left is in the first pair. This pair must be separated, thus the pair of the second gene from the left is nestled inside the first pair. The second pair must be separated, and thus the pair of the third gene from the left must be nestled inside the second pair, etc. As there are finitely many genes, we end up with a pair of genes (nestled inside other pairs) that is not separated, a contradiction.

We will now prove by mathematical induction on the number n of genes that the DNA that satisfies the conditions and uses n genes is $2n-1$ genes long. For $n = 1$ the statement is true, as longest DNA is $2-1 = 1$ gene long.

Assume that a DNA that satisfies conditions and uses n genes is at most $2n-1$ genes long. Now let S be a DNA sequence satisfying conditions that uses $n+1$ genes; we need to prove that it is $2(n+1)-1 = 2n+1$ genes long.

From the first paragraph, there a gene g that occurs only once in S; we throw it away. The only violation that this throwing may create is that two copies of another gene are now adjacent—if so, we throw one of them away too. We get the sequence S' that uses only n genes. By inductive assumption, S' is at most $2n-1$ genes long. But S is at most 2 genes longer than S', that is, S is at most $2n+1$ genes long. The induction is complete.

All that is left is to demonstrate that the DNA length of $2n-1$ is attainable. But this is easy: just take a sequence $1, 2, \ldots, n-1, n, n-1, \ldots, 2, 1$.

Second Solution to 5(a). We will now prove by mathematical induction on the number n of genes that the DNA that satisfies the conditions and uses n genes is $2n-1$ genes long. For $n = 1$, the statement is true, as the longest DNA is $2-1 = 1$ gene long.

Assume that for any positive integer k, $k < n$, a DNA that satisfies the conditions and uses k genes, is at most $2k-1$ genes long. Now let S be the longest DNA sequence that satisfies conditions and uses n genes; we need to prove that S is at most $2n-1$ genes long.

Let the first gene of S be 1, then the last term must be 1 as well, for otherwise we can make S longer by adding a 1 at the end. Indeed, assume that the added 1 has created a forbidden DNA. This means that we now have a subsequence $a, \ldots, 1, \ldots, a, \ldots, 1$ (with the added 1 at the end); but then the original DNA has already had the forbidden subsequence $1, \ldots, a, \ldots, 1, \ldots, a$.

Case 1. If there are no more 1's in the DNA, we throw away the first 1 and the last 1, and we get a sequence S' that uses $n-1$ genes (no more 1's). By inductive assumption, S' is at most $2n-1$ genes long. But S is 2 genes longer than S', that is, S is at most $2n-1$ genes long.

Case 2. Assume now that there is a 1 between the first 1 and the last 1. The DNA then looks as follows: $1, S', 1, S'', 1$. Observe that if a gene m appears in the sequence S', it may not appear in the sequence S'', for this would create the prohibited subsequence $1, \ldots, m, \ldots, 1, \ldots, m$. Let the sequence $1, S', 1$ use n' genes and the sequence $1, S'', 1$ use n'' genes. Obviously, $n' + n'' - 1 = n$ (we subtract 1 on the left side because we counted the gene 1 in each of two subsequences!). By inductive assumption, the length of the sequences $1, S', 1$ and $1, S'', 1$ are at most $2n'-1$ and $2n''-1$ respectively. Therefore, the length of S is $(2n'-1) + (2n''-1) - 1$ (we subtract 1 because the gene 1 between

S' and S'' has been counted twice). But $(2n' - 1) + (2n'' - 1) - 1 = 2(n' + n'') - 3 = 2(n + 1) - 3 = 2n - 1$ as desired. The induction is complete.

This proof allows us to find a richer set of examples of DNAs of length of $2n - 1$ (and even describe all such examples, if necessary). For example:

$$1, 2, \ldots, \mathbf{k}, k + 1, k, k + 2, k, \ldots, k, 2005, \mathbf{k}, k - 1, k - 2, \ldots, 2, 1.$$

Solution of problem 5(b). Assume S is the longest DNA string satisfying the conditions. Partition S into blocks of three terms starting from the left (the last block may be incomplete and have fewer than three terms, of course). We will call a block *extreme* if a number from the given set of genes $\{1, 2, \ldots, 2005\}$ appears in the block for the first or the last time. There are at most 2×2005 extreme blocks.

We claim that *there cannot be any complete (i.e. 3-gene) non-extreme blocks.*

Indeed, assume the block B, which consists of genes α, β, γ in *some* order, is not extreme. This means that the genes α, β, γ each appears at least once before and once after appearing in B. We will prove that then the DNA would contain the forbidden subsequence of the type $\alpha, \beta, \gamma, \alpha, \beta, \gamma$. Let A denote the ordered triple of the first appearances of α, β, γ (these three genes may very well come from distinct 3-blocks). Without loss in generality we can assume that in A the genes α, β, γ appear in *this* order. Let C denote the ordered triple of the last appearances of α, β, γ in *some* order. Let us look at the 9-term subsequence ABC and consider three cases, depending upon where α appears in the block B.

1. If α is the first gene in B (Fig. 2), then we can choose β also in B and γ in C to form α, β, γ which with α, β, γ from A gives us the forbidden $\alpha, \beta, \gamma, \alpha, \beta, \gamma$.
2. Let α be the second gene in B (Fig. 3). If β follows α then with a γ from C we get α, β, γ, which with α, β, γ from A produces the forbidden $\alpha, \beta, \gamma, \alpha, \beta, \gamma$. Thus, β must precede α in B. If the order of the genes β, γ in C is β, γ, then we can combine an α from B with this β, γ to form α, β, γ, which with α, β, γ from A gives us the

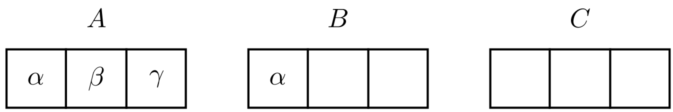


Figure 2

forbidden $\alpha, \beta, \gamma, \alpha, \beta, \gamma$. Thus, the order in C must be γ, β. We can now choose α, γ from A followed by β, α from B, followed by γ, β from C to get $\alpha, \gamma, \beta, \alpha, \gamma, \beta$, which is forbidden.

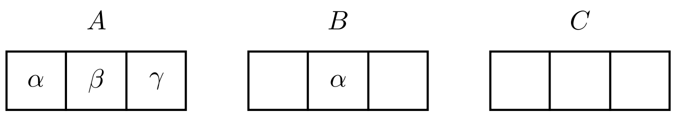


Figure 3

3. Let α be the third gene in B (Fig. 4), and is thus preceded by a β in B. If the order in C is β, γ, then we get α, β, γ from A followed by α from B and β, γ from C to get the forbidden $\alpha, \beta, \gamma, \alpha, \beta, \gamma$. Thus, the order in C must be γ, β, and we choose α, γ from A, followed by β, α from B, and followed by γ, β from C to form the forbidden $\alpha, \gamma, \beta, \alpha, \gamma, \beta$.

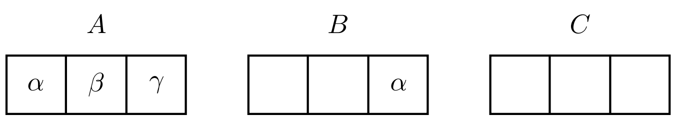


Figure 4

We are done, for the DNA sequence consists of at most 2×2005 extreme 3-blocks plus perhaps an incomplete block of at most 2 genes—or 12,032 genes at the most.

3 Crossing the Bridge into Research

In problem 5(a) we obtained the exact result, the maximum length of the DNA of bacterium *bacillus anthracis*. One cannot do better. The problem 5(b), however, produced only an upper bound for the length of the DNA of bacterium *bacillus amoris*. Can we obtain the exact result? Can we at least reduce the upper bound?

"Yes we can!" as the most inspiring USA Presidential candidate in a generation, Barack Obama, says. Let us allow the 3-gene blocks to overlap by their end terms. We can then use the same argument as in the solution of 5(b) above, and reduce the upper bound from $6n + 2$ (n is here the number of available genes) to $4n + 2$. This is exactly what Martin Klazar of Charles University, Prague, presented in his 1996 talk.

Can we do better? Yes, with the clever observation of the starting and ending triples, Klazar was able to reduce the upper bound to $4n - 4$. In his 1996 talk, Martin even claimed the bound of $4n - 7$, proof of which required further cleverness. However, the problem of finding the exact maximum length of DNA remains open.

Open Problem. The DNA of bacterium *bacillus amoris* (causing love) is a sequence, each term of which is one of n genes. No three consecutive terms may include the same gene twice, and no three distinct genes can reappear in the same order. That is, if distinct genes α, β, and γ occur in that order (with or without any number of genes in between), the order $\alpha, \ldots, \beta, \ldots, \gamma$ cannot occur again. Find the maximum length the DNA of bacterium *bacillus amoris* may have.

Warnings

1. While bacterium *bacillus anthracis* (causing anthrax) does exist, I claim no knowledge of its structure. It has been simply used to captivate the imagination of the Olympians.
2. The existence of bacterium *bacillus amoris* (causing love) has not been established by science.

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Math Olympiads for the Public Schools in Brazil

Suely Druck & Michel Spira

Suely Druck holds a Ph.D. in Mathematics from the Pontifícia Universidade Católica do Rio de Janeiro, in the area of Differential Topology. She was the president of the Brazilian Mathematical Society in 2001–2005, and is presently a Professor of Mathematics at the Universidade Federal Fluminense and academic director of the OBMEP.

Michel Spira holds a Ph.D. in Mathematics from the University of California, Berkeley, in the area of Quadratic Forms and Galois Theory. He is the co-author of a math textbook for middle school and has worked extensively with teacher preparation. Currently he is a Professor of Mathematics at the Universidade Federal de Minas Gerais and head of the Exam Committee of the OBMEP.

Abstract. In this article we describe an ongoing Math Olympiads project in Brazil, its motivation, aims and structure.

1 Introduction

There are many aspects involved in the teaching of Mathematics for young people, but certainly the most important is to get students interested in the subject. Where there is interest, natural curiosity, properly nurtured and taken care of, does the job. But this has to be done right and at the right time; otherwise, as we know unfortunately too well, Mathematics turns out to be a subject charged with authoritarian overtones, full of algorithms and ideas one does not understand. It is too easy to turn a student into the "I hate Math"—or, at least, in the "I don't understand Math"—type. In what follows we will describe a project whose aim is, simply put, to reach all children from 6th grade (ages 11 on) and up in the public schools of Brazil and to get them to enjoy Mathematics; we call this project the OBMEP (short for *Olimpíada Brasileira de Matemática das Escolas Públicas – Brazilian Mathematics Olympiads for the Public Schools*).

We are quite on the way of reaching our aim, as the following table shows. The number of schools and students in the "number" line refers to those able to take part in the OBMEP; as for municipalities, all are counted. To keep all these numbers and what follows in perspective, it should be noted that Brazil is a third world country of continental dimensions, with all the attending problems of resources and communication. We also remark that those taking exams in 2008 represent around 10% of the country's population.

	schools	**students**	**municipalities**
number	55 949	±24 000 000	5 564
2005	31 030 (55.5%)	10 520 830 (43.8%)	5 198 (93.5%)
2006	32 655 (58.4%)	14 181 705 (59.1%)	5 259 (94.5%)
2007	38 450 (68.7%)	17 341 732 (72.3%)	5 461 (98.1%)
2008	40 377 (72.2%)	18 317 779 (76.3%)	5 493 (98.7%)

To finish this introduction, a word about the system of public schools in Brazil. They fall in three main categories: federal (mostly military and technical schools), state and municipal. Federal schools are not significant in numerical terms; besides, their better resources and level of teaching and learning set them apart from state and municipal ones.

For this reason, in this article we are mainly concerned with schools and students in the state and municipal systems.

2 The motivation

We won't elaborate on the causes and consequences of the serious situation of the teaching of Mathematics in the public schools in Brazil, specially in what concerns social inclusion and the scientific and technological development of the country. It is enough to say that, in spite of the efforts of past and present governments, the reality is dismal; in international tests such as PISA and TIMMS, Brazil has always been at the bottom of the list. What we discuss here is what the OBMEP can do and has done to change this situation.

Brazil has its share of traditional Math Olympiads; there are many at the state level and a major national one, the *Olimpíada Brasileira de Matemática* (OBM). This is an academic upper level competition, aimed at talented kids, and is directly responsible for Brazil's strong showing in international mathematics olympiads. However, the public schools shy away from it, since their students, with rare exceptions, are not up to their level and are usually eliminated at the first stages. As a result the OBM became, by and large, a competition for students of private, federal and military schools. We felt that something should be done to attract the interest of the students of the public schools, who did not have the preparation to compete in this kind of olympiad.

We take as given that all students (and teachers) want to learn more and desire a better and richer study environment. As a consequence, a well made and interesting exam was sure to attract the public schools and their students. But one has be careful not to frustrate them by asking for more than they can do with what they learn at school (we will, though, gradually increase the level in ϵ-increments as time goes by). As a consequence, our *modus operandi* is to propose problems which can be tackled without deep techniques, strong theorems or tricks; all one needs is basic arithmetic, algebra, geometry, counting, pattern recognition and logic skills. The idea is to emphasize good thinking skills and avoid formulae, standard problems and rote algorithms. Besides getting the students interested, this has a very important side effect, which is to give teachers different ideas to develop in the classroom. As a matter of fact, to provoke the teachers into exploring new problems and new ways of thinking about Math in the classroom is another important aim of this project.

We point out that to find talent is not our only objective. Of course we do find talented kids in very large numbers, and we take care of them (more on this later on). But our main target is the average student, which has no particular leaning for Mathematics. For these, Math and its methods of thought are tools which will help them to succeed. If, in the future, we have a generation of engineers, artists, lawyers and doctors who tell their children that Math was important in their lives (and better yet, that they like Math!), then our mission will be accomplished.

We also note that evaluation or comparisons of any sort, be it of students or schools, is an explicit non-objective of the OBMEP. In particular, it is our promise that no school will have the scores of its students made public, be it individually or on the average; the same goes for cities and states. Only national averages are published. This policy was well received by the school system and we believe it is one of the factors directly responsible for the success of the OBMEP. Relating to this last point, one can answer right now a natural question: yes, on average, the students do not do well on the exams. This is as it should be. The level of Math proficiency in the public school system is very low, as we said before. OBMEP scores only reflect this reality; we hope our project will be a tool, among others, to change this situation.

All in all, our motivation can be stated very simply. We want to change the stereotyped way Math is taught and learned in the public schools in Brazil. This is, of course, a long-term project; however, what we have attained so far shows clearly that we are headed in the right direction.

3 A short history

In 2002 the Brazilian Mathematical Society (SBM) presented the Ministries of Science and Technology (MCT) and of Education (MEC) an ambitious project. Its main idea was that, in order to interfere positively in the teaching of Math in public schools, actions were needed which ranged from teacher training and stronger ties between higher learning institutions and the schools all the way up to increasing research scholarships in pure Math. Among the actions proposed, there was one about a special Math Olympiad for the public schools. This was not an original idea; very successful similar projects were underway in some states, notably the *Numeratizar* project in the state of Ceará and the State Olympiads of the state of Sergipe. After some comings and goings, in 2004 the president and the vice-president of the SBM presented the OBMEP project directly to Brazil's president, Luís Inácio Lula da Silva. The president became (and still is) very enthusiastic about the idea and immediately embraced it. From the beginning the OBMEP has been funded by the MEC and the MCT. Our hope is that the OBMEP will become a permanent project, with future governments recognizing its scope and importance.

4 The exams

We will now describe the structure of the OBMEP in what relates to the exams.

