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MATHEMATICS COMPETITIONS

JOURNAL OF THE
WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

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The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the President

There are several events in 2008 which are of importance for the *World Federation of National Mathematics Competitions* (WFNMC). Number one, no doubt, is the *International Congress on Mathematical Education* (ICME) that will take place in Monterrey, Mexico, from 6–13 July. The program and all activities associated with the Congress are gradually taking shape and it is a proper time to finalize the plans for participation in the Congress. Helpful information concerning the Congress is contained in its website: http://www.icme11.org.mx/icme11/. The event is organized by the *International Commission on Mathematical Instruction* (ICMI) to which our Federation has been an *Affiliated Study Group* since 1994. In 2008 ICMI becomes 100 years old. It was established under the name *International Commission on the Teaching of Mathematics* during the fourth *International Congress of Mathematicians* (Rome, 6–11 April, 1908). In this connection a Symposium will be organized in Rome (5–8 March, 2008) under the title:

The First Century of the International Commission on Mathematical Instruction (1908–2008)

Reflecting and Shaping the World of Mathematics Education

The website of this interesting event is at
http://www.unige.ch/math/EnsMath/Rome2008/.

It has become a tradition for the Federation to organize its *Formal General Meeting* during the ICME's. This is in accordance with Article 6 of the Constitution of the Federation which stipulates that: "At least once every four years the Federation will hold (usually at one of its two-yearly conferences) a Formal General Meeting open to all its members". At the meeting in Monterrey, Mexico, the new *Executive Committee* of the Federation as well as the new *Standing Committees* will be elected. The Officers to be elected are the *President*, 3 *Vice-Presidents* (one of whom will be designated *Senior Vice-President*), the *Publications Officer* and the *Secretary*. The Standing Committees to be determined are:

the *Program Committee*, the *Award Committee* and the *Committee of Regional Representatives*. Nominations for these positions are welcome.

Let me remind you also, that there is a special procedure (Article 12 of the Constitution) how to change the by-laws regulating the life of the Federation. Any amendments may be proposed in writing to the President at least sixty days before the meeting of the Federation. Such amendment, alteration, or repeal of the constitution will be placed before the members only if confirmed by a majority of votes cast at an Executive Committee Meeting. Any amendment, alteration, or repeal must be approved by two-thirds of the members present at a duly constituted Federation meeting.

Petar S. Kenderov
President of WFNMC
November 2007

From the Editor

Welcome to *Mathematics Competitions* Vol. 20, No 2.

Again I would like to thank the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:
The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

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Jaroslav Švrček,
December 2007

Creating an Olympiad Tradition in Colombia: What Went Right
(Plenary session at the WFNMC Conferece)

María Falk de Losada

At the invitation of the WFNMC Conference (July 26, Cambridge 2006) organizers, we attempt to analyze the strategies and choices, the plans, projects and outcomes, that have enabled Colombia (and several of its neighbors) to create and sustain a tradition of enrichment and competitions that have impacted the educational and mathematical communities and the development of mathematics in Colombia and the region.

1 Presentation

When asked by Tony Gardiner about a year ago to give a plenary talk at this conference trying to explain the Math Olympiad movement in Colombia, the performance of Colombian students and teams at the IMO and other international problem-solving competitions, and what lies behind it, I immediately accepted although I was not so sure of what I would have to say or of what value it might be for this highly qualified audience. In 2006 we are commemorating the twenty-fifth anniversary of the Colombian Mathematics Olympiads and this opportunity to speak at the WFNMC Conference is one of the ways to celebrate.

I suspect that I was interested in finding out for myself what arguments might be presented to support an explanation, and at the same time felt highly complimented that the organization of this conference saw something special in the gains made by Colombian students and the Colombian Math Olympiads.

What follows is then a first, tentative, laying out of the contribution of people, policies and strategies to the Colombian participation in problem solving competitions with special attention paid to the international level. This will inevitably lead us to Olympiads in all the Iberoamerican countries, for this highly supportive community is one of the keys to understanding all of our activities.

In preparing for this talk we were fortunate to attend a National Congress of Mathematics in August, 2005 that featured many ex-Olympians as speakers and we were fortunate in hosting, in September 2005, the XX Olimpiada Iberoamericana de Matemáticas, an event that brought together as organizers and coordinators, many ex team members and ex leaders and deputy leaders, both from the IMO and from the Iberoamerican and Central American Olympiads. This enabled us to appreciate firsthand the careers that many of our students have followed, and to see the impact on both the students themselves and those who have followed in their footsteps, and on the mathematics community as a whole.

2 Getting Started

When one begins an entirely new activity, as was the case of the Math Olympiads in Colombia, one makes many plans and choices without expertise or guidance, so that intuition and general good judgment play a part in the ensuing failure or success. In other words, we consider that we were lucky or that we made many "happy" decisions.

Getting Started: Looking For and Sticking to International Standards and Being Lucky

For several years during the 1970's Professor Ricardo Losada had to no avail written a series of projects to get financing from the appropriate Colombian sources to organize a national math olympiad. These sources were a governmental organization devoted to educational testing, the Colombian Ministry of Education and COLCIENCIAS, the Colombian, very junior, counterpart to NSF. Then ICME was to be held in Berkeley in 1980 providing an opportunity to speak directly with the organizers of the 1981 IMO to take place in Washington. A plan was hatched to look for the IMO organizers at ICME and solicit directly an invitation to the 1981 IMO. This was to be the mechanism for jump-starting olympiad activity in Colombia—and it worked! If we were unable to begin with a national olympiad, we would begin with international competition.

Preparation got underway. We began by forming an interinstitutional group, Universidad Nacional de Colombia (National University) represented by the chairman of its Mathematics Department, Colombian Math Society and Universidad Antonio Nariño, each represented by its President. The effort was led by Rafael Mariño who would be the team leader and myself who would become the first deputy, but a large portion of the math department of the National University got involved. This was the first stroke of luck or happy coincidence. There was a base of people willing to learn about the IMO. We began by publishing the problems of former IMOs and asking professors to contribute solutions. We worked hard on them ourselves, without referring to the official solutions, trying to get a real sense of the level of the competition. None of us was particularly good at it at the beginning, we had not come up through the ranks as olympiad competitors ourselves, we had to learn about problem solving on the IMO level, and we succeeded, at least partially. We went about this during the "year" between ICME-4 and IMO Washington. It was an important choice that would enable us to train a Colombian team for the competition.

But where were we to get a team? We invited schools from Bogotá to take part in a local olympiad-type competition. One hundred ten students from thirty-three schools did so. This was a second stroke of luck or circumstance, there were many very good (mostly private) schools in Bogotá willing to take the risk of allowing their students to be challenged in mathematics. Not having a broad base, we decided on a long series of weekly "tests" to choose our team. After about four months of working twice weekly with students on problems and solutions, as well as having them take tests, we chose a team of eight. Three from the Colegio San Carlos, which has produced one or more of our team members for the IMO just about every year since 1981, including our present team leader, Federico Ardila. Two from the Liceo Francés Louis Pasteur (one a girl), one from the Colegio Hebreo, one from the Colegio Antonio Nariño and one from the Colegio Santo Tomás de Aquino. This long process was another good decision (although completely untried). We really did have an excellent team assembled. Nevertheless, there were to be no medals at the Washington IMO, a team score of 93 and an average of 12 points was disappointing, but not discouraging, and whet our appetite for continuing to compete at the IMO.

Other lucky coincidences were to appear. The Colombian Minister of Education at the time was a lawyer, but more pertinently a frustrated mathematician whose father did not allow him to study mathematics. He put the Ministry of Education behind this first effort and financed the students' travel. He became directly involved with our project of creating a Colombian Math Olympiad, and supervised its approval when presented the following year. The Ministry of Education was subsequently behind the Colombian Olympiad for its first twelve years of existence.

Going Directly to the Students

Meanwhile, another excellent decision was made. We sought, and obtained, newspaper space to take the problems to the people. The director of one of the Bogotá dailies, El Espectador, was another aficionado of math problems and was willing to take the risk. Thus began eight years of a Sunday column of problems with readers' solutions published, and with a short introductory essay on one or more fun topics in math. Popularization was linked directly to the Colombian Math Olympiad and taken to the students themselves without having to go through the teachers and the schools, which would have drawn out the process for several more years, and maybe have prevented many students from hearing about the Olympiad or getting involved.

Receiving the Help and Following the Lead of Outstanding International Figures

Undoubtedly one of the most important ingredients in starting out on the right track and on firm footing, was the friendship with Sam Greitzer that began with the meeting in Berkeley, was nourished by the IMO experience in Washington, and that flourished in the following years as Sam made a series of trips to Colombia to accompany, advise and encourage us over the following years. He went there to train students, and we learned a lot about training, and to talk to us at length about the way he envisioned olympiads and teaching gifted students. This partnership, with the Colombia olympiad organizers still as very junior partners, was to lead to the founding of the Iberoamerican Olympiad, a step that was important for Colombia and crucial to the development of math Olympiad activities in all of Iberoamerica.

It was Sam Greitzer who intoduced us to Walter Mientka that same summer of 1981 in Pittsburgh. It was to be a good friendship and a fruitful professional relationship through the next twenty years.

Not only were individuals to work together with us. Our early, and wise, decision was that the first round of our competitions would remain demanding with regard to international standards. Due to the relationship with Walter Mientka and later Titu Andreescu, as well as Peter O'Halloran and Peter Taylor, we have taken part in the American Math Competition, the American Elementary School Olympiad, and the Australian Math Competition as a policy and as part of our philosophy. We have always believed that our students can be as good as students from any other country in the world (a truism), but we must both find them and challenge our schools to improve constantly. So we have always taken part in international challenges.

3 Going Forward

Building a Team of Doers with Young People Willing to Learn

The first Colombian Math Olympiad in 1982 attracted 1000 students and the second 2500, during the past few years the figure has stabilized around 75000–80000, a lull in growth that has us rather worried, given the surge in Olympiad participation especially in Brazil (twelve million) and Peru (two and a half million). But there are important choices that have been made and that have strengthened the Colombian hand. One of these has been building a team of organizers and trainers composed of a constantly-changing group of young ex-Olympians.