Students are divided in three levels, aptly named 1st, 2nd and 3rd, corresponding respectively to 6th and 7th, 8th and 9th (both middle school) and 10th, 11th and 12th (high school) grades. Each level has its specific exams, which are given in two stages. In the first stage, the exam is given to all students and consists of 20 multiple-choice questions; its function is to help the school select its 5% higher scoring students to take part in the second stage. The exam of the second stage consists of six open questions, with two to four items each.

We pause here to remark that this way of selecting students for the second stage means that many students of top schools are left out, even with very good scores, while students with low scores from other schools will make it to the second stage. This is on purpose and reflects our philosophy of not looking just for talent. We want every single school to have students in the second stage. In this way, all schools take part in the whole process and, being exposed to the exams and additional OBMEP materials, feel stimulated to improve their teaching conditions. In all exams we try (not always succeeding, as we often learn *a posteriori*, but we are getting better at it) to order the questions in increasing order of difficulty. In particular, the first five questions of the first stage exams are very easy, so that the students won't feel discouraged right at the beginning. In a similar way, each open question in the second stage begins with a very easy item, just to check if the statement of the problem was understood. In this way we hope to motivate the student to try the subsequent items which are, of course, more difficult.

All exams are concocted by a committee, aptly named (again) the Exam Committee, which consists of eight people. Six of those have a Ph. D. in Pure Mathematics, one is a former International Olympiad medalist, currently teaching middle and high school, and the last is formerly and presently the captain of many Brazilian teams competing in international Math olympiads. The committee begins its work for each stage by looking at a large number of proposed questions. As an example, in 2008, the 60 questions needed for the first stage were selected, after intense discussion, out of a list of around 220 which took close to three months to compile. Besides this committee, we have outside readers who help us debug the exams; they tackle everything from grammar and Math to the perceived difficulty and the order of questions. An anthropologist (who knows Math!) reads the exams carefully in order to avoid inadequate contextualizations and use of language, among other things. And we do have a special visually impaired student team; they take care of the Braille versions of the exams, as well as the written description of the figures for the students which do not read Braille (for those, the exam is read aloud and an assistant writes the answers down).

Finally, we note that the exam committee has no clearly stated criteria for selecting problems. Aesthetics, of course, is one of our main parameters. Not being a typical classroom or textbook question is another. What clinches matters most of the time, though, is when we decide that a given problem will attract the curiosity of the students. We are not always successful in this regard, of course. But quite often we find written messages in the exams saying things like "I loved this problem" or "I did not get it, but it was fun to try". This is exactly what we want!

5 How it is done

The OBMEP begins each year with promotions aimed at schools and students. This is done via mail and internet, and also by short advertisements on national television. Students are enrolled by their schools only by internet. This is not as trivial as it seems, since internet and computers are still not available in the majority of Brazilian schools. All this process is supervised by 72 regional coordinators, mostly university professors, who contact and engage the schools, as well as municipal, state education and political authorities, who help in this process.

There follows the exams, of which we have already talked. First stage exams are given and corrected in the student's own school. The correction of these exams is done voluntarily by teachers of all disciplines; in 2008 we had around 120 000 of them doing so. As for the second stage exams, they are given in 7400 different locations in 5353 municipalities, involving around 45 000 people (all those are 2008 numbers). We make it a point that every single school which wants its students to take part in the OBMEP can do so. This involves, on occasion, sophisticated logistics for schools in faraway places and in providing for disadvantaged students.

Even sending the exams to the schools is something of a military operation, done in cooperation with the Brazilian Post Office. Exams are sent sometimes 45 days in advance so that they can reach their destination in time (paddle boats and mules are not uncommon means of transportation).

Grading of the second stage exams is a major effort, which involves university teachers and Math students in 70 grading centers. And this is not the end. The 15 000 best exams are sent to a central location in Rio de Janeiro, were they are again graded by a 70-people team.

The grading of the second stage exams is done with the help of a solution set and a grading standard, both written by the Exam Committee. Again, the making of this document is a major effort, involving in particular a two-day meeting with all regional coordinators.

We can then give out prizes. Students are given 300 gold, 900 silver and 1800 bronze medals, as well as 30 000 honor certificates. Schools whose students do well get a library of Math and Science, as well as computers and multimedia systems; their teachers get Math and Science books, as well as honor certificates. Municipal and state secretaries of education also get trophies, according to the performance of students and schools under their jurisdiction. We do so because we believe that prizes and recognition should go not only to the students, but also to the community in which they are inserted.

Our pet prize, however, goes to the medalists. Besides their medals, they are given a one-year Scientific Initiation scholarship by the National Council of Reseach of Brazil (CNPq). This scholarship allows the receipt of mathematical instruction, on weekends, above and beyond what they get in their schools. These activities take place in selected universities and, in far-away places, in schools. We mention that these activities are open to selected non-medalists and interested teachers. Besides the scholarships, the students also get transportation, food, specially written texts and (not always, but increasingly) long distance tutoring between meetings. Again we won't get into the details of the extraordinarily difficult logistics of this program, in particular due to the fact that Brazil is a very, very large country; suffice it to say, canoe river crossings and 500-mile trips are routine.

Three special meetings close the OBMEP each year. Around 120 teachers from all over the country are selected, based on their students' performance at the OBMEP, and meet at the Institute of Pure and Applied Mathematics (IMPA) in Rio de Janeiro for a week of special courses and talks. Gold medalists meet for a week for an intensive program of talks, short courses and—of course, they are still kids—social interaction and mathematical talk (and boy, do they talk!). Both meetings are taken care of by specially selected university professors. These meetings have, besides their mathematical function, the aim of creating and strenghtening ties between students, teachers, the community of Math researchers and higher learning institutions. The third meeting is a national presentation ceremony for the gold medalists, which last year took place at the Teatro Municipal do Rio de Janeiro and was attended by the president of Brazil and the ministers of Science and Technology and of Education.

As we have already mentioned, financial support for the OBMEP comes from MEC and MCT. It is gratifying to say that the IMPA has sponsored the OBMEP from its very beginning, putting its physical and administrative resources at our disposal. (This is another example of the strong interaction between the OBMEP and higher learning institutions.) And, of course, the SBM is still a strong partner of the OBMEP.

6 Publications

In this short section we talk about the publications generated by the OBMEP, which fall into two categories.

When the OBMEP began, we wanted to make the public schools know that the exams would not be of the standard academic type. To this end, we sent them a collection of solved problems, so that they could understand our philosophy and also to give teachers classroom material. To our surprise, messages poured in large numbers demanding more and more problems. This showed us that the public schools and their teachers were avid for this type of Math teaching material, and that it was up to us to keep on delivering it. As of now, every year a big booklet of solved problems with solutions is sent to the schools, well before the exams. Problems are either original or culled from national and international Math Olympiads. Solutions are written very carefully and are meant for the teacher's use in the classroom. The demand for these booklets has increased steadily; they are a huge and unexpected editorial success, and—again to our surprise—are being used in undergraduate Math teacher preparation programs.

There are also the texts for the scientific initiation students. They are written on demand and with great care by experienced mathematicians. So far texts have been written on cryptography, number theory, combinatorics, induction, plane geometry, and so on.

All these texts, as well as exams and solutions, are available to the general public at our homepage `www.obmep.org.br`.

7 What have we got

As we have already said, our main aim is to change the reality of Math teaching in the public schools in Brazil, giving to students of those schools access to the same challenges as those in the private schools. We hope to foster social inclusion and opportunities for the students, as well as give better conditions for the teaching of Math in the public schools. And of course we want and do locate talent, which would otherwise go unnoticed.

All these aims have been accomplished to an extent which transcends our wildest expectations. Students who do well on the OBMEP are entering universities in higher and higher numbers and, while still at public school, are courted by private schools with scholarships. Universities, schools and teachers have developed ties which have resulted in uncountable programs for teacher improvement, based on our problem books, scientific initiation texts and exams. Uncountable also are the number of schools in which students and teachers decided to create Math clubs and/or meet outside the classroom to study Math.

As far as the interaction between the OBMEP and universities goes, a very exciting development has just happened. In conjunction with the MEC and the MCT, a program for giving two-year scientific initiation scholarships followed by two-year Math graduate studies scholarships to OBMEP medalists entering undergraduate school will begin in 2009. It should be mentioned that these scholarships will be given to all medalists who enter universities, not only to those who follow Math. This program aims to increase the number of mathematicians in Brazil and offer better mathematical training to students of other scientific and technological areas.

Some state governments have offered several hundreds of scientific initiation scholarships in addition to those given by the OBMEP, and have sponsored large ceremonies for the medalists. And in some states the school system has made a place for an "OBMEP Day" in their yearly planning.

We have managed to reach schools all over the country: frontier posts, villages deep in the jungle, and indian and *quilombola* (slave descendents) communities. We have even reached schools inside the prison system, including those for minor offenders.

We believe that we have been able to change the traditional view on what a Math Olympiad is. The OBMEP is seen not only as a competition which gives prizes, but as an educational project. The exams are perceived as teaching tools and have the general approval of students and teachers alike, as well as of the mathematical community in Brazil. All in all, we can say that our initial aims have been accomplished beyond our expectations, quantitatively and qualitatively speaking. It is still too early to measure objectively the results of the OBMEP as far as social inclusion is concerned but we have already preliminary data showing that taking part in the OBMEP substantially increases one's chances of passing the entrance exams of the best Brazilian universities.

In closing, we would like to say that we would be happy to share our experience with those interested, as well as to provide further details about the OBMEP.

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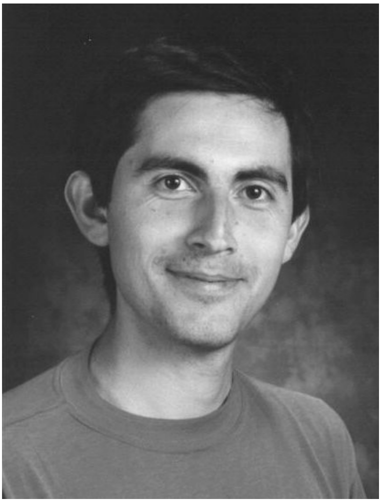
Second Mathematical Competition *30 Questions in 30 Languages*

Ivaylo Kortezov



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The Second Mathematical Competition *30 Questions in 30 Languages* took place on 25 May 2008. It involved students from Bulgaria, the Czech Republic, Greece and Romania. The Czech contestants and some of the Greek ones competed by correspondence. The event was hosted by the National High School of Mathematics and Natural Sciences in Sofia, Bulgaria. Each of the 30 teams consisted of (not more than) four students, who worked together in a separate room. The contestants had three hours to solve 30 multiple-choice questions, each written in a different live European language, without the aid of any type of literature or electronic devices. They had to find the correct answer and to determine the language in which the problem was written.

The first competition, held two years ago, involved only Bulgarian students. They liked the format of the competition very much and so stimulated the reappearance of this competition. The idea to make the competition international came from the St. Cyril and St. Methodius International Foundation in Bulgaria; it supported the participation of the teams from Romania and Greece and the overall organization of the competition as well. The set of 30 questions is given below.

1. En påse innehåller blå, röda och gröna kulor. Summan av de blå och röda kulorna är 23. Summan av de röda och de gröna kulorna är 34 och summan av de gröna och blå kulorna är 47. Hur många kulor fanns det i påsen?

A) 18 B) 29 C) 47 D) 52 E) 57

2. Діагоналі ромба дорівнюють 15 см і 20 см. Знайдіть радіус кола, вписаного у ромб.

A) 6 см B) 8 см C) 9 см D) 10 см E) 12 см

3. Në një pako ndodhen 3 fjalorë, 7 libra të matematikës dhe 15 romane. Sa është probabiliteti që një libër i nxjerrë të jetë roman?

A) 30 % B) 40 % C) 50 % D) 60 % E) 70 %

4. Vi har en sekskant $ABCDEF$ som er omskrevet en sirkel, dvs. at sidene tangerer sirkelen. Dersom $AB = 4$, $BC = 5$, $CD = 6$, $DE = 7$, $EF = 8$, hvor lang er da den siste siden?

A) 6 B) 7 C) 8 D) 9
E) ikke entydig bestemt

5. W trójkącie równoramiennym ABC ramiona tworzą kąt ACB, który ma $36°$. Dwusieczna kąta CAB przecina bok BC w punkcie D. Odcinek CD ma długość 6. Jaką długość ma bok AB?

A) 3 B) 4 C) 6 D) 8 E) 9

6. Предната гума на еден велосипед се излижува после изминати 600 km, а задната по изминати 400 km. После колку изминати километри треба да се променат местата на гумите за да тие се излижат истовремено?

A) 230 B) 240 C) 250 D) 260 E) 270

7. Într-o clasă sunt f fete şi b băieţi, numerele f şi b fiind direct proporţionale cu 4 şi 3. Un elev sau o elevă este transferat(ă) în altă clasă şi astfel numărul băieţilor a devenit 80 % din numărul fetelor. Să se afle diferinţa $f - b$.

A) 2 B) 3 C) 4 D) 5 E) 6

8. In una elezione si presentano solo tre partiti: repubblicani, anarchici e socialisti. Gli anarchici ottengono 2/5 dei voti dei socialisti e questi 5/6 dei voti dei repubblicani. Quanti voti ottiene il partito anarchico in un seggio di 910 votanti?

A) 130 B) 140 C) 210 D) 260 E) 350

9. Prvním přívodem se naplní bazén za 18 hodin, druhým za 12 hodin. Oba byly otevřeny současně v 6:15. V 9:15 byl otevřen třetí, kterým by se bazén naplnil za 9 hodin. V kolik hodin byl plný ze 70 %?

A) 10:23 B) 10:24 C) 10:25 D) 10:26 E) 10:27

10. Bestimmen Sie zwei Zahlen x und y mit dem Produkt 0,1 so, daß die Summe ihrer Kehrwerte gleich 7 ist. Dann $x^2 + y^2 =$

A) 0,13 B) 0,1225 C) 0,29 D) 0,7 E) 1,01

11. Як далёка бачна пры пагодзе з самалёта, які ляціць на вышыні 8 км над паверхняй акіяна? Радыус зямнога шара лічыць роўным 6396 км.

A) 80 км B) 120 км C) 160 км D) 240 км E) 320 км

12. Lad a, b, c, d, e, f være ikke-negative reelle tal som opfylder $a + b + c + d + e + f = 6$. Find den størst mulige værdi af $abc + bcd + cde + def + efa + fab$.

A) 6 B) 7 C) 7,5 D) 8 E) 9

13. Enakokraki trapez je sestavljen iz treh enakostraničnih trikotnikov. Srednjica trapeza meri 12 cm. Izračunaj obseg trapeza.

A) 20 cm B) 32 cm C) 40 cm D) 48 cm E) 64 cm

14. Body A, B rozdeľujú kružnicu k na menší a väčší oblúk. Obvodový uhol nad väčším oblúkom sa rovná stredovému uhlu nad menším oblúkom. Vypočítajte veľkosť obvodového uhla nad menším oblúkom.

A) 30° B) 60° C) 90° D) 120° E) 150°

15. Одредити реалан параметар p, тако да збир квадрата решења једначине $x^2 + 2px + 3p - 2 = 0$ буде минималан.

A) 0,25 B) 0,375 C) 0,75 D) 1,25 E) 1,5

16. Apskaičiuokite $x^4 + y^4 + z^4$, kai $x + y + z = 0$ ir $x^2 + y^2 + z^2 = 1000$.

A) 100000 B) 250000 C) 500000 D) 750000 E) 1000000

17. Sea x una fracción entre $\frac{8}{9}$ y $\frac{222}{223}$. Si el denominador de x es 2007 y el único factor común del numerador y el denominador es 1, ¿cuántos valores posibles hay para x?

A) 139 B) 140 C) 141 D) 142 E) 143

18. Номерата на колите от една серия са от 0001 до 9999. Колко най-много коли от тази серия можем да вземем, така че да няма две, чийто сбор от номерата се дели на 2008? (Водещите нули се игнорират.)