Perhaps the international trend in this direction that can be perceived is due to Colombia. Many other countries followed suit. Others have not renewed their Olympiad teams, or have done so only under extreme circumstances.

Problem Posers

One of the most fruitful and correct decisions we made was to begin to work on forming a team of problem posers. We encouraged our young students to become problem posers, and began by offering a $1000 prize for the first problem selected for use in the IMO and proposed on behalf of Colombia. This was another high international goal we set ourselves and our students from the beginning. Our first problem short-listed was in 1981 and our first problem selected came more than twenty years later, created by Federico Ardila.

In the meantime our young ex-Olympians, working in the organization of the Colombian National Olympiad and the training of gifted young students in January and June of each year, began posing new problems that were accepted for the Iberoamerican Math Olympiad, the Asian-Pacific Math Olympiad, and more recently for the Math Olympiad of Central America and the Caribbean. Although at first we adapted good problems from around the world, we soon began to formulate our own problems for the Colombian Olympiad, starting with short answer problems, then moving on to two levels each of problems requiring complete solutions for children of three different grade levels.

Posing original problems truly enabled our young ex-Olympians (still at university) to become great trainers of younger students, not only because it required them to get to know the problems that were being posed in other competitions, but also because they were able to exercise their own creativity and special problem talents and imprint a specially Colombian flavor on their problems. The team of young ex-Olympians gradually turned into a school. What are the particular advantages? Renewal is a positive word indicating a positive step. One of the strong points to mathematical problem solving is the orginality of the problems and the creativity they elicit from students.

Why has this been important? Novel ideas continually crop up. Different people get interested in different areas of mathematics and different areas of problem solving. This creates a constant evolution, a good variety in the training sessions and types of problems posed on the olympiad tests. The entire organization becomes richer and a student who spends three or four, or more, years of his life participating in the olympiad or related activities constantly encounters new material and challenges, keeping his interest honed and rounding-out his mathematical “repertoire”.

Growing and Diversifying

Math Olympiad for Primary Schools

It soon became apparent that it would be necessary and important to serve other groups of young students. Our first initiative then was to establish a competition for primary school students, grades 3 to 5.

This was seen as important to encourage students to love mathematics and develop their mathematical interests, creativity and talents in problem solving from the time they had learned to read. So the first primary school olympiad was held in 1984 and has continued ever since with wonderful results year after year.

Here are photos of one of the early winners of the competition for third graders in the figure.

Figure 1: Federico Ardila and María de Losada

Figure 2: Federico Ardila. First place among third graders. Colombian Math Olympiad for Primary School.

Figure 3: Another first place finish in the primary school Olympiad.

Math Olympiad for University Students

Although it was also apparent that those students who had taken part in the Math Olympiad in secondary school sometimes felt let down not

Figure 4: María de Losada and Federico Ardila in the organization with Angelica Osorno, Candidate. PhD. in mathematics, MIT. Deputy leader, Colombian team, Centroamerican and Caribbean Math Olympiad 2003–2005.

to find the same challenging atmosphere at university, it was to be several more years, not until 1997, that the University Olympiad was first organized. We quickly moved to founding the Iberoamerican Math Olympiad for University Students, as a correspondence type competition in 1998 and shortly thereafter were able to begin taking part in the International Math Competition for University Students in 2002. This move to internationalize is fundamental in creating and maintaining the interest of students and universities.

Future Olympians

Looking for ways to draw ever more students into challenging mathematics, we were not afraid to innovate, looking to alternative forms of representation of problems as an avenue of access for those intimidated or stymied by verbal and symbolic presentation of mathematics and mathematics problems, but still very able to develop and exercise mathematical thinking. The Future Olympians competition ever since has attempted to reach these students and encourage them to do mathematics.

Building International Groups on the Regional Level

One of the most important projects proposed early on was the organization, by Colombia, of an olympiad on the regional level. After several options were discussed, between Latin American, Pan American and Iberoamerican, this third alternative was chosen, and it also proved to be an excellent choice.

Several currents of thought coincided in 1984 leading to the founding of the OIM. There was the hope of the Colombian Ministry of Education to find constructive means of renewing and making more relevant the OEI (Organización de Estados Iberoamericanos para la Educación, la Ciencia y la Cultura) over which Colombia was then presiding. There was the idea of Samuel Greitzer, founder of the USA Mathematical Olympiad, to found a similar event on the Panamerican scene. And there was the dynamism of the Colombian Mathematical Olympiads (OCM), that had recently obtained the first medals for Colombia at the International Mathematical Olympiad (IMO), dynamism that was looking for a way to spill over and encourage and help to prepare other Iberoamerican countries to take part at the international level, by getting their first experiences on the regional level of competition.

Thus was conceived a joint project to organize the first Iberoamerican mathematical Olympiad in Colombia. The project was approved by the Assembly of Ministers of Education of the Iberoamerican Countries, meeting in Bogotá in the spring of 1985, and the first version of this regional olympiad took place in December of the same year with the participation of the ten founding countries mentioned above.

In 1984 when the project began only four Iberoamerican Nations had taken part in the IMO on a regular basis; Cuba, starting in 1972, Brazil (1979), Colombia (1981) and Spain (1983). The success of the original objective of leading a greater number of Iberoamerican countries toward international participation in an event such as the IMO of such high prestige and great import is also clear today. 13 Iberoamerican countries take part in the IMO on a regular basis, while others (notably Chile) have participated sporadically.

We wanted to show a path that would transcend the limitation of our mathematical schooling and lead to the conquest of a respectable international level in mathematics education consequent with the demands of a citizen of our times. The Iberoamerican Math Olympiad would also serve as the culmination of a vibrant activity of problem solving and development of mathematical thought beginning with the earliest stages of primary education and continuing on to the university level in each participating country. This indeed has been the case in the majority of the Iberoamerican countries; olympiad activity on the regional or international level is the culmination of events with popular participation on all school levels, events that challenge a large number of students to think creatively in mathematics and give them and their teachers orientation that enables them to enrich their mathematical education and attain higher personal levels of development of problem-solving ability and consequential mathematical thought.

Regional Integration Leads to Competitivity on the International Level

With this project the countries of the Iberoamerican region have shown leadership, in mathematics and mathematics education, with respect to tendencies that have since been proven to be of transcendental importance: they have achieved regional unity and cooperation as a prelude to becoming globally competitive. Thus the story of the Iberoamerican Mathematics Olympiad (Olimpiada Ibero-americana de Matemáticas—OIM) is the history of the insertion of students from Latin America and the Iberian Peninsula into the international arena of mathematics competitions.

Looking at the results of IMO 2004 held in Athens with the participation of some 85 countries, we find facts that may be surprising to many. Iberoamerican students have become and remained highly competitive on the international level.

The IMO distinguishes individual students and does not officialy rank countries; furthermore results vary from year to year. However, the unofficial team scores of 2004 do provide perspective. To illustrate we extract only some of the scores of European and Latin American countries. The highest team score was China with 220 points, followed by the USA with 212.

United Kingdom	134
Brazil	132
Germany	130
Colombia	122
México	96
France	94
Argentina	92
Italy	69
Spain	57
Netherlands	53
Perú	49
Ireland	48
Uruguay	47

These results clearly show that Iberoamerican teams not only take part in the IMO, they have reached a respectable international standard of excellence.

IMO results vary from year to year, and among the Iberoamerican countries it is still possible to perceive a rather wide variation. The results of 2006 might be said to be similar, but the names of the countries occupying these positions has changed. I believe that in good measure these wide variations are the consequence of an Olympiad tradition that is still relatively immature when judged by the standards of many other countries.

Impact on Students, Math Educators and the Mathematical Community

There are many aspects in which these olympiads impact the Iberoamerican educational and mathematical communities.

The math olympiad experience, starting from the elementary level, allows many students to develop their mathematical thought and problem-solving ability, striving to reach personal bests. It allows teachers to see how carefully planned problems with many different avenues of approach and solution can appeal to their students' imagination and enrich their understanding of mathematical concepts and relationships.

By making mathematics both challenging and fun, it becomes an attractive career choice, and also allows students to build a much stronger foundation for higher studies in other related areas such as physics, engineering or economics. The mathematical communities have been renewed by a greater number of talented young people entering the profession after having taken part in olympiads and having developed many of the skills required to do successful research in mathematics, such as a broad and deep grasp of fundamental mathematics, creative and flexible ways of thinking mathematically, ability to relate different areas of mathematics in new ways, and extraordinary capability for solving challenging and original problems.

Additionally, teachers have learned that many of their students are capable of mapping out their own strategies and thought patterns in mathematics when their desire to solve a particularly attractive new problem leads them to concentrate their mathematical strength and energy, laying a foundation for more ambitious school curricula.

Math educators can point to their research concerning the way in which the school math experience gradually drains many students' confidence in their own capacity to think consequentially about a problem, leading them by adolescence to strike haphazardly at a problem by attempting to apply pre-established formulas and methods drilled in school, results which necessarily imply a rethinking of the school mathematics experience.

The Iberoamerican Math Olympiad, and the enormous amount of mathematical activity it has unleashed or given direction to, has changed the face of school, university and professional mathematics in Iberoamerica and has contributed significantly to the development of mathematics in the region.

Having only two mutually comprehensible languages made this competition, though potentially large, quite an easy one to handle from the logistics standpoint. No need for specialists in many languages among the coordinators, every leader able to express himself in his own language in jury meetings and before coordinators. A mutually supportive group of people with several similarities in their cultural backgrounds, who have even read the literature and seen the films produced in each of the other participating countries, gives grounds for understanding, eases the possibility of exchanges, and just opens up the future for all taking part. Colombia, among other things, has been able to shine in the Iberoamerican group whenever it has brought a strong team with the intention of standing out (there are times when it is deemed more important for younger students to get some experience at the OIM before tackling the IMO, and so a less-experienced team with less than shining results may be taken to the OIM).

The Iberoamerican Math Olympiad celebrated its twentieth version in 2005 in Cartagena, Colombia; for the second consecutive year teams from all 22 Iberoamerican countries took part. As a prelude to the Olympiad, a three-day Problem-solving Seminar was held for all participating teams and local students, featuring such luminaries as Andy Liu, Titu Andreescu and Patricia Fauring. The strength of the Seminar is both motivational and academic on the one hand, and far more importantly strikes a chord of cooperation and harmony in the task of giving students what they need to face the challenge of the competition successfully.

The Iberoamerican community is there. We built it ourselves, and have no unhealthy rivalries, only healthy competition. We can find support among our group when we need it. Olympiad organizers from one country, among them many Colombians, will often take part in the organization of the Iberoamerican or Centroamerican Olympiad when it takes place in another country, as coordinators, members of the problem selection committee, or as speaker in the pre-Olympiad seminar.

Our students train each year with some students from other countries. Students from Costa Rica, Ecuador, Honduras, Panamá, Perú and Venezuela (and, yes, even Switzerland) have trained with the Colombian team. When a murmur of protest has arisen in Colombia that we are training our rivals in competition, we are quick to point out to our students that they will have to prove themselves best, on their own merits, not because they were privileged, perhaps, to get better training. This has set us straight as to the nature of an Olympiad competition. Our students finally realize that they are competing against the problems (and perhaps with themselves).

Olimpiada de Mayo y Centroamericana

The Iberoamerican group has been the cradle for other subregional competitions, or competitions for students of different age groups. Colombia takes part in two of these. The Olimpiada de Mayo is a competition run by Argentina (though founded by the entire Iberoamerican group) for students in two age groups, up to age 13 and up to age 15, that challenges our young students on the regional level without leaving their home towns, and gives them a glimpse of problems conceived with the Argentine twist. The Olimpiada de Matemáticas de Centro-América y el Caribe is for students up to 16 years of age and who, for Colombia, Cuba, Mexico and Venezuela have not yet had experience at the OIM or IMO. Organized in a similar way to other international olympiads, it is an excellent place to start international competition.

Reengineering Competitions, Training and Leaders

Perhaps most importantly, the OCM have evolved, filled out, added events, changed their training schemes, tried untried deputies, nurtured new leaders, shared responsibilities. The OCM has not been dominated by a single team leader or deputy at the IMO. Many different people have had the experience, especially of being a deputy leader. Then the deputies have had the opportunity to be leader, if not at the IMO, then at the OIM or OCC. This renewal is also positive in every sense of the term.

The Colombian leader at the IMO has always been a person fluent in English and able to take an active role in problem selection and the other duties of the jury, as well as speak confidently with coordinators and be able to get his or her point across. This is a far more important ingredient than many may think. It is to take part on equal footing with colleagues from around the world, not to accept a secondary role. This attitude helps our students, who are naturally quite reticent about their place on the international scene, to have more confidence in themselves, although they seem never to have quite enough.

IMO—AB

We have been fortunate in recent years to have such a fine young mathematician as Federico Ardila as our leader, a person who has not lost his keen appreciation for the Olympiad and the way it allows young students to flourish and shine. He has been fortunate to have the opportunity to be part of the IMO-AB and to give voice to the perspectives of participants from the Iberoamerican and other developing countries there.

Olympiad and the Math Society

There is no doubt that the Colombian Math Olympiads have renewed the Colombian mathematics community, forming an entire new generation of mathematicians who have revitalized university math departments, schools and the Math Society itself.

Tha Colombian Math Society has (almost) always supported the Olympiad program and initiative in Colombia. That is especially true with the current president of the Society, Carlos Montenegro, who has stated:

There does not exist another activity that has produced more benefit to mathematics in Colombia than the Mathematical Olympiads.

It is a program carried out with limited resources and with the conviction that is required to overcome common obstacles that have stopped other initiatives; it already has more than 25 years and has expanded to include university Olympiads.

The impact of the Olympiads is the creation of an entire generation of mathematicians that form a young community that possesses a sense of identity and a network of contacts and affinity given that they belong to this exclusive group of young people with great talent and motivation.

The number of mathematicians who have graduated in Colombia in the past 15 years has increased considerably and the process of selection and motivation is due in great measure to the Olympiads.

The generation renewal in Colombia can count on a great resource and this is the generation of young mathematicians who have come out of the

Olympiads. These are presently in all the different stages of formation as researchers (competing in school, in the university, studying in doctoral programs, young professors and postdoctoral students). The challenge we have is to create a milieu in Colombia so they can fully develop their profession here, and return after completing their studies abroad.

We hope that in a few years they will be the ones who will be found in our national universities researching and leading a vibrant mathematical community. They will have gotten their start in the Olympiads.

His support has consolidated the academic status of the Olympiad; there is no doubt of the importance of Olympiads to the mathematics community in Colombia.

Olympiads and the International Mathematical Union

The same is true of IMU, and one of the reasons that Nicolai Dobilin, Petar Kenderov and myself were elected by the IMU general assembly as members-at-large of the ICMI Executive Committee. Competitions are seen by IMU as an important means of promoting meaningful and challenging mathematics in the schools, replenishing aspirants to follow a career in mathematics at university and renewing the world-wide community of mathematicians.

The core agenda of IMU (through its Commission on Development and Exchanges) is to foster and nurture pockets or centers of excellence in mathematics research in developing countries. While this is distinct from the development of quality programs in mathematics education, the two goals have natural links, and there is a possibility of synergy between the two efforts that would give added leverage to both. One area of intersection of capacity building in both mathematics and mathematics education is the International Mathematical Olympiad movement. An example of dramatic progress in this domain is the growth through dedicated regional organization, of the participation of Ibero-American countries from four, in 1985, to virtually all twenty-two of them at present.

[From the Report and Recommendations of the Ad Hoc Sub-committee of the Executive Committee of the International Mathematical Union on Supporting Mathematics in Developing Countries, September 2003]

There is in IMU a group called the Developing Countries Strategic Group which has further focused its attention on developing countries, with some altruism, but with a stated goal of attracting talented young people from developing countries into mathematics, given that talented students from the developed world may choose other more lucrative or technology based alternatives.

Mathematics is in a period of rapid change, as are the educational systems that provide much of the human capital essential to the health and advancement of the profession. In many countries of the world, especially those with developed or flourishing economic systems, young people are perhaps less drawn to mathematics or basic science than in generations past. In part this is because of increasingly market-driven educational systems, and in part because of an increasing variety of lucrative career alternatives.

In strife-torn and economically underdeveloped countries, the academy in general and mathematics in particular has another face. There, academic pursuits, in general, and the study of mathematics, in particular, are often a refuge of comity and order in an otherwise disordered world. And too, academic distinction is also among the few avenues to social advancement and relative academic prosperity. All societies are fertile sources of the raw material of human intelligence and ingenuity, in roughly comparable measures. But, given the inceased intensity of the appeal of mathematics to the intelligent young people in such environments, and the lesser costs of education there, one can at least plausibly argue that the marginal return on investment may actually be greater in the developing world than elsewhere. Such a plausibility argument leads to the conclusion that, if for no other reason than its own future health and growth, the mathematics profession has a vital stake in advancing the mathematical sciences in the developing world.

[From the same source.]

Ex-participants in Schools and Universities

The olympiad has helped to replenish mathematics education in many schools and universities. We often find that a math teacher who is especially good at motivating his or her students to participate successfully in the Olympiad is someone who fondly remembers his own experience in olympiad competition.

At the university level, one of the reasons for the incipient success of the Colombian Math Olympiad for University Students and the respectable results of the Colombian team in the IMC is rooted in the ex-Olympians who man the math departments.

Working Hard

There is no substitute for working hard; for taking stock, assessing results, redesigning, innovating, learning constantly from others. It has always been my idea that true "development" has its roots in being inclusive (not excluding any person, school or initiative) and exercising leadership. We believe this has been an important idea for the Colombian math olympiads.

Recapitulating

I believe that in any case that we were lucky. We were lucky in the schools that began taking part in the Olympiad, although there was no tradition along these lines, We were lucky in the personalities of our first successful students, who were always willing to share what they had learned and the expertise only they could share (exam strategies, how to handle the stress, how to relate to kids from other teams). We were lucky in the people who became interested in working with us: Sam Greitzer, Walter Mientka, Peter O'Halloran, Peter Taylor, Andy Liu, Titu Andreescu, in the support of our Iberoamerican colleagues: Patricia Fauring, Angelo Barone, María Gaspar, Ceferino Ruíz, Carlos Bosch, Uldarico Malespina, Rafael Sánchez. We were lucky in the first Minister of Education who set the tone for over ten years of Ministry support.

We have been lucky in finding echo in the Iberoamerican community, in creating lasting friendships among those dedicated to math Olympiads in the different countries, in having the continued and inspired collaboration of fine young mathematicians and problem solvers, in being able to rely on the continued support of the Colombian Math Society and above all that of the Universidad Antonio Nariño as the principal institutional organizer of Olympiads (math, physics, informatics) in Colombia.

4 Looking Forward

Two keys to the spirit of the times, twenty-five years later:

- Continue to question, assess and revitalize what we are doing, year after year.
- Continue to see the appearance of young people of great genius, enough of them with true modesty, truly thankful for the opportunities they've had, willing to give some of their time and thought and creativity to the new generations.

Some new groups of people discover the Olympiad, year after year, become enthusiastic, respect the students and the organizers, "defend" and "sell" the olympiad to others.

Challenges

I am deeply impressed by the work being done in other Latin American countries that certainly is not a threat, but will just as certainly prove to be an enormous challenge for Colombia and other countries to keep pace.

In 2005 Brazil began the Brazilian Olympiad for Public Schools that had, as we mentioned, just under 12 million participants. The idea is positively brilliant. Each school is responsible for the inscription of its students in the Olympiad and is encouraged to have all its pupils take part. It is a one hour 10 question multiple choice and short answer exam, with some questions that require more analysis than creativity and others that require less routine ways of thinking spatially or in other mathematical terms.