A) 4013 B) 4014 C) 5015 D) 5016 E) 5017

19. Mitu reaalarvude paari (x, y) rahuldavad võrrandit $(x + y)^2 = (x + 5)(y - 5)$?

A) 0 B) 1 C) 2 D) 3
E) lõpmatult palju

20. Skaitļi 1, 2, 3, 4, 5, 6, 7, 8, 9 sadalīti trīs grupās pa trim skaitļiem katrā. Katrai grupai $\{a, b, c\}$ aprēķināts tajā ietilpstošo skaitļu reizinājums abc. Apzīmēsim lielāko no šiem reizinājumiem ar M. Kāda ir mazākā iespējamā M vērtība?

A) 60 B) 64 C) 66 D) 72 E) 80

21. Hány azok az n természetes számok, amelyekre igaz, hogy $0{,}2 < \frac{n}{n+96} < 0{,}25$?

A) 7 B) 8 C) 9 D) 10 E) 11

22. Hoeveel getallen zijn de som van zeven verschillende getallen uit de getallen $1, 2, \ldots, 2008$?

A) 7056 B) 14008 C) 28016 D) 56032 E) 2017036

23. A semicircunferência de centro O e diámetro $AB = 4$ é dividida em dois arcos AC e BC na relação 2:1. M é o ponto médio do raio OA. Seja T o ponto do arco AC tais que a área do quadrilátero $OCTM$ é máxima. Calcular a área do este quadrilátero.

A) $\sqrt{8}$ B) $\sqrt{7}$ C) $\sqrt{6}$ D) $\sqrt{5}$ E) 2

24. Ann holds six strings in her hand; their ends are sticking out above and below her palm. Betsy ties the upper ends in pairs. Then Cathy ties the lower ends in pairs, too. What is the probability that the six strings are now connected in a single ring?

A) 7/15 B) 2/5 C) 1/3 D) 3/5 E) 8/15

25. On considère les parties A de l'ensemble $\{1, 2, 3, \dots, 2008\}$ possédant la propriété suivante: si x appartient à A, alors $3x$ n'appartient pas à A. Déterminer le nombre maximal d'éléments d'une telle partie A.

A) 1339 B) 1488 C) 1502 D) 1504 E) 1506

26. $ABCD$ bir paralelkenar ve $AB = 25$ cm dir. A ve D köşerinden çizilen içaçıortaylar E noktasında, $AE = 12$ cm, $DE = 9$ cm ise. Alan $ABCD$ kaç cm^2 dir?

A) 180 B) 240 C) 270 D) 360 E) 540

27. Ratkaise epäyhtälön $|x^2 - 6x + 7| < 2$.

A) $(1; 5)$ B) $(3; 5)$ C) $\{1; 5\}$ D) $(1; 7)$
E) $(1; 5) \setminus \{3\}$

28. Говядина без костей стоит 92 рублей за килограмм, говядина с костями – 80 рублей за килограмм, а кости без говядины – 12 рублей за килограмм. Сколько костей в килограмме говядины с костями?

A) 125 г B) 150 г C) 175 г D) 200 г E) 225 г

29. Σε κύκλο ακτίνας $\sqrt{12}$ cm είναι περιγεγραμμένο ισόπλευρο τρίγωνο. Να υπολογίσετε την πλευρά του.

A) 6 cm B) 8 cm C) 9 cm D) 12 cm E) 18 cm

30. Koliko postoji prirodnih brojeva n takve da postoji ceo broj m za koji je $3^m = 30 \cdot n! + 2007$?

A) 0 B) 1 C) 2 D) 3 E) 4

The students attacked the problems enthusiastically. Form a team and try to solve them! The outcome was that many of the teams managed to decode and solve a great deal of the problems. What was very important was the team work: each student found different pieces of the puzzle. At the end the students—including the ones who missed out on an award—were happy that they had overcome together a seemingly impossible task. And all of them were asking when would be the next such event.

Among the students competing in Bulgaria, the best result was achieved by a combined team of students from the Sofia Math School and the German Language School in Sofia, who did 27 out of 30 questions correctly and determined the language of 26 out of 30. Among the students who competed by correspondence, the best result was achieved by the team of the Copernicus High School in Bilovec, Czech Republic, who found the correct answers to 20 problems and managed to correctly

*	Probl.	Language
English	24	English
Svenska	1	Swedish
Français	25	French
Türkçe	26	Turkish
Deutsch	10	German
Русский	28	Russian
Italiano	8	Italian
Polski	5	Polish
Български	18	Bulgarian
Español	17	Spanish
Македонски	6	FYR Macedonian
Ελληνικά	29	Greek
Português	23	Portuguese
Shqip	3	Albanian
Nederlands	22	Dutch
Slovenščina	13	Slovene
Română	7	Romanian
Slovenčina	14	Slovak
Norsk	4	Norwegian
Српски	15	Serbian
Українська	2	Ukrainian
Dansk	12	Danish
Беларуская	11	Belarusian
Eesti	19	Estonian
Hrvatski	30	Croatian
Česky	9	Czech
Magyar	21	Hungarian
Latviešu	20	Latvian
Lietuvių	16	Lithuanian
Suomi	27	Finnish

Table 1: Test answer keys

determine the language of all 30 questions. Here are the answer keys (Tables 1 and 2), the English translation of the problems and the solutions.

1: D	2: A	3: D	4: A	5: C	6: B
7: C	8: B	9: A	10: C	11: E	12: D
13: C	14: B	15: C	16: C	17: D	18: E
19: B	20: D	21: A	22: B	23: B	24: E
25: E	26: D	27: E	28: B	29: D	30: B

Table 2: Test answer keys

1. A bag contains blue, red and green balls. The sum of the blue and red balls is 23. The sum of the red and green balls is 34 and the sum of the green and blue balls is 47. How many balls are there in the bag?

Answer D. $2b + 2r + 2g = 23 + 34 + 47 = 104$, so $b + r + g = 52$.

2. The diagonals of a rhombus have lengths 15 cm and 20 cm. Find the inradius of the rhombus.

Answer A. The side of the rhombus is $\frac{25}{2}$ cm, so $\frac{15}{2} \cdot 10 = \frac{25}{2}r$ and $r = 6$ cm.

3. In a parcel there are 3 dictionaries, 7 math books and 15 novels. What is the probability that a chosen book is a novel?

Answer D. $15/25 = 60\,\%$.

4. We have a hexagon $ABCDEF$ that is circumscribed about a circle, i.e. its sides are tangent to the circle. If $AB = 4$, $BC = 5$, $CD = 6$, $DE = 7$, $EF = 8$, how long is the sixth side? [E) not uniquely determined]

Answer A. We have $AB + CD + EF = BC + DE + FA$, hence $FA = 6$.

5. The legs of an isosceles triangle ABC form an angle $ACB = 36°$. The bisector of angle CAB intersects the leg BC at point D. The segment CD has length 6. What is the length of AB?

Answer C. Calculate the angles to see that there are two smaller isosceles triangles, $AB = AD = DC = 6$.

6. The front tyre of a bike wears out after 600 km, while the rears tire after 400 km. After how many kilometers must we swap the two tires, so that they wear out at the same time?

Answer B. For x km the front tire wears $x/600$, and the rears $x/400$. To wear them out, we need to have $x/600 + x/400 = 1$. We get $2x + 3x = 1200$, whence $x = 240$.

7. In a class there are f girls and b boys, so that f and b are directly proportional to 4 and 3. A student has been transferred to another class and so the number of boys has become $80\,\%$ of the number of girls. Find the difference $f - b$.

Answer C. As $3 : 4 < 4 : 5 = 80\,\%$, a girl was moved. If $f = 4x$ and $b = 3x$, we have $5 \cdot 3x = 4(4x - 1)$, so $x = 4$.

8. In an election only three parties were present: republicans, anarchists and socialists. The anarchists obtained 2/5 of the votes of the socialists and the latter obtained 5/6 of the votes of the republicans. How many votes did the anarchists obtain in a poll of 910 votes?

Answer B. Let the republicans get $6x$ votes. Then the socialists get $5x$ votes and the anarchists $2x$ votes. In total we have $13x = 910$, hence $x = 70$ and $2x = 140$.

9. A pipe can fill a pool in 18 hours, and another pipe can fill it in 12 hours. They were both opened at 6:15. At 9:15 a third pipe was opened; it could fill the pool in 9 hours. At what time was the pool 70 % full?

Answer A. Let x be the time after 6:15. Then $x/18 + x/12 + (x - 3)/9 = 0.7$. We multiply both sides by 36 and get $2x + 3x + 4(x - 3) = 25.2$. Now $9x = 37.2$ and $x = 124/30 = 4\,\text{h}\,8\,\text{min}$. This means 10:23.

10. Find two numbers x and y with product 0.1, so that the sum of their reciprocals be 7. Then $x^2 + y^2 =$

Answer C. Let the reciprocals of x and y be a and b; their product is 10. Then they are the roots of the equation $z^2 - 7z + 10$, whose roots are 2 and 5. Then $x^2 + y^2 = 0.5^2 + 0.2^2 = 0.25 + 0.04$.

11. How far can we see from a plane flying at a height of 8 km above the ocean? Assume that the Earth's radius equals 6396 km.

Answer E. The required distance t equals the length of the tangent from the plane to the Earth. We have $t^2 = 8 \cdot 12800 = 102400$, so $t = 320$.

12. Let a, b, c, d, e, f be non-negative real numbers satisfying $a + b + c + d + e + f = 6$. Find the maximal possible value of $abc + bcd + cde + def + efa + fab$.

Answer D.

$$\begin{aligned} abc + bcd + cde + def + efa + fab &= (ab + de)(c + f) \\ \le (a + d)(b + e)(c + f) &\le \left(\frac{a + b + c + d + e + f}{3}\right)^3 = 8. \end{aligned}$$

This value is attained e.g. for $a = b = c = 2$, $d = e = f = 0$.

13. An isosceles trapezoid is formed by three equilateral triangles. The median of the trapezoid is 12 cm long. Find the perimeter of the trapezoid.

Answer C. The sides of the equilateral triangles are 8 cm long, so the perimeter is 40 cm.

14. The points A, B divide the circle k into a minor and a major arc. The inscribed angle of the major arc equals the central angle of the minor arc. Find the magnitude of the inscribed angle of the minor arc.

Answer B. Let the inscribed angle of the major arc and the central angle of the major arc be equal to $x°$. Then these arcs are $2x° + x° = 360°$, hence $x = 120$ and the inscribed angle measures $60°$.

15. Find the real parameter p, such that the sum of squares of the roots of the equation $x^2 + 2px + 3p - 2 = 0$ is minimal.

Answer C. We have $x_1 + x_2 = -2p$, $x_1x_2 = 3p - 2$, so $x_1^2 + x_2^2 = 4p^2 - 6p + 4$ is minimal when $p = 6/8 = 0.75$. In this case the equation is $x^2 + 1.5x + 0.25 = 0$, whose roots are real numbers.

16. Calculate $x^4 + y^4 + z^4$, if $x + y + z = 0$ and $x^2 + y^2 + z^2{=}1000$.

Answer C. We square the first identity and use the second to get $xy + yz + zx = -500$. We square this once again to get $x^2y^2 + y^2z^2 + z^2x^2 + 2xyz(x+y+z) = 250000$. Since $x+y+z = 0$, we get $x^2y^2 + y^2z^2 + z^2x^2 = 250000$. Squaring the second given identity yields $1000000 = x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + z^2x^2) = x^4 + y^4 + z^4 + 500000$ and so $x^4 + y^4 + z^4 = 500000$.

17. Let x be a fraction between $\frac{8}{9}$ and $\frac{222}{223}$. If the denominator of x is 2007 and the only common factor of the numerator and denominator is 1, how many possible values are there for x?

Answer D. The numerator is between $8 \cdot 223 = 1784$ and $9 \cdot 222 = 1998$, so there are 213 possible values. We have to exclude the 71 multiples of 3 $(1785, \ldots, 1995)$. There are no multiples of 223 there. We are left with $213 - 71 = 142$ numbers.

18. The plates of the cars of a given series have numbers from 0001 to 9999. What is the maximum possible number of cars we can take so that no two of them have a sum that is a multiple of 2008? (Ignore the leading zeroes.)

Answer E. We cannot take two cars with numbers i and $2008 - i$ for a given i. Thus the best we can do is to take all the numbers having remainders $1, 2, 3, \ldots, 1003$ modulo 2008, plus only one with remainder 1004 and only one with no remainder. Thus we have $5 \cdot 1003 + 2 = 5017$ numbers.

19. How many real pairs (x, y) satisfy the equation $(x + y)^2 = (x + 5)(y - 5)$? [E) Infinitely many]

Answer B. By replacing $x + 5 = a$, $y - 5 = b$ we have $(a + b)^2 = ab$ and so $a^2 + ab + b^2 = 0$ which is true only for $a = b = 0$. Thus $x = -5$, $y = 5$.

20. The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 are divided into three groups of three numbers each. For each group $\{a, b, c\}$ we calculate the product abc. Denote the greatest of these products by M. What is the least possible value of M?

Answer D. We can have $1 \cdot 8 \cdot 9 = 3 \cdot 4 \cdot 6 = 72$ and $2 \cdot 5 \cdot 7 = 70$. We cannot have M less than 72, as 71 is prime, while $9! > 70.70.70$.

21. How many natural numbers n satisfy $0.2 < \frac{n}{n+96} < 0.25$?

Answer A. Replacing by the reciprocals, we get $4 < 1 + 96/n < 5$, that is, $3 < 96/n < 4$, or $24 < n < 32$. There are seven possible values of n.

22. How many numbers are equal to the sum of seven different numbers among $1, 2, \ldots, 2008$?

Answer B. The least such number is $1 + 2 + \ldots + 7 = 28$, while the largest possible is $2002 + 2003 + \ldots + 2008 = 14035$. Any number between them can be obtained by an appropriate choice of the seven numbers, so there are 14008 possibilities.

23. A semicircle of center O and diameter $AB = 4$ is divided into two arcs AC and BC in ratio 2:1. M is the midpoint of the radius OA. Let T be a point on the arc AC such that the area of the quadrilateral $OCTM$ is maximal. Find the area of this quadrilateral.

Answer B. We have $OA = OB = OC = OT = 2$, $OM = 1$, $MC = \sqrt{7}$ (the latter can be obtained either by twice the Pythagorean Theorem, or directly from the Cosine Law). The area of a quadrilateral does not exceed the semiproduct of its diagonals, and equals it exactly when they are perpendicular to one another. Thus the maximal area of $OCTM$ is $2\sqrt{7}/2 = \sqrt{7}$.

24. Ann holds six strings in her hand; their ends are sticking out above and below her palm. Betsy ties the upper ends in pairs. Then Cathy ties the lower ends in pairs, too. What is the probability that the six strings are now connected in a single ring?

Answer E. WLOG the strings are tied above in pairs U-V, W-X, Y-Z. The probability that we have three rings is $(1/5)(1/3) = 1/15$. The probability that we have a ring of just U-V and a ring of four strings is $(1/5)(2/3) = 2/15$. The same holds for W-X and for Y-Z. Now the required probability is $1 - (1/15) - 3(2/15) = 8/15$.

25. We consider the subsets A of the set $\{1, 2, 3, \ldots, 2008\}$ with the following property: if x belongs to A, then $3x$ does not belong to A. Determine the maximal possible number of elements in such a subset A.

Answer E. Decompose A into the sets $\{a; 3a; 9a; \ldots\}$ for each $a < 2009$ that is not a multiple of 3. If such a set contains x elements, we can take in A at most $(x + 1)/2$ of them. This can be done in the following way: A contains all the positive integers up to 2008 that are:

- not multiples of 3 (they are $2008 - \lfloor 2008/3 \rfloor = 1339$);
- divisible by 9, but not by 27 (they are $[\lfloor 2008/9 \rfloor - \lfloor 2008/27 \rfloor = 223 - 74 = 149$);
- divisible by 81, but not by 243 (they are $\lfloor 2008/81 \rfloor - \lfloor 2008/243 \rfloor = 24 - 8 = 16$);
- divisible by 729 (they are $\lfloor 2008/729 \rfloor = 2$).

In total we have $1339 + 149 + 16 + 2 = 1506$ numbers in A.

26. $ABCD$ is a parallelogram and $AB = 25\,\text{cm}$. The angle bisectors of A and D intersect at point E, $AE = 12\,\text{cm}$, $DE = 9\,\text{cm}$. How many cm^2 is the area of $ABCD$?

Answer D. By Pythagoras we have $AD = 15\,\text{cm}$. Continue the line DE until it meets AB at point K; we have $AK = AD = 15\,\text{cm}$. Let DH be an altitude. Then $AK \cdot DH = AE \cdot DK$, so $DH = 12 \cdot 18/15 = 72/5\,\text{cm}$. So the area of $ABCD$ is $25 \cdot 72/5 = 5 \cdot 72 = 360\,\text{cm}^2$.

27. Solve the inequality $|x^2 - 6x + 7| < 2$.

Answer E. It is equivalent to $-2 < x^2 - 6x + 7 < 2$, that is, $0 < (x-3)^2 < 4$. Now $x-3$ is in $(-2; 2) \setminus \{0\}$, so x is in $(1; 5) \setminus \{3\}$.

28. A kilogram of beef without bones costs 92 rubles, a kilogram of beef with bones costs 80 rubles, and a kilogram of bones costs 12 rubles. How many grams of bones are there in a kilogram of beef?

Answer B. Let there be x kg of bones in a kilogram of beef. Then its price will be $12x + 92(1 - x) = 80$, hence $12 = 80x$ and $x = 12/80 = 0.150\,\text{kg} = 150\,\text{g}$.

29. An equilateral triangle is circumscribed about a circle of radius $\sqrt{12}\,\text{cm}$. Find its side.

Answer D. Its side equals $2 \cdot \sqrt{12} \cdot \sqrt{3} = 12\,\text{cm}$.

30. For how many natural numbers n does there exist an integer m such that $3^m = 30 \cdot n! + 2007$?

Answer B. If $n < 3$, the right-hand side is not divisible by 9. If $n > 5$, the right-hand side is divisible by 9, but not by 27. It remains to check $n = 3, 4, 5$, but only the first one gives an integer solution.

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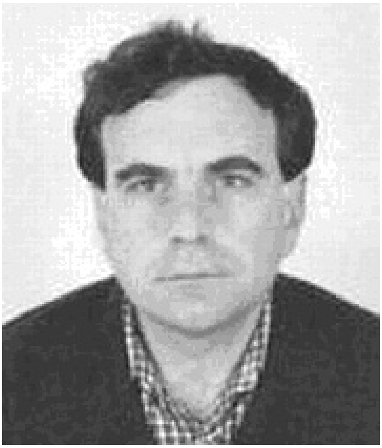
Risk-taking Behavior in Math Competitions

Dimitar Pavlov, Emil Kelevedjiev & Jordan Tabov



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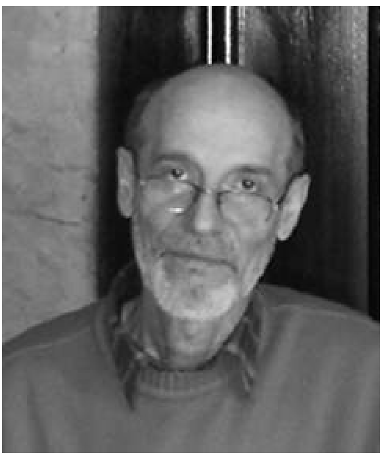

Jordan Tabov graduated with a Ph.D. from Moscow State University in 1974, and is now Head of the Department of Education in Mathematics and Informatics of the Institute of Mathematics of the Bulgarian Academy of Sciences. As a student he represented Bulgaria in the International Mathematical Olympiads in 1964 and 1965. He led the Bulgarian team on a number of occasions and has since had many years of close involvement with their training and success. In 1994 he was awarded The Paul Erdös Award of the WFNMC.

Prof. Dr. Dimitar Pavlov has led the most profound changes in the Bulgarian educational system and pedagogical research over the past 50 years. He harnessed the power of the internet and other educational technologies to shift the focus of Bulgarian education from uniformity to creativity and free individual expression and development. He developed for the first time in Bulgaria courses on educational technologies. He created and headed the Center of Educational Technologies at Sofia University. He was the chairman and national coordinator for international projects under the EU-sponsored TEMPUS program. Prof. Pavlov was Vice-rector for Curricula and Student Services at Sofia University "St. Kliment Ohridski" and Dean of its School of Education. He was the Director of the Central Institute for Teachers Development in Sofia.

The new century is clearly defining and outlining the so-called "information society." It is shaped by various characteristics, from information saturation, for example, to rapid increase of opportunities. If the past century was the time of the masses, the present one is "a century of the individual." This transition makes life preparation, the formation and development of a whole new system of personal qualities, and the emphasis on individual decision-making and responsibility crucial components of education as a whole.

This transition is defined by the individual's independence, efficiency, initiative, orientation in a complex multifactor environment, stability in chaos, fast reactions, decision-making skills, behavior strategies in conflict situations, clear definition of goals, measurement systems, analysis and synthesis skills, personal responsibility, competitiveness, helpfulness, fast concentration, transition from one field to another, unconventional thinking, original ideas and so on. *One of the most important factors underlying all these abilities and values is the capacity* *to measure and assess risks, and to choose the optimal risk reward tradeoffs.*

In this paper, we investigate the risk assessment and risk avoidance behavior of secondary and high school students using their performance on a particular competition in Mathematics. This setting provides for an objective measure of risk-taking behavior. We find that attitudes towards risk changes substantially with age. For instance, students tend to take a lot less risks as they age. Furthermore, we document that males tend to take on more risks then females, although the differences are relatively small.

Our work also opens the possibility to repeat the measurement in the future, and investigate whether attitudes towards risk change over time as students become more independent and face ever increasing sets of opportunities.

1 Choice, risk and risk behavior

Etymologically, the word "risk" comes from French (risqué), meaning a possible danger, an arbitrary action with expectations for success, a possible loss in a trade deal, or a danger that may require insurance. The verb "to risk" (Fr.—risqué) is associated with exposing oneself to a danger or a possible failure, or attempting to do something (Milev, et. al., 1958).

Taking a risk is also defined as: "1. A spontaneous action, committed by an individual without preliminary consideration; 2. An action, provoked by the individual's confidence that the probability to stay fit and safe is very high" (Keinan, 1984).

It is a common belief that men tend to exhibit risk-taking behavior more than women do. Usually this is explained by qualities traditionally assigned to men, such as bravery, self-esteem, independence, determination, strength and competitiveness.

With the development of democracy in Bulgaria the opportunities for choice, both personal and professional, continuously increase. A truly free person is the one who has the right and opportunity to choose.

Personal choice does not necessarily generate risk. On one hand, the right, needed, desired choice requires sufficient justification and guarantees. On the other, every personal choice is in direct correlation with the development of a sense of responsibility.

Risk is a complex phenomenon. It depends on a practically unlimited number of factors. Risk is often connected to the ability to compromise in a world with constantly decreasing security, where fewer decisions can eventually be classified as right or wrong with great certainty. Total uncertainty is most simply illustrated by the fact that when achieving something, we do not know what else we are missing in our time-constrained life.

2 Tests and Risk

The pedagogical options and limits of the multiple-choice tests have been examined extensively and the results have been described in pedagogical publications. Until now, too little attention has been given to the problem of risk and its influence on students during exams, multiple choice in particular.

Some student competitions (in math, chemistry, physics) use such tests extensively. There are certain rules to formulate the questions and the possible answers depending on the purpose, the values, the expectations, the evaluation model (forming, summing, by criteria, normative, sorting, etc.).

Often there are discussions about how to evaluate a blank or a wrong answer. If these two are counted in the same way, this would stimulate guessing and arbitrary choices. That kind of behavior stands out if there is weak motivation and not enough sense of responsibility. That is why in practice giving a wrong answer is penalized more severely than leaving a blank. This approach aims at increasing the accuracy of evaluation through eliminating the element of chance and guessing when giving an answer. It is expected that if somebody knows the answer, they will mark it right away. Those who do not know the answer will not try to guess it (Rowley and Leder, 1989; Forgasz and Leder, 1991).

To accept this thesis, we cannot ignore at least two important facts:

1. People are different. Personal traits influence the participants' choice as to whether they will actually refrain from guessing if the method of evaluation discourages that (Gronlung 1976, Mehrens and Lehmann 1991).

2. Fully denying the possibility of guessing may eliminate the means to reach the truth after all.

3 Specifics of the Aggregation of Students

The Chernorizets Hrabar Tournament is a Bulgarian math competition for students that uses a multiple-choice examination as the means to test the participants. In 2002, the students were separated in five different age groups, 3^{rd}–4^{th} grade, 5^{th}–6^{th} grade, 7^{th}–8^{th} grade, 9^{th}–10^{th} grade, and 11^{th}–12^{th} grade. Every group had a separate test with problems appropriate to the age of the participants and with difficulty consistent with the material covered at school. The number of problems varied from 15 for the youngest to 30 for the oldest students, and all of them had 90 minutes to solve them.

Most of the students that take part in the competition could be described as having "advanced mathematical skills" and typically rank in the top 15–20 % of all students. The participants are highly motivated to take part in the competition. For many of them, this is an excellent opportunity to stand out. Also, the fact that participants are charged a fee (sometimes paid by the school or by a sponsor but most commonly by the parents) increases the sense of responsibility among students. It is clear that if the students are not interested, it would be much easier for them to just miss the tournament.

As a long-lived tradition shows, participants in the competition work hard and do as much as they can to achieve the highest possible result. That is why they almost never show negligence or irresponsibility. This makes the competition results a valuable source of information that could give trustworthy conclusions if thoroughly analyzed.

The main subject of the present study is "risk-taking" by students with mathematical skills (and the participants in the competition play the role of a representative group of those students).

Of course, the study is not about introducing a quantitative scale that looks at individual cases but rather a comparison of the average "risk-taking" typical for boys and girls in the different age groups.

4 Test Specifications of the Chernorizets Hrabar Tournament

It is important to note that the tournament tests follow the principle that a wrong answer should be penalized. In other words, there is a difference between giving a wrong answer and leaving a blank. According to the rules of the tournament, a wrong answer on a given problem results in zero points for the participant, while not answering at all (leaving a blank answer) earns them three points.

In addition, the problems are separated in three groups according to their difficulty (for example, the 11^{th}–12^{th} grade test has 10 easy, 10 medium and 10 hard problems) and for every right answer the participant gets 5, 7 or 9 points, according to the difficulty of the problem. The 3-point "penalty" is designed to make systematic "guessing" a losing strategy. That is why participants usually fill in the answers only to the problems that they are convinced they have solved correctly.

5 Analysis of the Risk during Tests

Statistically, choosing wrong answers and, to a certain extent, leaving less blanks is a sign of "risk-taking". When the participant solves correctly a given problem, he or she selects the right answer. If he cannot solve it, however, they face a dilemma: should he mark an answer he thinks could be the right one, or should he leave a blank? Choosing to mark an answer brings the possibility of the 3-point penalty that the participant will lose compared to the "blank" answer that would not be penalized. It should be noted that marking the wrong answer could be the result of risk-taking of a different nature. It could be caused by one of the following factors:

- risk-taking in the case of uncertainty or hesitation
- risk-taking in the case of pure "guessing" of the answer

- a mistake during the solution of the problem (because of uncertain knowledge or bad concentration); or a hard test that predisposes the participant to make more mistakes.

The study is directed mainly to the first two hypotheses. For that purpose, the influence of each of the two abovementioned factors is analyzed based on the results of the test conducted on 1 November 2002.

Having in mind the studied group of students that participated in the tournament—that is, students with advanced mathematical skills, strong background and ambition, and having in mind their motivation to show their best, we could assume that the "pure guessing" of an answer is a much rarer event than guessing due to uncertainty or hesitation. Also, we could assume that the frequency of the "pure guessing" is proportional to the guessing due to uncertainty or hesitation.

This leads to the conclusion that the first two factors could be combined, assuming that the first one is the leading one. A typical situation when it could be observed is when the participant has eliminated 2 or 3 of the 5 possible answers as incorrect and has limited his choice to 2 or 3 answers. This is when the temptation to guess takes over and the participant enthusiastically reasons that rejecting several answers is a progress that should be "rewarded".

Making a mistake in the process of solving a problem is very uncommon among the group of students that is being studied. They have a good mathematical background and solid knowledge. The conditions under which the test is given are good and predispose the participants to concentrate well: usually, there are 10–15 students in a room, the discipline is good, the room is quiet and everybody is focused on solving the problems. The only factors that could influence the number of mistakes the students make are the difficulty of the test and the presence of problems with a great number of technical details that increase the chance of a mechanical mistake. That is why we should look at the difficulty of the test in our particular case.

Average number of points for a problem from the maximum possible			
Grade	Boys	Girls	All
3^{rd}–4^{th}	.4974	.4834	.4913
5^{th}–6^{th}	.5014	.5034	.5022
7^{th}–8^{th}	.4179	.4105	.4149
9^{th}–10^{th}	.4524	.4172	.4399
11^{th}–12^{th}	.5062	.4719	.4920

Table 1: Average number of points for a problem from the maximum possible

6 Results and Conclusions

Table 1 shows that the difficulty of the questions complied with the requirements: the average results in the different age groups vary from 41 % to 50 % of the possible maximum number of points. It is interesting to note that the average results of boys and girls in the first three age groups are practically the same and only the boys above 15 years old show 3 % better results than the girls of the corresponding group. In comparison, Curran (1995) studies the results of 11–12 year old students during a test in New Zealand. The data is for a large group of students, about 22,000, which for a country of that size, means that students who lacked advanced mathematical skills were also a part of the study. The results of the New Zealand study show that boys have better results than girls. There are also examples of problems where boys do better and problems where girls do better.

The most acceptable explanation for the difference between the studies in New Zealand and Bulgaria is that our study includes only students with advanced mathematical skills. Another factor might be the fact that the more rigorous Bulgarian educational system prepares all students better, giving the chance for girls to make up the (hypothetical) difference in capabilities as compared to boys.

Having in mind that the difficulty of the problems was suitable and relatively the same in all age groups, we should expect that the percentage of incorrect answers due to mistakes made by the participants is not too large and dominating. It is pretty much the same for all age groups. Based on that conclusion, we could assume that the difference in the average number of mistakes in the various age groups is a result of other factors, and not of mistakes in the solution of the problems. The indicators "average number of mistakes per participant in a given problem" and "average number of blank answers per participant in a given problem" for the different age groups are given in the last columns of Tables 2 and 3. Comparing those leads to the following conclusions:

Average number of wrong answers			
Grade	Boys	Girls	All
3^{rd}–4^{th}	.4525	.4530	.4527
5^{th}–6^{th}	.3132	.2933	.3054
7^{th}–8^{th}	.3281	.3408	.3332
9^{th}–10^{th}	.2040	.2307	.2134
11^{th}–12^{th}	.2022	.2063	.2039

Table 2: Average number of wrong answers

Average number of blank answers			
Grade	Boys	Girls	All
3^{rd}–4^{th}	.0752	.0954	.0841
5^{th}–6^{th}	.3284	.3551	.3388
7^{th}–8^{th}	.4111	.4009	.4070
9^{th}–10^{th}	.5420	.5669	.5508
11^{th}–12^{th}	.4794	.5333	.5017

Table 3: Average number of blank answers

The highest percent of incorrect answers around 45 %, and respectively the lowest percent of blank answers–8%, is found in the group of the youngest students, 3^{rd} and 4^{th} grade. The data for this age group is in a large contrast with the results from the other age groups. The conclusion is that the youngest participants are most likely to take risks with no restraints. Actually, they probably do not realize the risk they take and the negative results the "guessing" strategy may have on their overall results, and therefore on their rank in the tournament. The motivation in that age group is relatively low. They have been told:

- how good it is to solve the problems, but the advantage, the effect, the meaning of all that is somehow unclear to them
- that they can solve problems and that this is their "gift." Some participants accept such claims and have high self-esteem and look at the problems with too much confidence—"that's way too easy"
- how they should behave during a tournament. A lot of parents, misled by their own experience at school, tell their children not to leave blank answers–"maybe you will guess the correct one."