The exam is corrected by the teachers of each school and the names and papers of the first 5% of its students is sent to the organizers for a second round of questions requiring full solution. This meant that 600,000 students took part in the second round which was held in regional centers and corrected by professors from the math departments of the public universities all over Brazil.

There were medals and prizes for the best students, the prize being having earned the right to take part in a summer math school for talented students. There were prizes for the teachers of the best students; the prize was the right to attend an in-service course preparing them to teach more challenging mathematics to their students.

There were prizes for schools, according to the number of students from that school earning prizes; the prizes were laptop computers and math books for their school libraries.

There were prizes for the towns with the best schools, trophies and equipment.

The Brazilian Math Olympiad devised the problem sets.

This is an excellent model, and one which we will try to follow in the coming years, for if we do not evolve into challenging all of our students in all of our schools the positive results which we have been able to show, though modest on an international level, will no longer shine on the Iberoamerican scene.

5 Conclusion

This teaches us all that we'll simply have to learn to do better.

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Informatics olympiads: Approaching mathematics through code[1]

Benjamin A. Burton

Benjamin Burton is the Australian team leader for the International Olympiad in Informatics, and has directed the Australian training programme for the past eight years. He also has a keen interest in mathematics enrichment, having spent thirteen years teaching at the National Mathematics Summer School and five years helping train the IMO team. As a student he won a gold medal at the IMO in Moscow, 1992. His research interests include low-dimensional topology and information security.

Many readers are familiar with the International Mathematical Olympiad (IMO), a pinnacle in the yearly calendar of mathematics competitions. Here we introduce its cousin in computer science, the International Olympiad in Informatics (IOI).

The International Olympiad in Informatics is one of the newer Science Olympiads, beginning in 1989 in Bulgaria under the leadership of Petar Kenderov. In its short lifespan it has also become one of the largest, with 74 countries participating in 2006 [16]. Like the IMO, the competition is targeted at secondary school students who compete on an individual basis.

[1]This paper is based on a presentation by the author at the WFNMC Congress, 2006.

In this paper we take the reader on a brief tour of the IOI, paying particular attention to its relationships with traditional mathematics competitions. We begin in Section 1 with the structure of a typical IOI problem, including an outline of the mathematical ideas that lie behind such problems. For illustration, two problems are worked through in detail in Section 2. In Section 3 we compare the different ways in which mathematics and informatics contests encourage students to think mathematically. Section 4 closes with suggestions for students and teachers who wish to become involved in the IOI.

It should be noted that the IOI is not the only international competition of this type. Other examples include the ACM International Collegiate Programming Contest for university students [1], and the privately-run TopCoder contests [14]. For an overview and comparison of these and related competitions, see [7].

Thanks must go to Margot Phillipps, the IOI team leader for New Zealand, for her valuable comments on an earlier draft of this paper.

1 Structure of an IOI Problem

A typical informatics olympiad task requires competitors to create an algorithm that can solve some given problem. For instance, they might need an algorithm to calculate the nth Fibonacci number, or to compute the area of the intersection of two polygons. Any algorithm will not do, however; this algorithm must be both *correct* and *efficient*.

The concepts that lie beneath informatics problems are often mathematical in nature, and for harder problems students need a sound mathematical mindset to succeed. However, unlike a typical mathematics contest, students do not communicate their solutions by writing proofs—instead they communicate their solutions by writing computer programs.

In a similar vein, solutions are marked by computer. The contest judges prepare an *official data set*, consisting of several input scenarios that the students' programs must handle. Students receive marks according to whether their programs answer these scenarios correctly and within a given time limit (thereby testing correctness and efficiency).

The official data set essentially takes the place of a traditional marking scheme, and so the judges must take great care to construct it well. It typically consists a range of cases from easy to hard, including both small and large input scenarios to test for efficiency, and both "ordinary" and pathological input scenarios to test for correctness.

A typical problem statement has the following components:

- *The task overview.* This explains precisely what task the students' algorithms must solve, and often wraps it in a real-world (or fantasy) story, or *flavourtext.*
- *The input and output formats.* These explain the precise formats of the text files that students' programs must read and write. Each scenario from the official data set is presented to the program in the form of a given *input file*, which the program must read. Likewise, each program must write its corresponding solution to a given *output file*, which is given back to the judging system for marking.
- *Limits on time, memory and input data.* These allow algorithms to be tested for efficiency. Each program is given a small amount of time in which it must run (the *time limit*, typically no more than a few seconds) and a maximum amount of memory that it may use (the *memory limit*). Competitors are also given upper and lower bounds for the input data (which the scenarios in the official data set promise not to exceed), so they can estimate whether their programs are likely to run within the time and memory limits even for the most difficult scenarios.
- *Sample input and output files.* These sample files illustrate some simple input scenarios and their solutions. The problem statement should be perfectly understandable without them; they are merely given as examples so that students can be sure that they understand both the task and the input/output formats correctly.

Figure 1 shows a problem statement that, whilst simple, illustrates each of the components listed above. A sample solution using the Pascal programming language is given in Figure 2 (though this solution is not optimal, as seen in the following section).

Pascal's Triangle

Pascal's triangle is a triangular grid of numbers whose rows are numbered $0, 1, 2\ldots$ and whose columns are also numbered $0, 1, 2, \ldots$. Each number in the triangle is the sum of (i) the number immediately above it, and (ii) the number immediately above it and to the left. The numbers along the boundary all have the value 1. The top portion of the triangle is illustrated below.

$$\begin{array}{lcl} \text{Row } 0 & \rightarrow & 1 \\ \text{Row } 1 & \rightarrow & 1\ 1 \\ \text{Row } 2 & \rightarrow & 1\ 2\ 1 \\ \text{Row } 3 & \rightarrow & 1\ 3\ 3\ 1 \\ \text{Row } 4 & \rightarrow & 1\ 4\ 6\ 4\ 1 \\ & & \vdots \qquad\qquad \ddots \end{array}$$

Your task is to write a computer program that can calculate the number in row r and column c of Pascal's triangle for given integers r and c.

Input: Your program must read its input from the file `pascal.in`. This file will consist of only one line, which will contain the integers r and c separated by a single space.

Output: Your program must write its output to the file `pascal.out`. This file must consist of only one line, which must contain the number in row r, column c of Pascal's triangle.

Limits: Your program must run within 1 second, and it may use at most 16 Mb of memory. It is guaranteed that the input integers will be in the range $0 \le c \le r \le 60$.

Sample Input and Output: The sample input file below asks for the number in row 4, column 2 of the triangle. The corresponding output file shows that this number is 6 (as seen in the triangle above).

`pascal.in`:	`pascal.out`:
`4 2`	`6`

Figure 1: A simple problem that illustrates the various components

```
program Triangle;

var
   r, c : longint;

function pascal(i, j : longint) : longint;
begin
   { Compute the value in row i, column j of the triangle. }
   if (j = 0) or (i = j) then
      { We are on the boundary of the triangle. }
      pascal := 1
   else
      { We are in the interior; apply the recursive formula. }
      pascal := pascal(i - 1, j - 1) + pascal(i - 1, j);
end;

begin
   assign(input, 'pascal.in');
   reset(input);
   assign(output, 'pascal.out');
   rewrite(output);

   readln(r, c);
   writeln(pascal(r, c));

   close(input);
   close(output);
end.
```

Figure 2: A correct but inefficient solution to Pascal's Triangle

Correctness and Efficiency

As discussed earlier, algorithms for informatics olympiad problems must be both *correct* and *efficient*. This means that students must think about not only the theoretical concerns of obtaining the correct answer, but also the practical concerns of doing this without placing an excessive burden upon the computer.

For example, consider again the solution to Pascal's Triangle presented in Figure 2. Whilst it certainly gives correct answers, it is not efficient—in the worst case the recursive routine `pascal(i,j)` may call itself on the order of 2^r times as it works its way from the bottom of the triangle to the top. For the maximal case $r = 60$ this cannot possibly run within the 1 second that the program is allocated. A simple but inefficient solution such as this might score $\sim 30\%$ in a real competition (that is, approximately 30% of the scenarios in the official data set would be small enough for this program to run in under 1 second).

The solution can be improved by keeping a lookup table of intermediate values in different positions within the triangle. That is, each time `pascal(i,j)` finishes computing the value in some position within the triangle, it stores this value in a large table. Conversely, each time `pascal(i,j)` is called, the program looks in the table to see if the value in this position has already been calculated; if so, it avoids the recursive descent and simply pulls the answer from the table instead.

In this way we reduce the number of recursive calls to `pascal(i,j)` from the order of 2^r to the order of r^2 instead, since there are $\leq r^2$ distinct positions in the triangle that need to be examined en route from bottom to top. Using this lookup table we can therefore reduce an exponential running time to a (much better) quadratic running time. For the worst case $r = 60$ we are left with ≤ 3600 function calls which are easily done within 1 second on a modern computer. A solution of this type should score 100%.

Mathematical Foundations

In one sense the IOI is a computer programming competition, since students are required to write a computer program to solve each task. There are no handwritten or plain-language solutions; the computer programs are the only things that are marked.

In another sense however, the IOI is a mathematics competition. The tasks are algorithmic in nature—the difficulty is not merely in the programming, but in devising a mathematical algorithm to solve the task. The computer program then serves as a means of expressing this algorithm, much as a written proof is the means of expressing a solution in a traditional mathematics contest.

Looking through recent IOIs, the following mathematical themes all make an appearance in either the problems or their solutions:

- Case analysis and simplification
- Combinatorics
- Complexity theory
- Constructive proofs
- Cryptanalysis
- Difference sequences
- Induction and invariants
- Game theory
- Geometry
- Graph theory
- Optimisation
- Recurrence relations

This list is of course not exhaustive. Although the IOI does not have an official syllabus as such, moves are being made in this direction; see [17] for a recent proposal by Verhoeff et al. that to a large degree reflects current practice.

2 More Interesting Problems

Whilst the example problem given in Section 1 is illustrative, it is not overly difficult. It is certainly useful at the junior level for highlighting the benefits of lookup tables, but the mathematics behind it is relatively simple.

It is often the case that more interesting mathematical ideas do not surface in informatics olympiad problems until the senior level. This is partly because (at least in Australia) algorithm design is rarely taught at secondary schools, and so challenging algorithmic problems must wait until students have had a little training.

In this section we turn our attention to senior level problems with more interesting mathematics behind them. Two of the author's favourite problems are discussed in some detail in their own sections below. For other examples of real olympiad problems, see [16] for an archive of past IOI problems, or see [9] for a thorough discussion of one particular IOI problem including an analysis of several potential solutions.

Polygon Game

This problem is not from an IOI per se; instead it is from the 2002 Australian team selection exam. It is included here because of its relation to the Catalan numbers, a combinatorial sequence with which some mathematics olympiad students might be familiar.

The problem itself is outlined in Figures 3, 4. Essentially students are given a polygon to triangulate; each triangulation is given a "score", and students' programs must determine the largest score possible. Given the bound $n \leq 120$ and the time limit of 1 second (and the speeds of computers back in 2002), an efficient algorithm should run in the order of n^3 operations or faster.

Brute Force Solution

A simple "brute force" solution might be to run through all possible sequences of valid moves. This does guarantee the correct answer, but it certainly does not run within the order of n^3 operations—instead the running time is exponential in n (with some arithmetic it can be shown that 4^n is a reasonable approximation).

Indeed, even if we note that the order of moves does not matter, the worst case $n = 120$ still leaves us with $\sim 5 \times 10^{67}$ possible sequences to consider.[2] Assuming a modern machine and extremely fast code, this is unlikely to finish in 10^{50} years, let alone the 1 second that the program is allowed. Thus the brute force solution is correct but incredibly inefficient, and would probably score 5–10% in a real competition.

Greedy Solution

For a faster strategy we might use a "greedy" solution that simply chooses the best available move at each stage. That is, the program looks at all lines that can be legally drawn and chooses the line with the

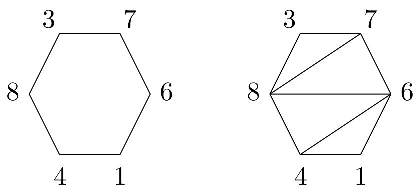
[2] The number of ways in which an n-gon can be triangulated is the Catalan number $C_{n-2} = \binom{2n-4}{n-2}/(n-1)$. Using the approximation $C_k \sim 4^k/k^{3/2}\sqrt{\pi}$, this gives $C_{118} \sim 4.86 \times 10^{67}$ for the case $n = 120$. See [15] for details.

Polygon Game

(B. Burton, G. Bailey)

You are given a regular polygon with n vertices, with a number written beside each vertex. A *move* involves drawing a line between two vertices; these lines must run through the interior of the polygon, and they must not cross or coincide with any other lines. The *score* for each move is the product of the numbers at the two corresponding vertices.

It is always true that after making precisely $n-3$ moves you will be left with a set of triangles (whereupon no more moves will be possible). What is the *highest possible score* that can be achieved through a sequence of valid moves?



As an example, consider the polygon illustrated in the first diagram above. Suppose we join the leftmost and rightmost vertices, the leftmost and top right vertices, and the rightmost and bottom left vertices, as illustrated in the second diagram. This gives a total score of $8 \times 6 + 8 \times 7 + 4 \times 6 = 128$, which in fact is the highest possible.

Input: The input file `polygon.in` will consist of two lines. The first line will contain the integer n, and the second line will list the numbers at the vertices in clockwise order around the polygon.

Output: The output file `polygon.out` must consist of one line giving the highest possible score.

Limits: Your program must run within 1 second, and it may use at most 16 Mb of memory. It is guaranteed that each input file will satisfy $3 \leq n \leq 120$.

Figure 3: The problem *Polygon Game* from the 2002 Australian team selection exam

Sample Input and Output: The following input and output files describe the example discussed earlier.

polygon.in:

```
6
3 7 6 1 4 8
```

polygon.out:

```
128
```

Figure 4: The *Polygon Game* input and output

greatest score (and then repeats this procedure until no more lines can be drawn).

This is certainly more efficient than brute force. For each move there are roughly n^2 possible lines to consider (more precisely $n(n-1)/2$), and this procedure is repeated for roughly n moves (more precisely $n-3$). Thus the entire algorithm runs in the order of n^3 operations, and is thereby fast enough for our input bounds and our time limit.

However, is this algorithm correct? Certainly it works in the example from Figures 3, 4. The best line that can be drawn inside an empty polygon has score 8×7, and the second best has score 8×6. The third best has score 7×4, but we cannot draw it because it would intersect with the earlier 8×6; instead we use the fourth best line, which has score 6×4. At this point no more lines can be drawn, and we are left with a total score of 128 as illustrated in the problem statement.

Nevertheless, a cautious student might worry about whether choosing a good move early on might force the program to make a *very* bad move later on. Such caution is well-founded; consider the polygon illustrated in Figure 5. The greedy solution would choose the 5×5 line (which is the best possible), and would then be forced to choose a 5×1 line for its second move, giving a total score of $25 + 5 = 30$ as illustrated in the leftmost diagram of Figure 5. However, by choosing a smaller 20×1 line for the first move, we open the way for another 20×1 line for the second move, giving a greater score of $20 + 20 = 40$ as shown in the rightmost diagram.

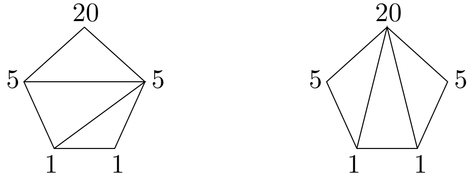


Figure 5: A case that breaks the greedy solution

This greedy solution was in fact submitted by a large number of students in the 2002 Australian team selection exam, where this problem originally appeared. It is a significant task when teaching informatics olympiad students to help them understand the difference between solutions that "feel good" (such as the greedy solution) and solutions that can be *proven correct* (such as the solution we are about to see). In an informatics olympiad where written proofs are not required, students need a healthy sense of self-discipline to spend time with pen and paper verifying that their algorithms are correct.

Correct Solution

It seems then that what we need is an algorithm that tries all possibilities—either directly or indirectly—but that somehow manages to identify common tasks and reuse their solutions in a way that reduces the running time from the exponential 4^n to the far more desirable n^3.

This is indeed possible, using a technique known as *dynamic programming*. Dynamic programming is the art of combining generalisation, recurrence relations and lookup tables in a way that significantly improves running time without any loss of thoroughness or rigour. We outline the procedure for Polygon Game below.

(i) *Generalisation:* Instead of solving a single problem (find the maximum score for the given polygon), we define an *entire family* of related "subproblems". Each subproblem is similar to the original, except that it involves only a portion of the original polygon. These subproblems become our common tasks that can be reused as suggested earlier.

Suppose the vertices of the given polygon are $v_1, v_2, \ldots, v_n$. For any $i < j$ we define the subproblem $P_{i,j}$ as follows:

> *Consider the polygon with vertices $v_i, v_{i+1}, \ldots, v_{j-1}, v_j$, as illustrated in Figure 6. Find the largest possible score that can be obtained by drawing lines inside this polygon (including the score for the boundary line $\overline{v_i v_j}$).*

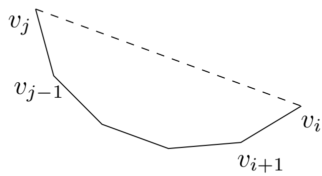


Figure 6: The polygon used in the subproblem $P_{i,j}$

In the cases where i and j are very close ($j = i + 1$ or $j = i + 2$), the subproblem $P_{i,j}$ becomes trivial since no lines can be drawn at all. On the other hand, where i and j are very far apart ($i = 1$ and $j = n$) we are looking at the entire polygon, and so $P_{1,n}$ is in fact the original problem that we are trying to solve.

Note that the boundary line $\overline{v_i v_j}$ causes some sticky issues; in most cases it receives a score, but in extreme cases (such as $j = i + 1$) it does not. We blissfully ignore these issues here, but a full solution must take them into account.

(ii) *Recurrence relations:* Now that we have our family of subproblems, we must find a way of linking these subproblems together. Our overall plan is to solve the smallest subproblems first, then use these solutions to solve slightly larger subproblems, and so on until we have solved the original problem $P_{1,n}$.

The way in which we do this is as follows. Consider the polygon for subproblem $P_{i,j}$. In the final solution, the line $\overline{v_i v_j}$ must belong to some triangle. Suppose this triangle is $\Delta v_i v_k v_j$, as illustrated in Figure 7. Then the score obtained from polygon $v_i \ldots v_j$ is the score

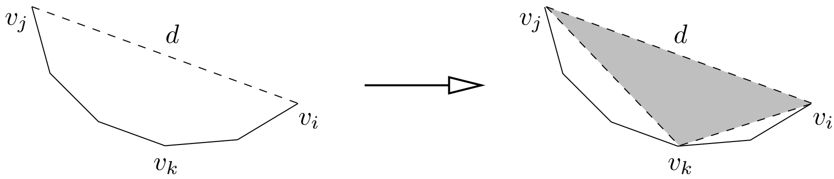


Figure 7: Splitting $P_{i,j}$ into smaller subproblems

for line $\overline{v_iv_j}$ plus the best possible scores for polygons $v_i \ldots v_k$ and $v_k \ldots v_j$. Running through all possibilities for vertex v_k, we obtain:

$$\begin{aligned}\text{Solution}(P_{i,j}) &= \text{Score}(\overline{v_iv_j}) \\ &\quad + \max_{i<k<j} \{\text{Solution}(P_{i,k}) + \text{Solution}(P_{k,j})\}.\end{aligned}$$

Students familiar with the Catalan numbers might recognise this recurrence relation, or at least the way in which it is obtained. The Catalan numbers arise from *counting* triangulations of polygons (rather than maximising their scores). The recurrence relation that links them is based upon the same construction of Figure 7, and so takes a similar form (the max becomes a sum, additions become multiplications, and the subproblem $P_{i,j}$ becomes the Catalan number C_{j-i-1}). The Catalan numbers are a wonderful sequence for exploring combinatorics and recurrence relations, and interested readers are referred to [15] for details.