Some more objective factors should be added to the reasons for increased risk taking among 3$^{\text{rd}}$ and 4$^{\text{th}}$ grade students. One of them is that insufficient time to perceive, think about and solve a problem leads to guessing. What happens when a child has 6 minutes to solve an arithmetic operation with 3 numbers? In the first two minutes he solves the problem. In the other 4 minutes he spends to double check if it is correct, coming back to it again and again, wondering. And with such doubts, he often takes another decision, the wrong one. Also, the capabilities of 9–10 year old children to fully concentrate are not that developed yet. A period of 90 minutes is too long for them. It is natural that the tired and unconcentrated child will start guessing the answers, searching to relieve the pressure by turning the competition into a guessing game.

The next group, 5$^{\text{th}}$ and 6$^{\text{th}}$ graders, shows a tremendous decrease in the risk-taking. The percentage of wrong answers falls to 30 %, and the blank answers increase to 34 %. It is not surprising that Sherrifs and Boomer (1954) found that students who lack confidence have difficulties if there is a penalty for giving a wrong answer. They sometimes leave a blank answer even if they know the correct answer. The decreased risk-taking among participants in the competition is obvious. Why? On one hand, the motivation is much stronger and clearer. "Where you will continue your education depends on your performance!" The child has ambitions. He carefully counts the points to be won or lost. He does not let himself risk as he tries to achieve a specific goal.

The tendency of decreasing risk-taking continues in the other age groups as well, but the difference is not that striking. The difference between 9^{th}–10^{th} and 11^{th}–12^{th} grade is almost negligible. At first sight one could even say that there is a 5 % backlash. However, Table 1 shows that the 11^{th}–12^{th} grade test is about 5 % harder than the one for 9^{th}–10^{th} grade, and Table 4 shows that the 11^{th} and 12^{th} graders have exactly 5 % more correct answers compared to the other group. In other words, the decreased number of blank answers among the older students is due to the larger number of solved problems and not to increased risk-taking. Along with the increasing motivation and personal responsibility, the reasons for the decreasing risk-taking could be found in:

- the increase of the analytical thinking capabilities and the evaluation of specific situations that a young person faces
- clearer understanding of cause-effect relations related to test-taking
- increased capabilities for evaluation and self-evaluation
- establishment of the Self.

As a whole, we are talking about smart young people who know their needs and the way they could achieve them.

The Chernorizets Hrabar Tournament encompasses students from all grades of secondary education in Bulgaria. From that point of view, the differences between boys and girls in terms of mathematical skills and tendency to take risks are quite interesting.

The data presented in Table 1 for the average number of points earned by the different groups leads to the following conclusions:

- In the groups of 3^{rd}–4^{th}, 5^{th}–6^{th} and 7^{th}–$^{\text{th}}$ grade, the performance of boys and girls is practically the same—50 %–48 % for the youngest, 50 %–50 % for 5^{th}–6^{th} grade and 42 %–41 % for 7^{th}–8^{th} grade. This means that math skills are equally accessible for everybody.

- The groups of 9^{th}–10^{th} and 11^{th}–12^{th} graders show that boys prevail—45 %–41 % and 50 %–47 % respectively. Gender differentiation is a factor and the change among young people is not purely biological. They form different goals, values, actions and behavior. The rational thinking of boys makes math knowledge more accessible for them than for the relatively more emotional psyche of the girls. Yet, the difference is only a few percent (3–4 %). This is due to all students having exceptional math skills to begin with. Prior studies have found similar, even more pronounced, gender differences. They find an age barrier (15–16 years old) when boys "pull forward" compared to girls in terms of math ability. Of course, these conclusions focus on the averages and do not exclude the possibility for girls to win competitions.

Swineford (1941) studied the tendencies for guessing and risk behavior among 9^{th} graders. He found that the tendency to guess is much greater among boys than among girls. It is even stronger if the tests include less known material.

All that is quite natural. Girls have passed puberty at that age. They treat themselves, their actions and behavior differently. They are more careful and responsible. They instinctively do not like risk, including risk in education.

Their fellow male students at the same age tend to be less emotionally mature. They have just entered puberty and try to express their independence, the establishment of the Self. The idea of "this is what I think!" makes them more susceptible to risk. The formation and establishment of their own opinion, even if it is momentary, spontaneous, and unfounded, is far more important to boys than the possible consequences of the risk they have taken. Often, they do not even think about the risk.

Some authors, Hardings (1981), Murphy (1978), Anderson (1989), note that the difference in results among boys is observed exactly during tests where wrong answers are penalized, while during other forms of testing (for example, short answer examinations) no such difference is noted.

Girls consider the threat of penalty points. This has a strong restraining effect on them.

On the contrary, boys are stimulated by bans, penalties, limits, anything they could overcome. The greater the danger, the greater the pleasure of challenging it. The thought in these circumstances is: "Nothing can scare me!"

The results of the 2002 study of Chernorizets Hrabar confirm common tendencies. The average number of right and wrong answers (Tables 2 and 4), and the number of blank answers (Table 3) among boys and girls of the first three groups (3^{rd}–4^{th}, 5^{th}–6^{th} and 7^{th}–8^{th} grade) are in the tolerance of 0.01 % to 0.02 %. These negligible limits show:

- the tendency for risk taking among students from primary and secondary schools decreases as they get older
- the penalty for a wrong answer given through guessing has an effect after 7^{th}–8^{th} grade. This is the time when students realize what it means and consider it (Table 3)
- more significant differences in the behavior of boys and girls are observed in the next age groups.

Table 3 shows that the average number of blank answers among 9^{th}–10^{th} grade girls is 3 %, while in the next group, 11^{th}–12^{th} grade, the percent rises to 5 %. These differences are with rather negligible absolute values as the study is conducted among students with advanced math skills, that is, the girls are also oriented towards mathematical knowledge. Yet, puberty among girls makes them more responsible and less likely to take risks. The results also show that the number of boys that restrained themselves from guessing in the next age group (11^{th}–12^{th} grade) increases to 6 %. On one hand, this is related to their sexual maturation. On the other, their sense of responsibility increases and they become more motivated and aware of their future prospects.

Table 4 demonstrates the great differences between the last two age groups. For example, the difference in the number of right answers between boys and girls in 9^{th}–10^{th} grade is 5 %, and in the next group

Average number of right answers			
Grade	Boys	Girls	All
3^{rd}–4^{th}	.4723	.4516	.4633
5^{th}–6^{th}	.4080	.4013	.4054
7^{th}–8^{th}	.2607	.2583	.2598
9^{th}–10^{th}	.2540	.2023	.2357
11^{th}–12^{th}	.3184	.2603	.2944

Table 4: Average number of right answers

(11^{th}–12^{th}grade) is 6 % in favor of the boys. The conclusion is that boys' rational thinking matches the specifics of math better. When we talk about girls (even the ones with special math skills), emotions are still the leading factor. Otherwise, success among both boys and girls in the last two age groups is significant and almost identical (7 % for boys and 6 % for girls). That fact proves the quality of math education both genders receive. Yet, boys are still ahead in their results and the differences, created by their psycho-physiological characteristics, remain. Looking carefully at the number of correct answers in Table 4, it is easy to see that those differences are visible even in the first age group (3^{rd}–4^{th} grade). In the next two groups (5^{th}–6^{th}, 7^{th}–8^{th}), girls have results closer to those of boys due to their quicker development, but as boys enter puberty, they increase the gap again and the differences stay relatively stable.

The idea that under specific circumstances any child is suitable for any task is not confirmed by reality. There are cases when some individuals have peculiar results. There are boys that find math to be an unbearable burden and girls whose results beat any boy's results. Yet, these individual cases cannot be the basis for general conclusions. Every child (a boy or a girl) is a unique individual and everybody working with them should study, analyze and respect that unique individuality. It is important to know common tendencies, but every child's uniqueness has an important defining role. The Chernorizets Hrabar Tournament is an excellent opportunity for such a differential study.

The skill of taking and controlling risk has an important role in the formation and development of a student's personality. Our study is only a start in this direction. Besides choice and the threat of being penalized, a student's risk behavior depends on several other factors:

- Who meets the students at the day of the tournament is influential, whether it is somebody they know, whether it is somebody who is smiling, a person with good intentions and positive attitude, or a cold, nervous person full of suspicions and negative emotions.
- The gender of the instructor in the room is important to the last two age groups (9^{th}–10^{th} and 11^{th}–12^{th} grade) as some students apprehend instructions better from a male voice, while for others it is better if they listen to a female voice.
- Disturbance of the comfort factors (i.e. heat, light, noise, clean air, etc.) provokes students to take risks only to get away from a critical, often unbearable situation.
- How the evaluation will be done—directly or indirectly through specific indicators—is pretty important and has a significant influence. Students' behavior is different if the evaluation is summative, normative, traditional or alternative.

This list could continue. It shows how complex the problem of risks taking among students is, and how significant the beginning of its study is, based on the experience gained from the Chernorizets Hrabar Tournament.

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Reach Problems

Teaching and assessing non-routine problem solving skills

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Andy Edwards taught mathematics in Australian secondary schools for 32 years. In 2001, he received a B. H. Neumann Award for his work on the Mathematics Challenge For Young Australians, to which he still—19 years later—contributes non-routine problems for students in years 5 to 10. Since 2004, he has worked for the Queensland Studies Authority writing test items for the state and national numeracy tests (years 3 to 9) and the Queensland Core Skills Test (year 12). His current interest is in the extent to which mathematics learning is retrospective. His permanent interests are integers and pattern recognition.

We teachers spend most of our time teaching students how to get the right answers. This is not a bad thing. The basic understandings and processes students need to make their way in a mathematical world are taught well by explanation and learned well by structured practice where the goal is to get the right answer.

But this doesn't teach them how to solve problems.

In senior maths, it has become fashionable to use modelling and problem solving to discriminate between the very good students and the very best ones. This trend extends to middle school and high school classes where problem solving is frequently regarded as an important component of the course. We have a moral obligation to teach students how to solve problems better than we do.

But learning anything is a complex and haphazard process. If we accept that everyone makes mathematical meaning out of situations in their own way, using the sum of their own experiences and recollections and metaphors, then we have to accept that learning how to solve problems is at least equally haphazard. And that teaching students how to solve problems is not nearly as susceptible to an "explain-and-practice" approach as teaching processes and mathematical knowledge is.

The first to respond to the challenge of teaching students how to solve problems were—predictably—the publishers of maths textbooks. They added little codas at the end of each chapter. These codas contained problems tied directly to the material just studied in the chapter. These are better termed "contextualized practice questions" than problems. They don't require anything much more than the application of recently-studied routines.

Good problem solvers can:

- engage with and interpret a problem successfully
- choose from their mathematical toolchest skills and knowledge that may help them unravel the problem and choose or devise a strategy
- recognise a failed attempt at solution and use it to inform their next attempt
- and show persistence.

The end-of-the-chapter problem-solving exercises fail on point two (the students know that this problem will be susceptible to solution using the techniques that have been the chapter topic) and they tend not to give them much practice on points three or four either. It makes contextualised questions the fast food of problem solving.

"Reach" problems aim to develop students' problem-solving power. They typically contain some elements which make them non-routine.

- The mathematical processes and knowledge required for them need to be familiar to the students who may have mastered them as much as two or three years before.

- They may require the development of a problem-specific strategy and several of these may work—some may be elegant and insightful but others may be workaday or even cumbersome but still yield the solution.
- They may have multiple solutions.
- They may require simple proofs or arguments.
- The context may be unfamiliar (though it must be comprehensible to the audience). Note that these contexts may be real life or real world but need not necessarily be so. The best ones are often purely mathematical or pure fantasy. What is *critical* is that they should be *engaging*.

The techniques needed to assess students' efforts on reach problems are different and even somewhat counter-intuitive. But they are engineered to reflect what we are seeking to develop in students by setting them.

We are putting a high value on students' willingness to:

- engage with and attempt to solve a problem using well-chosen processes
- develop and apply a strategy (possibly one specific to the problem)
- recognise a failed attempt and use it to inform later attempts
- be tenacious and not give up easily.

A lesser value is put on:

- the elegance of the method
- the orderliness or setting out of the solution, and even in this context
- the solution's completeness or correctness of the solution.

Student behaviour	**Score**	**Credit is for. . .**
Does not attempt problem or gives only a solution with no supporting evidence	0	Nothing
Attempts, fails, gives up	1	Engagement
Attempts, fails, attempts using same strategy, fails again, gives up	2	Engagement, persistence
Attempts, fails, attempts using changed strategy informed by the first try, fails again, gives up	5	Engagement, strategy refinement, persistence
Devises and applies a potentially functional strategy which produces an incorrect solution with some merit	6	Engagement, strategy development and application, partial solution
Devises and applies a functional strategy that produces a solution which is correct but incomplete, or contains an error which is mechanical only	7	Engagement, strategy development and application, persistence, partial solution
Devises and applies a functional strategy to produce a correct and complete solution	8	Engagement, strategy development and application, persistence, correct solution

Table 1: Marking scheme for reach problems

Note that the student's working out—no matter how disorganized it may be—must be submitted for any credit to be given. There must be evidence of the process towards the solution so that a judgement can be made (Table 1).

Notice that students receive a "passing" grade of 5 out of a possible 8 for a solution which is quite wrong. They do so because they have done something which has contributed to their development as problem solvers, however long this may take. Students ought to be encouraged to compete against the problems rather than against each other. Time limits, if set at all, should be generous.

If five reach problems are given to a class over a term or semester and scored as above, any student who makes an honest attempt at all five problems will outscore any student who engages with only three of them, however successfully.

The following problems may be considered as reach problems at the year levels indicated. The asterisked examples were originally published by the Australian Mathematics Trust in the Mathematics Challenge For Young Australians in the papers and years indicated. Some of these have been adapted to fit the purpose.

1. My Auntie Bertha is a bit shy about telling people her age.
But she gave me some clues about it the other day.
When you divide it by two there is one remainder.
When you divide it by three there is two remainder.
When you divide it by four there is three remainder.
When you divide it by five there is four remainder.
When you divide it by six there is five remainder.
How old is my Auntie Bertha?

[Target audience: students who have mastered division with remainders.]

***2.** My friend had finally collected the full set of basketball cards, numbered 1 to 10, from her favourite chewing gum. I tried some of the gum the other day and got card number 5. I asked her whether it was a good one. She explained that the higher numbers were rare but had better trading value than lower numbers. She offered to give me five extra 10s she had collected along the way. "There's a dude trading basketball cards every Sunday at the market," she told me. "You could try for a full set by trading with him."
It turns out that the card-trading dude has some serious rules.

- You give him two cards and ask him for three, which add up to the same as your two (e.g. you could give him 8 and 6 and ask for a 7, a 4 and a 3 or two 5s and a 4).
- You cannot ask for a higher card than the best one you give him (e.g. he won't trade your two 8s for a 10, a 5 and a 1).