(iii) *Lookup tables:* Now that we have a recurrence relation, our overall plan is straightforward. We begin by solving the simplest $P_{i,j}$ in which i and j are very close together ($j = i+1$ and $j = i+2$, where no lines can be drawn at all). From here we calculate solutions to $P_{i,j}$ with i and j gradually moving further apart (these are solved using our recurrence relation, which requires the solutions to our earlier subproblems). Eventually we expand all the way out to $P_{1,n}$ and we have our final answer.

At first glance it appears that this algorithm could be very slow, since the recurrence relation involves up to $(n-2)$ calculations inside the $\max\{\ldots\}$ term, each involving its own smaller subproblems. However, we avoid the slow running time by storing the answers to each subproblem $P_{i,j}$ in a lookup table, similar to what we did for Pascal's Triangle in Section 1.

There are roughly n^2 subproblems in all (more precisely $n(n-1)/2$). For each recurrence we simply look up the answers to the earlier subproblems in our table, which means that each new subproblem requires at most n additional steps to solve. The overall running time is therefore order of $n^2 \times n = n^3$, and at last we have our correct and efficient solution.

Utopia Divided

Our final problem is from IOI 2002, and indeed was one of the most difficult problems of that year's olympiad. Summarised in Figures 8, 9, this is a fascinating problem that could just as easily appear in a mathematics olympiad as an informatics olympiad. We do not present the solution here; instead the reader is referred to [10] for the details. We do however present an outline of the major steps and the skills required.

1. *Simplifying the problem:* The first crucial step is to simplify the problem to a single dimension. In this case the input becomes a sequence of n distinct positive integers (not $2n$ integers) and a sequence of n plus or minus signs (not n quadrants). Your task now is to rearrange and/or negate the n integers to create a one-dimensional path along the number line; this path must begin at 0 and travel back and forth between the + and − sides of the number line in the given order.

   For example, suppose the integers were $6, 8, 10, 11$ and the signs were $+, -, -, +$. A solution might be to use steps 8, −10, −6 and 11 in order; here the path begins at 0 and then runs to the points 8, −2, −8 and 3, which matches the signs $+, -, -, +$ as required.

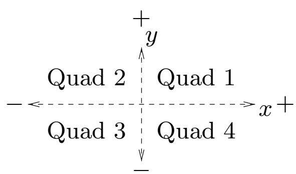
   Certainly the one-dimensional problem is simpler to think about than the original two-dimensional version. More importantly, the one-dimensional solution is easily generalised to two dimensions—arbitrarily choose n of your integers to be x coordinates and the remaining n integers to be y coordinates, solve the problems in the x and y dimensions independently, and then combine the x and y solutions to obtain your final n two-dimensional steps.

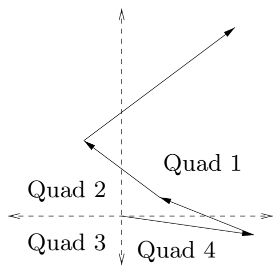
Utopia Divided

(S. Melnik, J. Park, C. Park, K. Song, I. Munro)

Consider the (x, y) coordinate plane, and number the four quadrants 1–4 as illustrated in the leftmost diagram below. You are given a sequence of $2n$ distinct positive integers, followed by a sequence of n quadrants. Your task is to create a path that begins at $(0, 0)$ and travels through these n quadrants in the given order.

You must create your path as follows. Your $2n$ positive integers must be grouped into n pairs; each of these pairs becomes a single (x, y) step along your path. You may negate some (or all) of these integers if you wish.





For example, suppose that $n = 4$ and you are given the sequence of integers `7 5 6 1 3 2 4 8` and the sequence of quadrants `4 1 2 1`. You can solve the problem by rearranging the integers into the following pairs:

$$(7, -1)\ (-5, 2)\ (-4, 3)\ (8, 6)$$

Following these steps from $(0, 0)$ takes you to the point $(7, -1)$, then $(2, 1)$, then $(-2, 4)$ and finally $(6, 10)$, as illustrated in the rightmost diagram above. This indeed takes you through quadrants 4, 1, 2 and 1 in order as required.

Figure 8: The problem *Utopia Divided* from IOI 2002

Input: The input file `utopia.in` will consist of three lines, containing the integer n, the sequence of $2n$ positive integers, and the sequence of n quadrants respectively.

Output: The output file `utopia.out` must consist of n lines, giving the n steps in order. Each line must list the x and y integer components of the corresponding step.

Limits: Your program must run within 2 seconds, and it may use at most 32 Mb of memory. It is guaranteed that each input file will satisfy $1 \leq n \leq 10\,000$.

Sample Input and Output: The following input and output files describe the example discussed earlier.

`utopia.in`:

```
4
7 5 6 1 3 2 4 8
4 1 2 1
```

`utopia.out`:

```
7 -1
-5 2
-4 3
8 6
```

Figure 9: The problem *Utopia Divided* input and output

2. *Broad plan:* After some thought one can formulate a broad strategy, which is to use larger numbers to change between + and − sides of the number line, and to use smaller numbers to stay on the same side of the number line. Indeed this is what we see in the example above—the smallest integer 6 is used when the path must run through the − side twice in a row, and the largest integers 10 and 11 are used when the path must change from + to − and from − to + respectively.

It then makes sense to split the sequence of integers into "large numbers" and "small numbers" (just how many large and small numbers you need depends upon how many times you must change sides). Whenever you need to change sides, you pull an integer from your large set; otherwise you pull an integer from your small set.

An important factor to consider in this plan is the *order* in which you use the large and small numbers. With some thought it can be seen that the large numbers should be used in order from smallest to largest, and the small numbers should be used in order from largest to smallest. This ensures that, as your position becomes more erratic over time, the large numbers are still large enough to change sides, and the small numbers are still small enough to avoid changing sides.

3. *Details and proofs:* Some important details still remain to be filled in; this we leave to the reader. In particular, when using a small number, the decision of whether to add or subtract requires some care.

   Moreover, once the algorithm is complete, it is not immediately clear that it works for all possible inputs—some proofs are required. One approach is to place upper and lower bounds upon the kth point along the path, and prove these bounds using induction on k. The details can be found in [10].

As mentioned earlier, this was a difficult IOI problem—it received the lowest average score of all six problems in IOI 2002 (and in the author's opinion, IOI 2002 was one of the most difficult IOIs of recent years). Nevertheless, like the solutions to so many mathematics problems, the algorithm is simple and sensible once seen. It is the process of identifying the algorithm and proving it correct that creates so much enjoyable frustration during the competition.

3 Benefits and Challenges

As discussed in Section 1, informatics olympiad problems are often mathematical in nature, with regard to both the content and the skills that they require. In the author's view, an informatics olympiad can indeed be viewed as a type of mathematics competition, in which solutions are communicated through code (computer programs) instead of the more traditional written answers.

With this in mind, it is useful to examine both the benefits that informatics olympiads offer over traditional mathematics competitions, and the difficulties that they present. We spread our discussion across three broad areas: enjoyment and accessibility, skills and learning, and judging.

The points made here merely reflect the author's own opinion and experiences; there are of course many variants of both mathematics and informatics competitions with their own different strengths and weaknesses. Readers are heartily encouraged to participate themselves and come to their own conclusions!

Enjoyment and Accessibility

Here we examine the ways in which students are encouraged to participate in different competitions, and the barriers that prevent them from doing so.

On the positive side, many students enjoy working with computers and find computer programming to be fun. In this way, informatics competitions can expose them to mathematical ideas using what is already a hobby—although the students are not explicitly setting out to study mathematics (which some of them might not even enjoy at school), they nevertheless develop what are essentially mathematical skills and ideas.

Likewise, a number of students are afraid of writing mathematical proofs. In an informatics olympiad they only need to submit computer programs, which for some students are rather less onerous to write—computer programs are concrete things that students can build, run, test and tinker with. Nevertheless, as problems become harder students must develop the underlying skills of proof and counterexample, in order to distinguish between algorithms that *feel* correct and algorithms that *are* correct.

On the negative side, there are some very clear barriers to involvement in informatics olympiads. The first (and highest) barrier is that students cannot participate unless they can write a working computer program from scratch within a few hours. Unfortunately this alone rules out many bright and otherwise eager students. Another barrier is the fact that informatics olympiads are difficult for schools to run—they require computers, compilers and sometimes network access. In general informatics olympiads cause far more trouble for teachers than a mathematics competition where all a student needs is a desk and an exam paper. Recent moves have been made by countries such as Lithuania [8] and Australia [5, 6] to work around these problems by offering multiple-choice competitions that test algorithmic skills in a pen-and-paper setting.[3]

Skills and Learning

Although competitions are intended to be fun, they are also intended to be educational. Here we examine the skills that students discover and develop from participating in such competitions.

A strong advantage of informatics competitions is that they expose students to fields that in many countries are rarely taught in secondary schools. Certainly in Australia, schools tend to focus on using computers as a tool—courses in computer programming are less common, and even then they tend to focus on translating ready-made algorithms from English into computer code. The *design* of algorithms is almost never discussed.

A positive side-effect of automated judging is that informatics olympiad problems are often well suited for self-learning. Several countries have online training sites [4, 11], where students can attempt problems, submit their solutions for instant evaluation and refine them accordingly. Even without online resources, students can critique their solutions by designing test cases to feed into their own programs.

Informatics olympiads also develop a sense of rigour, even in the easiest problems. This comes through the hard fact that students cannot score any points without a running program—even with slow or naïve algorithms, a certain attention to detail is required. Combined with the fact that official data sets typically include pathological cases, students learn early on that rigour is important.[4]

[3]Both contests are now offered outside their countries of origin; see [2] and [3] for details.

[4]In some events such as the ACM contest [1], students score no points at all unless their programs pass *every* official test case. Whilst this might seem harsh, it puts an extreme focus on rigour that has left a lasting impression on this author from his university days, and that has undeniably benefited his subsequent research into mathematical algorithm design.

A clear deficiency in informatics olympiads is that they do not develop communication skills. Whereas mathematics students must learn to write clear and precise proofs, informatics olympiad students write computer programs that might never be read by another human. Bad coding habits are easy to develop in such an environment.

Another problem is that, since solutions are judged by their behaviour alone, one cannot distinguish between a student who guesses at a correct algorithm and a student who is able to *prove* it correct. For harder problems guesswork becomes less profitable, but for simpler problems it can be difficult to convince students of the merits of proof when their intuition serves them well.

Judging

Competitions are, when it comes to the bottom line, competitive. In our final point we examine the ways in which students are graded and ranked in different competitions.

On the plus side, weaker students in informatics competitions can still obtain respectable partial marks even if they cannot find a perfect algorithm. For instance, students could still code up the brute force or greedy solutions to Polygon Game (Section 2) for a portion of the available marks.

On the other hand, a significant drawback is that students cannot score *any* marks without a running program. A student who has found the perfect algorithm but who is having trouble writing the code will score nothing for her ideas.

In a similar vein, the marking is extremely sensitive to bugs in students' programs. A perfect algorithm whose implementation has a small bug might only score 10% of the marks because this bug happens to affect 90% of the official test cases. It is extremely difficult to design official data sets that minimise this sensitivity but still subject good students to the expected level of scrutiny.

As a result of these sensitivities, exam technique is arguably more important in an informatics olympiad than it should be. In particular, students who have found a perfect algorithm might be tempted to code up a less efficient algorithm simply because the coding will be quicker or less error-prone. There has been recent activity within the IOI community to work around these problems (see [7] and [12] for examples), but there is still a long way to go.

4 Getting Involved

For students eager to become involved in the informatics olympiad programme, there are a number of avenues to explore.

- *Books:* Steven Skiena and Miguel Revilla have written an excellent book [13] specifically geared towards programming competitions such as the IOI. The book is very readable and full of problems, and includes outlines of the most prominent competitions in the appendix.
- *Online resources:* As discussed in Section 3, several countries have produced online training sites through which students can teach themselves. The USACO site [11] is an excellent resource with problems, reading notes and contest tips. The Australian site [4] is also open to students worldwide.
- *National contacts:* Students are encouraged to contact their national organisations for local contests and training resources. The IOI secretariat [16] has links to national organisations, as well as archives of past IOI problems and resources for other olympiads such as the IMO.

Informatics olympiads are often seen as belonging purely to the domain of computer science. However, when seen from a mathematical point of view, they offer new challenges and activities that complement traditional mathematics competitions. Certainly the author, trained and employed as a mathematician, has gained a great deal of pleasure from his involvement in informatics competitions; it is hoped that other teachers and students might likewise discover this pleasure for themselves.

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Mathematical Olympiads in Albania

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Artur Baxhaku graduated in 1983 at the University of Tirana, and since then he is a lecturer of that University, Mathematics Department. He is the author of some research papers (main field: semigroups) and textbooks for his students and mathematical Olympiad participants.

He received his Ph. D. at 1994 and from 2004 he is a professor of Mathematics. Since 2003 professor Baxhaku is the secretary of the Albanian Mathematics Associacion. He has participated at the leading and the training of the Albanian team in international olympiads, like Balkan Mathematical Olympiad, Junior BMO, and since 1999 is deputy leader of the Albanian IMO teams.

The Mathematical Olympiads are considered in Albania as very important extra-curricular activities, aiming to encourage the work and identify gifted students in mathematics. They also serve to exchange and enrich the experience between school teachers and students, between different regions within the country, contributing to the increase of the teaching level and math knowledge in general. Students of general, professional, public and private high schools participate in these Olympiads.

1 A Brief History

The first attempt to organize a National Olympiad of Mathematics for high school students in the Republic of Albania was made during academic year 1974–1975, under the care of the Ministry of Education in collaboration with the Department of Specialty Mathematics in the Faculty of Natural Sciences of Tirana University. This is recognized as the first national Olympiad of mathematics, and it was organized in a single phase with the participation of high schools students of 11th grade at country level.

Although, this was considered as a successful attempt, it took 5 years of a break and a new organization to run the second National Olympiad of Mathematics in Tirana during academic year 1980–1981, involving again high school students of 11th grade. From that year on, the National Olympiad of Mathematics is organized every year, coming to the 28th Olympiad in 2007.

High school students of second and fourth year (grade 10 and 12) were included in the competition of the Olympiad of Mathematics during 1981–1982, which was the 3rd National Olympiad, whereas first year (grade 9) students were included for the first time in 19th Olympiad, run in academic year 1997–1998.

2 Organization

The National Olympiad of Mathematics is organized by the Ministry of Education and Science, which for this reason engages the District Education Directories together with Department of Mathematics of the

Faculty of Nature Sciences of the University of Tirana. The Albanian Association of Mathematics collaborates in this activity as well.

The Olympiad is developed in three phases, for each of the four categories (grades 9, 10, 11 and 12) of high schools students, with special tests for each category.

Phase One, is organized based on one or several schools of a district (region), around December. In this phase, students who score 8 and above (in a system of 10 scores) in mathematics are invited to participate voluntarily or based on math teachers' suggestion. This phase is organized by the District Education Directory, in cooperation with the respective schools. Phase Two, is organized on regional basis during February. In this phase there participate students who have accumulated at least 70% of the points in the first phase. It is organized by the District Education Directory which at the end awards prizes to the winners. The problems given for solution to students are more difficult compared to the first phase.

Phase Three, the final phase, is organized on country basis, either in Tirana or any other city appointed by the Ministry of Education and Science, usually in March. In this phase, there participate students who have accumulated over 50% of the points in the second phase, and were already awarded prizes. Usually in the third phase there participate 50–60 students for each category, which means 200–250 students in total. For the best students of each category, the Ministry of Education awards one First Prize, one Second and one Third prize. The awarding is based on the following criteria:

- the first prize goes to the student who has the highest score, but not less than 75% of the total score;
- the second prize goes to the student who scores not less than 65% of the total score and comes right after the first;
- the third prize goes to the student who scores not less than 55% of the total score, and comes after the second one.

So, it might happen that not all three prizes are awarded.

The same criteria of prize awarding is used during the second phase with regional winners. In both these cases, it is the District Education

Directory which awards the prizes. The Ministry of Education and Science, District Education Directory or School Directories, if their budget allows, may also reward the winners materially.

3 The Test

The tests for the first phase are compiled by commissions of specialists, established by the District Education Directories. The evaluating commission consists of math teachers of the respective schools in the region, as well as the responsible person for the organization of the Olympiad of mathematics from the school directorate. The test includes 4–5 problems that ask for full answers. The test for the second phase is compiled by the Commission established by the Ministry of Education and Science, with math teachers from the University and high schools. This test is unique for the whole country. The grading is carried out by a special commission established near the respective District Education Directory. This test includes 4–5 problems that also ask for full answers. Each exercise is evaluated with 10 points.

The test for the third phase is compiled by a Commission established by the Ministry of Education and Science, with University Professors and eventually any high school math teacher. The test of the third phase includes 5 problems from the subjects of algebra, geometry, number theory and combinatorics, which require full answers. The full solution of each problem is evaluated by 8 points. The evaluation commission engaged in the third phase is set up by the Ministry of Education. This commission announces the winners of the three prizes. It has the right to propose the award of encouraging prizes for special cases.

4 Selection of students who participate in International Olympiads (regional and world)

After the completion of the National Olympiad (usually mid-March), with the best 5–6 students from each of the last three grades (grades 10, 11, and 12) together with 1–3 best students of grades 9-s, thus a total of around 20 students (this figure might change according to the quality of competitors), the Albanian Association of Mathematics runs a selection procedure named "Spring Contest *.1". In 2007 it was organized the Spring Competition 6.1), and the top students make up a team that participates in the Balkan Olympiad of Mathematics (BMO). This Olympiad, since 1984, is organized on rotation basis, in one of the participating countries (Bosnia-Herzegovina, Bulgaria, Greece, Monte Negro, Macedonia, Moldova, Cyprus, Rumania, Serbia, Turkey and Albania), at the beginning of May (Albania started to participate in this event in 1989. In this competition, different from the previous phases, there is only one common test for all students' categories, because at the Balkan Olympiad where they are preparing to participate, they will have only one test. The test, and the evaluation is carried out by a commission of university lectors, set up by the association, which, based on results, announces the team (made of at most 6 students) as ready to participate at the Balkan Olympiad.

After the Balkan Olympiad, by mid-May, there is organized another selection (Spring Contest $*.2$), with the 6 competitors of the previous Balkan Olympiad as well as some other students who had good results in the Spring Competition $*.1$. The organizers are usually the same as in the first competition. During testing there are used problems from other countries' Olympiads, as well as from the Balkan and World Olympiads. The participatory team of the International Mathematics Olympiad (IMO), is selected from this competition. This selection procedure for International Olympiads started to be applied since 2002 and has already proved to be successful. It leaves students longer time to prepare and make them prepare more seriously for the competitions.

From the students of grade 9-s, a team of 5–6 students participates in another Balkan Mathematics Olympiad for Juniors up to 15 and a half years old (*Junior BMO*). This Olympiad started to be organized with the participating countries of the Balkan Olympiad for "seniors", since 1997.

Albanian Mathematical Society – Spring Contest 6.2

Problems for the Selection of the representative team at the IMO 2007, 24.05.2007.

1. At the beginning of the gymnastic class all the n students have to be ordered in only one order, the by-height one. The teacher admits only one break of that rule, that is he allows any ordering where

only one student has the right neighbor shorter than himself. What is the number of these orderings?

2. Find the greatest positive integer N such that the number of the multiples of 3 in the set $\{1, 2, 3 \ldots, N\}$ is equal to the number of the multiples of 5 and/or 7 in that set.

3. Find all the functions $f\colon \mathbb{R} \to \mathbb{R}$ that

$$\forall x, y \in \mathbb{R} \qquad f(y - f(x)) = f(f(x)) + yf(x) + f(y).$$

4. Find the greatest number n such that there exist $n+4$ points A, B, C, D, X_1, $X_2, \ldots, X_n$ in a plane where $AB \neq CD$ and such that for any $i = \overline{1, n}$ the triangles ABX_i and CDX_i are equal.