- He never does a two-for-one swap (e.g. if you give him a 9, he won't give you a 7 and a 2—this is not going to happen!).
- You can't scam him either, by trying to give him a 9 and a 6 for a 7, a 6 and a 2.

I was careful and managed, by trading alone, to get a complete set. Show how I might have done this, clearly writing down the set you have after each trade.

[Adapted from Basketball Cards, question 3, 2006 Junior Paper. Target audience: Years 7 and 8.]

3. Three two-digit numbers are made up of six different digits. The biggest number is three times as big as the smallest one. If the three numbers all add up to 100, what are they?

[Target audience: Years 4 to 6.]

***4.** A rectangular prism has dimensions x cm, y cm and z cm, where x, y and z are integers. The surface area of this prism is $A\,\text{cm}^2$.

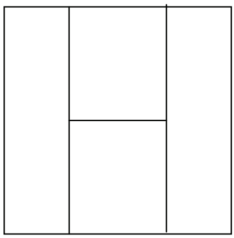
(a) Find the dimensions of all rectangular prisms for which $A = 100$.

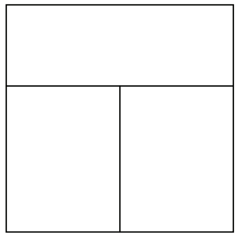
(b) There are some even numbers which A cannot be. Explain why all such numbers greater than 6 must be divisible by 4 and find the four smallest even integers greater than 6 that A cannot be.

[Adapted from Boxes, problem 1, 1996 Intermediate Paper. Target audience: Years 9 and 10.]

5. At the Hotel Wiltshire, they bake their own loaves of bread. The pieces of toast they cut from this bread are all square (12 cm by 12 cm). They have a set of toast-cutting irons which can be pressed onto the toast so that it is cut into pieces exactly the same in area.

For example, the H-iron and T-iron cut the toast like this:





(a) Calculate and write the measurements of these pieces on the diagrams.

(b) Using only straight lines or semicircles to make the blades, design toast-cutting irons for the letters E, D, A and R. Draw a diagram for each piece and calculate and write the measurements correct to the nearest millimetre.

[Target audience: students who are familiar with methods for the calculation of areas of rectangles, circles, trapeziums, triangles and composite shapes—Years 9 and 10.]

6. What are the last two digits of the number you get when you take 3 to the power of 2009?

[Target audience: students with experience of index numbers—Years 7 to 9. Calculators are recommended here. They help establish patterns to be recognised and extrapolated. Thought is required to get the right answer.]

7. Sophia and Ann competed in a 1200 metre cross-country run which was run on an out-and-back course (where runners turned at the halfway mark and ran back to finish where they started). Sophia aimed to finish in 60 minutes and Ann's target time was 65 minutes. Assume each girl ran at a steady speed and achieved her target time. At some point in the race, Ann met Sophia running in the opposite direction as Sophia headed for the finish.

(a) For how long had Ann been running at this point?

(b) How far was Ann from the halfway mark at this point?

[Adapted from Runners, problem 4, 1994 Junior paper. Target audience: Years 7 to 9.]

8. The number 324 has the unusual property that it is divisible by each of its digits and also the sum of its digits.

(a) What is the smallest four-digit number which has this property?

(b) What is the smallest number made up of four *different* digits which has this property?

[Target audience: students with short division skills. Some idea of the rules for divisibility by 2, 3, 5 and 6 helps. Years 5 to 7.]

9. "To reach the fair damsel in yonder castle," said the old man, "thou must pass by the four dragons which guard the only road. They are immortal and cannot be killed even by the likes of thee, Sir Knight. Aye, many a gallant knight have I seen eaten by these ferocious, fire-breathing monsters. The only way past them is to bribe them with precious jewels, for they have great stores of treasure in the caves and are always greedy for more."

"First thou must pay the Green Dragon 20 emeralds and she'll let thee pass. Then thou mayest trade with her and she'll give thee one ruby every two emeralds."

"The Red Dragon is thy friend for 15 rubies and will give two diamonds for every five rubies after she receiveth payment."

"Next is the Rainbow Dragon. Thou mayest pass by her on payment of 20 jewels, at least five each of emeralds, rubies and diamonds. She will not trade with thee."

"Nor will the Black Dragon, the last and fiercest of all. Diamonds are all she will want for her trove—20 of them, and be sure to hand them over quickly."

"Now I can sell thee jewels, Sir Knight, but emeralds be all I have. How many emeralds wouldst thou want to buy from this old emerald seller, Sir?"

Legend has it that the knight bought the least number of emeralds, passed each dragon once and had no jewels left over when he reached the fair damsel. How many emeralds did he buy and how did he trade with the dragons?

[Adapted from Dragons and Jewels, problem 3, 1998 Junior paper. Target audience: Years 7 to 9.]

10. A Fibonacci sequence is one in which each term is the sum of the two preceding terms. The first two terms can be any positive integers. Two examples of Fibonacci sequences are: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ (the "standard" Fibonacci sequence) and $15, 11, 26, 37, 63, 100, 163, \ldots$

(a) Find a Fibonacci sequence which has 2000 as its fifth term.

(b) Find a Fibonacci sequence which has 2000 as its eighth term.

(c) Find the greatest value of n such that 2000 is the nth term of a Fibonacci sequence.

[Fibonacci, problem 3, 2000 Intermediate paper. Target audience: Years 9 and 10.]

Quick Answers—not too many marks for most of these as they are not well enough explained!

1. Auntie Bertha is 59. (At 119, Auntie Bertha would be one of the three oldest people in the world and *everyone* would know her age!)

2. One of many solutions is:
$\{10, 10, 10, 10, 10, 5\}$—six cards.
Trade $10 + 10 = 8 + 6 + 6$ gives $\{10, 10, 10, 8, 6, 6, 5\}$—seven cards.
Trade $10 + 6 = 9 + 4 + 3$ gives $\{10, 10, 9, 8, 6, 5, 4, 3\}$—eight cards.
Trade $10 + 5 = 8 + 6 + 1$ gives $\{10, 9, 8, 8, 6, 6, 4, 3, 1\}$—nine cards.
Trade $8 + 6 = 7 + 5 = 2$ gives $\{10, 9, 8, 7, 6, 5, 4, 3, 2, 1.\}$—full set.

3. $19 + 24 + 57 = 100$.

4. $\{1, 2, 16\}$ and $\{2, 4, 7\}$ have $A = 2(2+16+32)$ and $2(8+14+28) = 100$. The smallest possible cube has $x = y = z = 1$ so all others have $A > 6$. Even numbers which are not multiples of 4 are of the form $4n + 2$, n an integer. Cuboids with dimensions $\{1, 1, n\}$ will have $A = 4n + 2$. The four smallest even integers that A cannot be are 8, 12, 20 and 36.

5. Many solutions for each letter. The E falls into three parts each with area 48 cm^2.

The D cannot be made with a 12 cm-rectangle and a semicircle with area 48 cm^2, so the toast must be cut into two parts each of 72 cm^2. A 10×3.27 cm rectangle with a 5 cm radius semicircle attached to the right leaves 72 cm^2 outside.

The A must have the horizontal line drawn $\sqrt{72} \approx 8.49$ cm from its apex so that the triangle's base is $\sqrt{72}$ and the trapezium and triangle formed both have areas of 36 cm^2.

The R falls into four pieces of area 36 cm^2 each. Some care needs to be exercised so that the upper D-shaped part does not touch the right edge to create a fifth part and that the oblique strut intersects the D-shaped part where it is straight, not curved. Otherwise the bottom part will not be a trapezium.

6. The last two digits of 3^n cycle through a pattern with 20 elements—(0)3, (0)9, 27, 81, 43, 29, 87, 61, 83, 49, 47, 41, 23, 69, 07, 21, 63, 89, 67, 01, 03, etc. The last two digits of 3^{2009} will therefore be the same as those of 3^9, i.e. 83.

7. At the point where they meet, the girls have run a total distance of 12 000 m as Sophia is as far past the halfway mark as Ann is short of it. Their combined speed is $12000/60+12000/65$ metres per minute or 384.615 m/min. So it has taken them $12000/384.615 = 31.2$ minutes (or 31 min 12 *sec*) to reach that point. Ann has run $31.2 \times 12000/65 = 5760$ m which is $6000 - 5760 = 240$ m away from the halfway mark.

8. (a) For a number to be divisible by *all* its digits, it may not contain any zeros. The smallest such four-digit number is 1111 but this does not divide by its digit sum (4) so is rejected. 1112, 1113, 1114, 1115 also fail at least this test but 1116, being even *and* with a digit sum of nine, satisfies all the conditions.

(b) The smallest four-digit number with distinct non-zero digits is 1234 which is indivisible by 3 (and 4 and 10.) Its nearest larger neighbour divisible by 3 is 1236, which, being an obvious multiple of 12, satisfies all constraints.

9. For safe passage the knight will need 20 emeralds, then 15 rubies, then 5 of each and 5 other jewels, and then 20 diamonds. To get 25 diamonds from the red dragon he will need 62.5 rubies which is not possible. But 26 diamonds can be bought for 65 rubies. (The extra diamond can be used as an "other" jewel for the Rainbow Dragon.) Twenty rubies will be handed over to the Red and Rainbow dragons. So altogether, 85 rubies will need to be bought from the Green Dragon. In addition the Green Dragon will want 20 emeralds and the Rainbow Dragon will want 5, plus another 4 "other" jewels to go with the extra diamond. Eighty-five rubies will cost 170 emeralds. Another 29 will need to be paid in bribes. The total is 199.

10. (a) Many answers are possible. Two possibilities are 400, 400, 800, 1200, 2000,... (by multiplying each term in the "standard" Fibonacci sequence by 40) and by working backwards from the fifth term 2000, 1300, 700, 600, 100.

(b) Using the fact that for any two consecutive terms m and n in a Fibonacci sequence the value of m/n tends to $\varphi \approx 1.618034\ldots$ we can see that $2000/\varphi \approx 1236$ is a good choice for the seventh term which gives us $2000 - 1236 = 764$ for the sixth, 472 for the fifth, 292 for the fourth, 180 for the third, 112 for the second and 68 for the first i.e. 68, 112, 180, 292, 472, 764, 1236, 2000... In fact, it can be shown algebraically that any seventh term between 1231 and 1249 will yield one of the 19 valid sequences.

(c) The sequence began in (b) can be continued 44, 24, 20, 4, 16 before it becomes negative (the next term would need to be -12). This adds another five terms for $n = 13$. This is maximal, which can be shown by experimenting with 2000, 1237... and 2000, 1238... and demonstrating that they become negative at $n = 13$.

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Madrid, Spain

The 49th International Mathematical Olympiad (IMO) was held 10–22 July in Madrid, Spain. It was the largest IMO to be held to date. A record number of 535 contestants from a record 94 countries participated.

The IMO competition consists of two exam papers held on consecutive days. This year the dates of the competition were Wednesday and Thursday 16–17 of July. To qualify for the IMO, contestants must not have formally enrolled at a university and be less than 20 years of age at the time of writing the second exam paper. Each country has its own internal selection procedures and may send a team of up to six contestants along with a team Leader and Deputy Leader. The Leaders and Deputy Leaders are not contestants but fulfill other roles at the IMO.

Most Leaders arrived in Spain on July 10th. They stayed at a hotel in La Granja, Segovia where their first main task was to set the IMO paper. For a number of months prior to this meeting, countries had been submitting proposed questions for the exam papers. The local Problem Selection Committee had considered these proposals and composed a shortlist of 26 problems considered highly suitable for the IMO exams. Over the next few days the Jury of team Leaders discussed the merits of the problems. Through a voting procedure they eventually chose the six problems for the exams. This year three of those 26 problems had to be excluded from consideration because they were known in the public domain. The final problems in order of selection for the exam are described as follows:

1. An easy geometry problem proposed by Russia. The problem is concerned with proving that a certain hexagon associated with an arbitrary triangle is cyclic. For a well-trained student virtually any sensible approach leads to a solution.
2. An inequality proposed by Austria. This question ended up being more difficult than anticipated. Many students fell into the trap of spending much time trying to solve it using the Cauchy–Schwartz

inequality. It turns out that the solution is much more elementary. The inequality could easily be transformed into the form $W^2 \geq 0$ by completing the square, although this is made easier by making a change of variables.

3. A number theory problem proposed by Lithuania. This problem required proving the existence of infinitely many integers of a particular form which have a prime factor satisfying a certain inequality.
4. A rather easy and standard functional equation proposed by South Korea. However, there was a trip that could stumble inexperienced students about halfway through the solution.
5. An appealing combinatorics problem proposed by France. This problem is a classic for those familiar with bijective combinatorics. It can be solved by finding a simple many-to-one map whose pre-images all have equal cardinality.
6. A very difficult geometry problem proposed by Russia. A sound technique in Euclidean methods including a working ability to consider and handle dilations was imperative for this problem.

Each exam paper consists of three questions. The contestants have $4\frac{1}{2}$ hours on each exam in which to provide carefully written proofs to the three challenging mathematical problems. Each contestant writes the exam in his or her own language so the Jury must also spend time to ensure uniform translations of the exams into the required languages. After this marking schemes are discussed with representatives of a team of local markers called Coordinators. The Coordinators ensure fairness and consistency in applying the marking schemes to contestants' scripts.

Most contestants and Deputy Leaders arrived in Madrid, Spain on 14 July. The Deputy Leaders look after their contestants until the exams are over.

On 15 July the opening ceremony was held at Teatro Circo Price in Madrid. This was one of the most entertaining IMO events I have attended. There was a multilingual Master of Ceremonies who may as well have also been the ringmaster of the circus-like atmosphere and performances. There was the parade of the national teams, or should I say, a representative from each team. There were speeches, all of which were relatively short, by representatives of the Spanish Government, the IMO and the Royal Mathematical Society of Spain. Then there was the entertainment: juggling with feet, curtain acrobatics, a comical music act and a strong men display.

The next two days were the competition days. After the exams the Leaders and their Deputies spend about two days assessing the work of the students from their own countries. They are guided by marking schemes discussed earlier. The Coordinators also assess the papers. They are also guided by the marking schemes but may allow some flexibility if, for example, a Leader brings something to their attention in a contestant's exam script which is not covered by the marking scheme. There are always limitations in this process, but nonetheless the overall consistency and fairness by the Coordinators was very good. Only two disputes made it to the Jury room.

The outcome was largely as expected. Question 1 was the easiest question on this IMO. The average mark was 5.0, with 321 complete solutions. Question 6 was the most difficult question on the paper. The average mark was just 0.3, with 12 complete solutions. The medal cuts were set at 31 points for gold, 22 points for silver and 15 points for bronze. Most gold medalists essentially solved at least 5 questions, most silver medalists solved 3 or 4 questions and most bronze medalists solved 2 questions.

We were privileged to have HRH The Prince of Asturias speak at the closing ceremony and present some of the medals. There were 267 (= 49.9 %) medals awarded, the distributions being 120 (= 22.4 %) bronze, 100 (= 18.7 %) silver and 47 (= 8.8 %) gold. There were three students, Xiaosheng Mu (China), Dongyi Wei (China) and Alex (Lin) Zhai (USA) who achieved the most excellent feat of a perfect score. Of those who did not get a medal, a further 103 contestants received an honourable mention for solving at least one question perfectly. The Prince encouraged all students to continue exploring mathematical endeavours and then officially declared the 49th IMO closed.

The 2008 IMO was supported by the Government of Spain and by the Royal Mathematical Society of Spain.

The 2009 IMO is scheduled to be held in Bremen, Germany.

1 IMO Paper

First Day

1. An acute-angled triangle ABC has orthocentre H. The circle passing through H with centre the midpoint of BC intersects the line BC at A_1 and A_2. Similarly the circle passing through H with centre the midpoint of CA intersects the line CA at B_1 and B_2, and the circle passing through H with centre the midpoint of AB intersects the line AB at C_1 and C_2. Show that A_1, A_2, B_1, B_2, C_1, C_2 lie on a circle.

2. (a) Prove that

$$\frac{x^2}{(x-1)^2} + \frac{y^2}{(y-1)^2} + \frac{z^2}{(z-1)^2} \geq 1$$

for all real numbers x, y, z each different from 1, and satisfying $xyz = 1$.

(b) Prove that the equality holds above for infinitely many triples of rational numbers x, y, z each different from 1, and satisfying $xyz = 1$.

3. Prove that there exist infinitely many positive integers n such that $n^2 + 1$ has a prime divisor which is greater than $2n + \sqrt{2n}$.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points

Second Day

4. Find all functions $f\colon (0,\infty) \to (0,\infty)$ (so, f is a function from the positive real numbers to the positive real numbers) such that

$$\frac{(f(w))^2 + (f(x))^2}{f(y^2) + f(z^2)} = \frac{w^2 + x^2}{y^2 + z^2}$$

for all positive real numbers w, x, y, z satisfying $wx = yz$.

5. Let n and k be positive integers with $k \geq n$ and $k - n$ an even number. Let $2n$ lamps labelled $1, 2, \ldots, 2n$ be given, each of which can be either on or off. Initially all the lamps are off. We consider sequences of steps: at each step one of the lamps is switched (from on to off or from off to on). Let N be the number of such sequences consisting of k steps and resulting in the state where lamps 1 through n are all on, and lamps $n+1$ through $2n$ are all off. Let M be the number of such sequences consisting of k steps resulting in the state where lamps 1 through n are all on, and lamps $n+1$ through $2n$ are all off, but where none of the lamps $n+1$ through $2n$ is ever switched on.

Determine the ratio $\frac{M}{N}$.

6. Let $ABCD$ be a convex quadrilateral with $|BA| = |BC|$. Denote the incircles of triangles ABC and ADC by ω_1 and ω_2 respectively. Suppose that there exists a circle ω tangent to the ray BA beyond A and to the ray BC beyond C, which is also tangent to the lines AD and CD. Prove that the common external tangents of ω_1 and ω_2 intersect on ω.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points

2 Statistics

Some Country Scores		
Rank	Country	Score
1	China	217
2	Russia	199
3	USA	190
4	South Korea	188
5	Iran	181
6	Thailand	175
7	North Korea	173
8	Turkey	170
9	Taiwan	168
10	Hungary	165
11	Japan	163
12	Vietnam	159
13	Poland	157
14	Bulgaria	154
15	Ukraine	153

Some Country Scores		
Rank	Country	Score
16	Brazil	152
17	Romania	141
17	Peru	141
19	Australia	140
20	Serbia	139
20	Germany	139
22	Canada	135
23	United Kingdom	133
24	Italy	132
25	Kazakhstan	128
26	Belarus	125
27	Israel	120
28	Hong Kong	107
29	Mongolia	106
30	France	104

Distribution of Awards at the 2008 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Albania	53	0	0	1	1
Argentina	85	0	1	3	0
Armenia	56	0	0	0	4
Australia	140	0	5	1	0
Austria	63	0	0	1	4
Azerbaijan	74	0	0	3	0
Bangladesh (4 members)	33	0	0	0	1
Belarus	125	0	3	2	0
Belgium	61	0	1	1	1
Bolivia (5 members)	5	0	0	0	0
Bosnia & Herzegovina	68	0	0	3	1
Brazil	152	0	5	1	0
Bulgaria	154	2	1	3	0

Distribution of Awards at the 2008 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Cambodia	25	0	0	0	1
Canada	135	0	2	4	0
Chile (3 members)	49	0	1	1	1
China	217	5	1	0	0
Colombia	77	0	2	0	1
Costa Rica	65	0	0	2	3
Croatia	86	0	0	3	2
Cuba (1 member)	27	0	1	0	0
Cyprus	42	0	0	1	1
Czech Republic	85	0	1	1	3
Denmark	66	0	2	0	1
Ecuador	26	0	0	0	1
El Salvador (4 members)	31	0	0	0	3
Estonia	41	0	0	1	1
Finland	40	0	0	1	1
France	104	0	1	4	1
Georgia	84	0	0	5	1
Germany	139	1	2	3	0
Greece	85	0	0	2	4
Guatemala (4 members)	16	0	0	1	0
Honduras (2 members)	17	0	0	0	2
Hong Kong	107	0	3	1	1
Hungary	165	2	3	1	0
Iceland (5 members)	31	0	0	1	0
India	103	0	0	5	1
Indonesia	88	0	1	2	2
Ireland	45	0	0	0	2
Iran	181	1	5	0	0
Israel	120	1	1	2	2
Italy	132	0	3	3	0
Japan	163	2	3	1	0
Kazakhstan	128	1	2	3	0
Kuwait (5 members)	3	0	0	0	0
Kyrgyzstan (5 members)	28	0	0	0	1

Distribution of Awards at the 2008 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Latvia	58	0	1	0	2
Liechtenstein (2 members)	16	0	0	0	1
Lithuania	92	0	1	2	3
Luxembourg (5 members)	60	0	0	2	2
Macau	58	0	0	2	1
Macedonia (FYR)	61	0	0	2	0
Malaysia	65	0	1	0	4
Mexico	87	0	1	1	4
Moldova	74	0	1	0	2
Mongolia	106	0	2	1	2
Montenegro (3 members)	24	0	0	0	2
Morocco	58	0	0	1	2
Netherlands	94	0	2	2	0
New Zealand	42	0	0	0	3
North Korea	173	2	4	0	0
Norway	62	1	0	0	1
Paraguay (4 members)	24	0	0	1	0
Peru	141	1	3	2	0
Philippines (3 members)	23	0	0	1	0
Poland	157	2	3	1	0
Portugal	55	0	0	2	1
Puerto Rico (3 members)	9	0	0	0	0
Romania	141	0	4	2	0
Russia	199	6	0	0	0
Saudi Arabia	8	0	0	0	0
Serbia	139	1	3	0	2
Singapore	98	0	1	3	1
Slovakia	76	0	0	3	1
Slovenia	68	0	0	2	1
South Africa	79	0	1	0	4
South Korea	188	4	2	0	0
Spain	82	0	0	3	3
Sri Lanka	29	0	0	0	1
Sweden	67	0	1	0	3
Switzerland	68	0	1	1	2

Distribution of Awards at the 2008 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Taiwan	168	2	4	0	0
Tajikistan	60	0	0	1	3
Thailand	175	2	3	1	0
Trinidad & Tobago	28	0	0	1	0
Tunisia (4 members)	20	0	0	0	1
Turkey	170	3	1	2	0
Turkmenistan	76	0	0	4	1
Ukraine	153	2	2	2	0
U.A.E. (4 members)	5	0	0	0	0
United Kingdom	133	0	4	2	0
United States of America	190	4	2	0	0
Uruguay (5 members)	22	0	0	0	1
Uzbekistan	94	0	0	4	0
Venezuela (2 members)	16	0	0	0	1
Vietnam	159	2	2	2	0
Total (535 contestants)		**47**	**100**	**120**	**103**

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ICMI Study: Mathematical Challenges

Why should we teach mathematics? Traditionalists and politicians tend to answer this question in a practical way: to provide people with skills needed for life and to prepare students for careers. However, many educators decry such a narrow vision. They feel that their appreciation for their discipline can be more widely shared. Even accepting technical proficiency as an important goal, teachers realize that simple exposition and practice are often insufficient to foster utility, depth and fluency; students must somehow be engaged. While this idea is not new in theory or in practice, only recently has it attained widespread currency in facilities for teacher training, classrooms and public presentations around the world. This is due to several factors:

1. the ferment in mathematical education brought on by the need to appeal to a wide variety of students in the modern school and the realization that schools have frequently failed to achieve mathematical literacy
2. research developments in mathematics and its pedagogy
3. technological advances that have engendered access to information, communication among widely dispersed individuals and the ability to explore mathematics to some depth
4. a need to better inform the general public about a subject that (whether it realizes it) plays a large role in its wellbeing.

Into this environment comes the Study Volume for ICMI Study 16, *Challenging Mathematics in and Beyond the Classroom*, commissioned by the International Commission on Mathematics Instruction in 2002. The first fifteen studies, appearing about once a year, have embraced a range of issues, from the different areas of the curriculum to the impact of history, psychology and culture. This one is broadly based, touching on many areas of mathematics, as well as different styles of pedagogy and cultural milieux.

Peter Taylor of the Australian Mathematics Trust and Ed Barbeau were invited to chair the Study and were consulted in the appointment of the International Programme Committee. The IPC met in Modena, Italy to determine the scope of the Study and to issue an invitation for participation in the Study Conference held in Trondheim, Norway in July, 2006. In all, forty-five scholars from around the world gathered there to lay the foundation for the Study Volume. Over the next two years, teams of authors produced the individual chapters, and the book, entitled *Challenging Mathematics In and Beyond the Classroom*, is due to appear at the very end of 2008. The reader is invited to visit the site for the Study, www.amt.edu.au/icmis16.html, where the papers submitted by the participants in advance of the Study Conference along with a list of participants and the discussion document (the last in four languages) can be found.

To many, the words "mathematical challenge" evoke competitions. Indeed, the two chairmen were concerned that submissions from those in this area would predominate. The discussion paper was designed to make sure that the Study would attract a broader range of participants. A challenge arises in any situation in which "people are faced with a problem whose resolution is not apparent and for which there seems to be no standard method of solution." The committee strongly felt that the facing and surmounting of challenges was an essential component of mathematical learning, so that challenges should be present in all sorts of settings that have nothing to do with competition. In particular, if the classroom is to be the locus of challenge, then this has many implications for the design of the syllabus, formation of teachers and assessment.

In the end, about a third of the participants were "competition types," while the rest represented a host of activities including professional development of teachers, educational research, operation of mathematical exhibitions, design of internet material, work with groups of students in research, and investigation of mathematical problems, and assessment. At the Study Conference, three working groups were constituted to prepare chapters for the Study Volume. One dealt with challenges beyond the classroom (including competitions) while the others looked at challenges within the classroom from the viewpoint of the student and of the teacher. They were inspired by two keynote speakers, Jean-Pierre Kahane, from France, and Alexei Sossinsky, from Russia, both of whom remained for the entire time at the conference and participated actively.

The "Beyond the Classroom" group worked under the direction of Petar Kenderov from Bulgaria. Their deliberations led to the first three chapters of the Study Volume. Chapter 1 provides many examples with brief commentaries of challenges that arise from contests of all levels, as well as from traditional puzzles and adaptations of textbook material. Alexander Karp ties the chapter together with a discussion of how such examples can be constructed and used. It is abundantly clear that the creative impulses in twentieth century mathematics are not restricted to frontier research; the insatiable appetite for material of contests, projects and investigations has led to the recent creation of problems that are novel, ingenious and compelling. Consider for example this recent IMO problem: Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P. Show that the sum of the areas assigned to the sides of P is at least twice the area of P. There are easy cases that serve as entry points, but the full solution is difficult.

The second chapter surveys all the settings where challenges can be found. Along with competitions, students, teachers and the public have recourse to journals and books, conferences and camps, clubs and circles, exhibitions and houses, public lectures, open houses at universities, correspondence programmes, workshops and fairs, and finally websites. People can work individually or in teams, in a competitive or cooperative environment. The challenge can take the form of a problem with a single answer or a written-out solution or it can be an extended investigation. The sheer variety of contests is illustrated by the examples discussed, the *Australian Mathematics Competition*, *Euromath*, *Kapp Abel*, *Ontario Math Olympics*, *Tournament of the Towns* and the *A-Lympics*. Mathematical publications directed to the young have burgeoned over the last century from humble beginnings in Eastern Europe to books and magazines in every region of the world. There are now many opportunities for students to make their own investigations in countries such as Germany (*Jurgend Forscht*), the USA (*Research Science Institute*) and Bulgaria (*High School Students' Institute for Mathematics and Informatics*). For the general public, many countries boast mathematics exhibitions, such as *Mathematikum* in Giessen, Germany. The numerous activities described in the chapter that illustrate the different settings for challenge show that the time is ripe to take stock, analyze their structure and account for their successes. An appendix of the chapter discusses in more detail the Mathematical

Houses in Iran and the *Archimedes* mathematics organization in Serbia; the Iran case is interesting in its involvement of the local and national governments.

Of course, any discussion of challenges would be incomplete without a discussion of technology, which has become incredibly powerful and pervasive during the last two decades. Computers have not only allowed the dissemination of material and communication over long distances, but have affected how we perceive the whole learning environment, the classroom in particular. We can provide guidance that is tailor-made and designed to bring challenges to within Vygotski's Zone of Proximal Development, that is, to the point where the past experience of the solver can be brought to bear. The third chapter takes up the sort of challenge best suited to the technology environment, the tools that can be employed, the support that technology can provide in more traditional settings, psychological issues and cost. A section treats in detail the characteristics of the Internet-based *Numerical Working Spaces* that can bring together in one place resources for the users. There are numerous cases detailed in the chapter that illustrate design of material and types of context.

Now we move to the classroom with Chapters 4 and 5. In the first of these, the focus is on challenging problems and how they can be organized into strands. It is through the orchestration of challenges that students become aware of underlying structure and gain the ability to articulate their reasoning. Many examples in different settings are described. The fifth chapter opens with a discussion of the social learning environments, time constraints, instruments and objects, and pedagogical method. More examples are discussed, in this case, of activities for school students apart from the regular curriculum, such as visits to museums and art galleries, rallies, additional courses and original research. All of these illustrate how highly students can aim with the proper leadership.

We have to rely on teachers themselves to provide much of the leadership; Chapter 6 addresses the issue of professional development. Teachers must consciously consider the nature of mathematics and the importance of mathematical challenges in the classroom, understand what makes a suitable challenge, and be aware of factors, both psychological and mathematical, that might undermine success. It concludes with what research tells us about the affect of teacher knowledge and beliefs, psychological development and appropriate pedagogy. Chapter 7 offers some examples to show how classroom tasks can be designed in order that students are willing to accept and appreciate challenges. The final chapter of the Study Volume treats assessment issues. Singapore, in particular, has mandated challenge as part of the curriculum and designed its tests to incorporate challenge items along with procedural ones. Norway too has recently revamped its syllabus with attendant modifications to its examination system. Many questions in this area are posed that require research.

The Study occurred at a critical time, when considerable experience is available to build upon, but when systematic research into the role and use of challenge is only beginning. We expect that the Study Volume will become a standard reference that will help to focus this research.

1 Appendix

The Study Volume is really a related collection of papers, each written by a lead author with the collaboration of a team of participants. Here is a list of the chapter titles along with the authors. The lead author is given in italics.

1. Challenging Problems: Mathematical Contents and Sources (41 pages) Vladimir Protassov (Russia), Mark Applebaum (Israel), Alexander Karp (USA), Romualdas Kašuba (Lithuania), Alexey Sossinsky (Russia), Ed Barbeau (Canada), Peter Taylor (Australia)
2. Challenges Beyond the Classroom—Sources and Organization Issues (44 pages) *Petar Kenderov* (Bulgaria), Ali Rejali (Iran), Maria G. Bartolini Bussi (Italy), Valeria Pandelieva (Canada), Karin Richter (Germany), Michela Maschietto (Italy), Djordje Kadijevich (Serbia), Peter Taylor (Australia)
3. Technological Environments beyond the Classroom (29 pages) *Viktor Freiman* (Canada), Djordje Kadijevich (Serbia), Gerard Kuntz (France), Sergey Pozdnyakov (Russia), Ingvill Stedoy (Norway)

4. Challenging Tasks and Mathematical Learning (38 pages) *Arthur B. Powell* (USA), Inger Christin Borge (Norway), Gemna Inez Fioriti (Argentina), Margo Kondratieva (Canada), Elena Koublanova (USA), Neela Sukthankar (Canada/PNG)
5. Mathematics in Context: Focussing on Students (33 pages) *Maria G. Bartolini Bussi* (Italy), Sharade Gade (India), Martine Janvier (France), Jean-Pierre Kahane (France), Vince Matsko (USA), Michela Maschietto (Italy), Cecile Ouvrier-Buffet (France), Mark Saul (USA)
6. Teacher Development and Mathematical Challenge (38 pages) *Derek Holton* (New Zealand), Kwok-cheung Cheung (Macau), Sesutho Kesianye (Botswana), Maria Falk de Losada (Colombia), Roza Leikin (Israel), Gregory Makrides (Cyprus), Hartwig Meissner (Germany), Linda Sheffield (USA), Bharath Sriraman (USA), Ban Har Yeap (Singapore)
7. Challenging Mathematics: Classroom Practices (40 pages) *Gloria Stillman* (Australia), Kwok-Cheung Cheung (Macau), Ralph Mason (Canada), Linda Sheffield (USA), Kenji Ueno (Japan)
8. Curriculum and Assessment that Provide Challenge in Mathematics (31 pages) *Maria Folk de Losada* (Colombia), Ban-Har Yeap (Singapore), Gunnar Gjone (Norway), Mohammad Hossein Pourkazemi (Iran)

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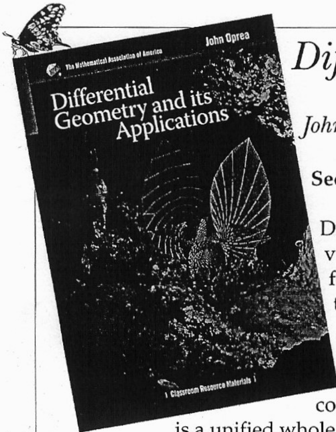
New from the Mathematical Association of America



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John Oprea

Second Edition

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Table of Contents: *Preface • Chapter 1. The Geometry of Curves • Arclength Parametrization • Frenet Formulas • Non-Unit Speed Curves • Some Implications of Curvature and Torsion • Green's Theorem and the Isoperimetric Inequality • The Geometry of Curves and Maple • Chapter 2. Surfaces • The Geometry of Surfaces • The Linear Algebra of Surfaces • Normal Curvature • Surfaces and Maple • Chapter 3. Curvatures • Calculating Curvature • Surfaces of Revolution • A Formula for Gauss Curvature • Some Effects of Curvature(s) • Surfaces of Delaunay • Elliptic Functions, Maple and Geometry • Calculating Curvature with Maple • Chapter 4. Constant Mean Curvature Surfaces • First Notions in Minimal Surfaces • Area Minimization • Constant Mean Curvature • Harmonic Functions • Complex Variables • Isothermal Coordinates • The Weierstrass-Enneper Representations • Maple and Minimal Surfaces • Chapter 5. Geodesics, Metrics and Isometries • The Geodesic Equations and the Clairaut Relation • A Brief Digression on Completeness • Surfaces not in* $\mathbb{R}^3$*• Isometries and Conformal Maps • Geodesics and Maple • An Industrial Application • Chapter 6. Holonomy and the Gauss-Bonnet Theorem • The Covariant Derivative Revisited • Parallel Vector Fields and Holonomy • Foucault's Pendulum • The Angle Excess Theorem • The Gauss-Bonnet Theorem • Applications of Gauss-Bonnet • Geodesic Polar Coordinates • Maple and Holonomy • Chapter 7. The Calculus of Variations and Geometry • The Euler-Lagrange Equations • Transversality and Natural Boundary Conditions • The Basic Examples • Higher-Order Problems • The Weierstrass E-Function • Problems with Constraints • Further Applications to Geometry and Mechanics • The Pontryagin maximum Principle • An Application to the Shape of a Balloon • The Caluclus of Variations and Maple • Chapter 8. A Glimpse at Higher Dimensions • Manifolds • The Covariant Derivative • Christoffel Symbols • Curvatures • The Charming Doubleness*

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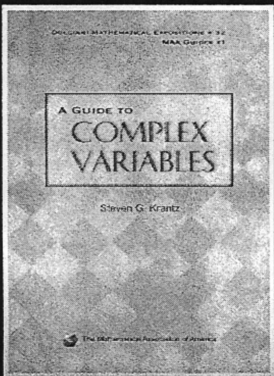

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Useful Problem Solving Books from AMT Publications

These books are a valuable resource for the school library shelf, for students wanting to improve their understanding and competence in mathematics, and for the teacher who is looking for relevant, interesting and challenging questions and enrichment material.

To attain an appropriate level of achievement in mathematics, students require talent in combination with commitment and self-discipline. The following books have been published by the AMT to provide a guide for mathematically dedicated students and teachers.

Bundles of Past AMC Papers

Past Australian Mathematics Competition papers are packaged into bundles of ten identical papers in each of the Junior, Intermediate and Senior divisions of the Competition. Schools find these sets extremely valuable in setting their students miscellaneous exercises.

AMC Solutions and Statistics
Edited by PJ Taylor

This book provides, each year, a record of the AMC questions and solutions, and details of medallists and prize winners. It also provides a unique source of information for teachers and students alike, with items such as levels of Australian response rates and analyses including discriminatory powers and difficulty factors.

Australian Mathematics Competition Book 1 1978–1984
Edited by W Atkins, J Edwards, D King, PJ O'Halloran & PJ Taylor

This 258-page book consists of over 500 questions, solutions and statistics from the AMC papers of 1978-84. The questions are grouped by topic and ranked in order of difficulty. The book is a powerful tool for motivating and challenging students of all levels. A must for every mathematics teacher and every school library.

Australian Mathematics Competition Book 2 1985–1991
Edited by PJ O'Halloran, G Pollard & PJ Taylor

Over 250 pages of challenging questions and solutions from the Australian Mathematics Competition papers from 1985-1991.

Australian Mathematics Competition Book 3 1992–1998
W Atkins, JE Munro & PJ Taylor

More challenging questions and solutions from the Australian Mathematics Competition papers from 1992-1998.

Australian Mathematics Competition Book 3 CD
Programmed by E Storozhev

This CD contains the same problems and solutions as in the corresponding book. The problems can be accessed in topics as in the book and in this mode is ideal to help students practice particular skills. In another mode students can simulate writing one of the actual papers and determine the score that they would have gained. The CD runs on Windows platform only.

Australian Mathematics Competition Book 4 1999-2005
W Atkins & PJ Taylor

More challenging questions and solutions from the Australian Mathematics Competition papers from 1999-2005.

Problem Solving via the AMC
Edited by Warren Atkins

This 210-page book consists of a development of techniques for solving approximately 150 problems that have been set in the Australian Mathematics Competition. These problems have been selected from topics such as Geometry, Motion, Diophantine Equations and Counting Techniques.

Methods of Problem Solving, Book 1
Edited by JB Tabov & PJ Taylor

This book introduces the student aspiring to Olympiad competition to particular mathematical problem solving techniques. The book contains formal treatments of methods which may be familiar or introduce the student to new, sometimes powerful techniques.

Methods of Problem Solving, Book 2
JB Tabov & PJ Taylor

After the success of Book 1, the authors have written Book 2 with the same format but five new topics. These are the Pigeon-Hole Principle, Discrete Optimisation, Homothety, the AM-GM Inequality and the Extremal Element Principle.

Mathematical Toolchest
Edited by AW Plank & N Williams

This 120-page book is intended for talented or interested secondary school students, who are keen to develop their mathematical knowledge and to acquire new skills. Most of the topics are enrichment material outside the normal school syllabus, and are accessible to enthusiastic year 10 students.

International Mathematics — Tournament of Towns (1980–1984)
Edited by PJ Taylor

The International Mathematics Tournament of the Towns is a problem-solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. This 115-page book contains problems and solutions from past papers for 1980-1984.

International Mathematics — Tournament of Towns (1984–1989)
Edited by PJ Taylor

More challenging questions and solutions from the International Mathematics Tournament of the Towns competitions. This 180-page book contains problems and solutions from 1984-1989.

International Mathematics — Tournament of Towns (1989–1993)
Edited by PJ Taylor

This 200-page book contains problems and solutions from the 1989-1993 Tournaments.

International Mathematics — Tournament of Towns (1993–1997)
Edited by PJ Taylor

This 180-page book contains problems and solutions from the 1993-1997 Tournaments.

International Mathematics — Tournament of Towns (1997–2002)
Edited by AM Storozhev

This 214-page book contains problems and solutions from the 1997-2002 Tournaments.

Challenge! 1991 – 1998
Edited by JB Henry, J Dowsey, AR Edwards, L Mottershead, A Nakos, G Vardaro & PJ Taylor

This book is a major reprint of the original Challenge! (1991-1995) published in 1997. It contains the problems and full solutions to all Junior and Intermediate problems set in the Mathematics Challenge for Young Australians, exactly as they were proposed at the time. It is expanded to cover the years up to 1998, has more advanced typography and makes use of colour. It is highly recommended as a resource book for classes from Years 7 to 10 and also for students who wish to develop their problem-solving skills. Most of the problems are graded within to allow students to access an easier idea before developing through a few levels.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

Australian Mathematical Olympiads 1979 – 1995
H Lausch & PJ Taylor

This book is a complete collection of all Australian Mathematical Olympiad papers from the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads Book 1 1981–1993
A Liu

This book contains the papers and solutions of two contests, the Chinese National High School Competition and the Chinese Mathematical Olympiad. China has an outstanding record in the IMO and this book contains the problems that were used in identifying the team candidates and selecting the Chinese team. The problems are meticulously constructed, many with distinctive flavour. They come in all levels of difficulty, from the relatively basic to the most challenging.

Asian Pacific Mathematics Olympiads 1989–2000
H Lausch & C Bosch-Giral

With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981–1995
ME Kuczma & E Windischbacher

Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
Edited by T Andreescu & Z Feng

This book contains one hundred and one highly rated problems used in training and testing the USA International Mathematical Olympiad team. The problems are carefully graded, ranging from quite accessible towards quite challenging. The problems have been well developed and are highly recommended to any student aspiring to participate at National or International Mathematical Olympiads.

Hungary Israel Mathematics Competition
S Gueron

The Hungary Israel Mathematics Competition commenced in 1990 when diplomatic relations between the two countries were in their infancy. This 181-page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions. The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Chinese Mathematics Competitions and Olympiads Book 2 1993–2001
A Liu
This book is a continuation of the earlier volume and covers the years 1993 to 2001.

Bulgarian Mathematics Competition 1992-2001
BJ Lazarov, JB Tabov, PJ Taylor & A Storozhev

The Bulgarian Mathematics Competition has become one of the most difficult and interesting competitions in the world. It is unique in structure combining mathematics and informatics problems in a multi-choice format. This book covers the first ten years of the competition complete with answers and solutions. Students of average ability and with an interest in the subject should be able to access this book and find a challenge.

Mathematical Contests – Australian Scene
Edited by PJ Brown, A Di Pasquale & K McAvaney

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

Mathematics Competitions
Edited by J Švrcek

This bi-annual journal is published by AMT Publishing on behalf of the World Federation of National Mathematics Competitions. It contains articles of interest to academics and teachers around the world who run mathematics competitions, including articles on actual competitions, results from competitions, and mathematical and historical articles which may be of interest to those associated with competitions.

Problems to Solve in Middle School Mathematics
B Henry, L Mottershead, A Edwards, J McIntosh, A Nakos, K Sims, A Thomas & G Vardaro

This collection of problems is designed for use with students in years 5 to 8. Each of the 65 problems is presented ready to be photocopied for classroom use. With each problem there are teacher's notes and fully worked solutions. Some problems have extension problems presented with the teacher's notes. The problems are arranged in topics (Number, Counting, Space and Number, Space, Measurement, Time, Logic) and are roughly in order of difficulty within each topic. There is a chart suggesting which problem-solving strategies could be used with each problem.

Teaching and Assessing Working Mathematically Book 1 & Book 2
Elena Stoyanova

These books present ready-to-use materials that challenge students understanding of mathematics. In exercises and short assessments, working mathematically processes are linked with curriculum content and problem solving strategies. The books contain complete solutions and are suitable for mathematically able students in Years 3 to 4 (Book 1) and Years 5 to 8 (Book 2).

A Mathematical Olympiad Primer
G Smith

This accessible text will enable enthusiastic students to enter the world of secondary school mathematics competitions with confidence. This is an ideal book for senior high school students who aspire to advance from school mathematics to solving olympiad-style problems. The author is the leader of the British IMO team.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Innovation, Industry, Science and Research) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

Newton Enrichment Student Notes
JB Henry

Recommended for mathematics students of about Year 5 and 6 as extension material. Topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

Dirichlet Enrichment Student Notes
JB Henry

This series has chapters on some problem solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory. It is designed for students in Years 6 or 7.

Euler Enrichment Student Notes
MW Evans & JB Henry

Recommended for mathematics students of about Year 7 as extension material. Topics include elementary number theory and geometry, counting, pigeonhole principle.

Gauss Enrichment Student Notes
MW Evans, JB Henry & AM Storozhev

Recommended for mathematics students of about Year 8 as extension material. Topics include Pythagoras theorem, Diophantine equations, counting, congruences.

Noether Enrichment Student Notes
AM Storozhev

Recommended for mathematics students of about Year 9 as extension material. Topics include number theory, sequences, inequalities, circle geometry.

Pólya Enrichment Student Notes
G Ball, K Hamann & AM Storozhev

Recommended for mathematics students of about Year 10 as extension material. Topics include polynomials, algebra, inequalities and geometry.

T-SHIRTS

T-shirts of the following six mathematicians are made of 100% cotton and are designed and printed in Australia. They come in white, Medium (Turing only) and XL.

Leonhard Euler T–shirt

The Leonhard Euler t-shirts depict a brightly coloured cartoon representation of Euler's famous Seven Bridges of Königsberg question.

Carl Friedrich Gauss T–shirt

The Carl Friedrich Gauss t-shirts celebrate Gauss' discovery of the construction of a 17-gon by straight edge and compass, depicted by a brightly coloured cartoon.

Emmy Noether T–shirt

The Emmy Noether t-shirts show a schematic representation of her work on algebraic structures in the form of a brightly coloured cartoon.

George Pólya T–shirt

George Pólya was one of the most significant mathematicians of the 20th century, both as a researcher, where he made many significant discoveries, and as a teacher and inspiration to others. This t-shirt features one of Pólya's most famous theorems, the Necklace Theorem, which he discovered while working on mathematical aspects of chemical structure.

Peter Gustav Lejeune Dirichlet T–shirt

Dirichlet formulated the Pigeonhole Principle, often known as Dirichlet's Principle, which states: "If there are p pigeons placed in h holes and $p>h$ then there must be at least one pigeonhole containing at least 2 pigeons." The t-shirt has a bright cartoon representation of this principle.

Alan Mathison Turing T-shirt

The Alan Mathison Turing t-shirt depicts a colourful design representing Turing's computing machines which were the first computers.

ORDERING

All the above publications are available from AMT Publishing and can be purchased on-line at:

www.amt.edu.au/amtpub.html or contact the following:

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The Australian Mathematics Trust

The Trust, of which the University of Canberra is Trustee, is a non-profit organisation whose mission is to enable students to achieve their full intellectual potential in mathematics. Its strengths are based upon:

- a network of dedicated mathematicians and teachers who work in a voluntary capacity supporting the activities of the Trust;
- the quality, freshness and variety of its questions in the Australian Mathematics Competition, the Mathematics Challenge for Young Australians, and other Trust contests;
- the production of valued, accessible mathematics materials;
- dedication to the concept of solidarity in education;
- credibility and acceptance by educationalists and the community in general whether locally, nationally or internationally; and
- a close association with the Australian Academy of Science and professional bodies.