5 Training

The training of students is initially done by the school teachers and some selected teachers in the biggest districts, who are recognized by the Ministry of Education, and are entitled reduced teaching hours (about half of work load). Private schools are better organized and pay special teachers or university lectors to train their students. Once a year the Mathematics Association organizes several-days-training with the teams who participate in International Olympiads. In most cases training is delivered on voluntarily basis, without any payment for university lectors or teachers who accompany the students in these activities.

Mathematics Olympiads are also organized with 8-years school students (now 9-years), with the initiative and financial support of the private schools of Albania, and in cooperation with the District Education Directory.

6 Participation in International Olympiads

The first International Olympiad where the Albanian team participated was the 6th Balkan Mathematics Olympiad in 1989. Since then, the Albanian team has regularly participated in the Balkan Mathematics Olympiads, often winning medals. Some rare exception from the "participation rule" has been caused by financial difficulties as a result of economic or social situation of the country during the last years. In 2003, Albania was the hosting country of the 20th Balkan Mathematics Olympiad.

The beginning of the '90-s, with the change of the political systems and the country's breaking through isolation, marks a deadline even for the mathematics Olympiads; on one hand there is an increasing participation in international activities, but on the other hand, the financial support from the central authorities is not increased at the same rate. A problem that participants in the Olympiads and their leaders face, is that this vacuum created in these new conditions in supporting them is not filled yet by other sources. Attempts to find other resources have remained a continuous challenge. The experience of international colleagues is helping a lot.

The lack of support is felt by the students especially due to the lack of attention by the state institutions during their training stage. This has mostly affected public schools as against private schools that finance these trainings themselves. This fact has led to the situation where students coming from private schools make the majority of the participants in our teams. This result does not reflect the real situation and it does not reflect the level of commitment or willingness to study math. Neither does it reflect the level of talent. In this way it happens that some very talented students are not selected, and this objectively affects the level of presentation of our teams in these events.

The participation in these Olympiads is considered to be very important, especially dictated by the presence of the two regional "superpowers" in Math Olympiads: Bulgaria and Rumania. This is considered as one of the main encouraging positive factors, and as a source of experience for participants in the Olympiads, for both competitors and their leaders.

Albania has participated in the Junior Balkan Mathematical Olympiad since 1989. This participation has been conditioned by the same factors as the Balkan Olympiads. In some cases attempts have been rewarded with medals. Some of the winners or participants at the Junior BMO-s continue the successful participation at the "senior" ones, but in the meantime some others transfer their interest to other Olympiads, such as Physics, Informatics, or elsewhere. It is programmed that in June

2008 Albania will be the organizing country of the 12-th Junior Balkan Mathematical Olympiad. About 11 teams are expected to participate in this event.

The participation of the Albanian team at International Mathematics Olympiad started in 1993, at the 34th Olympiad in Istanbul. Since then, we have participated in almost all Olympiads, except for the 35th, 36th and 39th, for above-mentioned reasons. During last years, we have been awarded medals, bronze and silver, taking in the "non-official order" a place somewhere near the middle.

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48th International Mathematical Olympiad 19–31 July 2007 Hanoi, Vietnam

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The 48th International Mathematical Olympiad (IMO) was held during the period July 19–31 in Hanoi, Vietnam. A total of 520 contestants from a record 93 countries participated in this year's IMO.

It is quite an organizational task running a successful IMO. This year almost one thousand people were directly involved in the event. A number of Vietnamese living abroad returned to take part in the marking and scoring process. A national selection and interviewing process was also involved in selecting some one hundred guides who would look after the visiting international teams.

The IMO competition consists of two exam papers held on consecutive days. This year the dates of the competition were Wednesday 25th and

Thursday 26th of July. To qualify for the IMO, contestants must not have formally enrolled at a university and be less than 20 years of age at the time of writing the second exam paper. Each country has its own internal selection procedures and may send a team of up to six contestants along with a team Leader and Deputy Leader. The Leaders and Deputy Leaders are not contestants but fulfill other roles at the IMO.

Most Leaders arrived in Vietnam on July 19th. They stayed at a hotel overlooking Ha Long Bay for most of the IMO. Their first main task is to set the IMO paper. Already for a number of months prior, countries had been submitting proposed questions for the exam papers. The local Problem Selection Committee had considered these proposals and composed a shortlist of 30 problems considered highly suitable for the IMO exams. Over the next four days the "Jury" of team Leaders discussed the merits of the problems and through a voting procedure eventually chose the six problems for the exams. This year five of those 30 problems had to be deleted from consideration because they were known in the public domain. The final problems in order to be selected for the exam are listed as follows.

1. An inequality proposed by New Zealand. It had two parts. The first part was to prove an inequality, but the technique required was not from the usual classical methods but had a very nice combinatorial feel about it. The second part was to prove that equality could be achieved, and this proved to be a tricky little exercise in algebra.
2. A challenging Euclidean geometry question proposed by Luxembourg.
3. An extremely difficult graph theory problem proposed by Russia. The question was about considering maximal cliques of subgraphs. There were many plausible ways of approaching this question but which end up in dead ends.
4. A rather easy Euclidean geometry question proposed by the Czech Republic.
5. A nice number theory problem proposed by the United Kingdom. It was essentially a Diophantine equation that could be tackled by infinite descent in a number of different ways.
6. A very difficult algebra problem proposed by the Netherlands.

> While on the surface it appeared to be a combinatorics problem, it was really a problem in Algebraic Geometry.

Each exam paper consists of three questions. The contestants have $4\frac{1}{2}$ hours on each exam in which to provide carefully written proofs to the three challenging mathematical problems. Each contestant writes the exam in his or her own language so the Jury must also spend time to ensure uniform translations of the exams into the required languages. After this, marking schemes are discussed with representatives of a team of local markers called “Coordinators”. The Coordinators ensure fairness and consistency in applying the marking schemes to contestants’ scripts.

Most contestants and Deputy Leaders arrived in Hanoi, Vietnam on July 23rd. The Deputy Leaders look after their contestants until the exams are over.

On July 24th the Opening Ceremony was held at Vietnam’s National Conference Centre. I must say that we were made to feel rather important because the buses transporting us there had a police escort that took us straight down the middle of the road between the two lanes of traffic and also through red traffic lights. Furthermore the Prime Minister of Vietnam, H.E. Mr Nguyen Tan Dung, attended the Opening Ceremony and addressed the international audience. There were other addresses by government officials and IMO officials welcoming everyone and looking forward to a “great feast of problem solving and intellectual endeavour”. We were treated to some music and dance interludes between speeches. In one memorable dance performance, each successive set of dancers were younger and younger until they were small children. Near the end of the Opening Ceremony there was the parade of participating national teams. Some teams were of particular note. The Austrian team treated us to a brief display of waltzing. The Bolivian team, consisting of two members, were both girls. The North Korean team made a comeback by being present at the IMO after a 16 year absence. Thus concluded the Opening Ceremony. The contestants would write the first exam the very next day.

After the exams the Leaders and their Deputies assess the work of the students from their own countries. They are guided by the marking schemes discussed earlier. The Coordinators also assess the exams.

They are also guided by the marking schemes but may allow some flexibility if, for example, a Leader brings something to their attention in a contestant's exam script which is not covered by the marking scheme. There are always limitations in this process, but nonetheless the overall consistency and fairness by the Coordinators was very good.

In the final outcome, question 4 was the easiest question on this IMO. The average mark was 5.7 and there were 363 complete solutions. At the other end of the scale, question 3 unexpectedly turned out to be the most difficult question ever in the history of the IMO if we take the number of complete solutions as the measure of difficulty. The average mark was just 0.3 and there were only 2 complete solutions; one by Mladen Radojevic from Serbia and one by Caili Shen from China. Question 6 also turned out to be one of the most difficult IMO questions ever with an average mark of 0.2 and just 5 complete solutions.

At the closing ceremony there were 253 (=48.7%) medals awarded. The distributions being 131 (=25.2%) bronze, 83 (=16.0%) silver and 39 (=7.5%) gold, the gold medals being awarded by the President of Vietnam, H.E. Mr Nguyen Minh Triet. Of those who did not get a medal, a further 149 contestants received an honourable mention for solving at least one question perfectly. Because of the difficultly of the paper, most gold medalists only solved four questions. There were no perfect scores and only 5 contestants managed to solve five questions. The top score was 37 by Konstantin Matveev from Russia.

In his concluding words, József Pelikan, President of the IMO Advisory Board thanked the Vietnamese Organizing Committee of the 2007 IMO for all their "hard work and warm hospitality." They were supported by the Ministry of Education and Training of Vietnam, the Institute of Mathematics, the Vietnamese Mathematical Society and Hanoi University of Science.

The 2008 IMO is scheduled to be held in Madrid, Spain.

1 IMO Paper

First Day

1. Real numbers $a_1, a_2, \ldots, a_n$ are given. For each i $(1 \leq i \leq n)$ define

$$d_i = \max\{a_j : 1 \leq j \leq i\} - \min\{a_j : i \leq j \leq n\}$$

and let

$$d = \max\{d_i : 1 \leq i \leq n\}.$$

(a) Prove that, for any real numbers $x_1 \leq x_2 \leq \cdots \leq x_n$,

$$\max\{|x_i - a_i| : 1 \leq i \leq n\} \geq \frac{d}{2}. \qquad (*)$$

(b) Show that there are real numbers $x_1 \leq x_2 \leq \cdots \leq x_n$ such that equality holds in (*).

2. Consider five points A, B, C, D and E such that $ABCD$ is a parallelogram and $BCED$ is a cyclic quadrilateral. Let ℓ be a line passing through A. Suppose that ℓ intersects the interior of the segment DC at F and intersects line BC at G. Suppose also that $EF = EG = EC$.

Prove that ℓ is the bisector of angle DAB.

3. In a mathematical competition some competitors are friends. Friendship is always mutual. Call a group of competitors a *clique* if each two of them are friends. (In particular, any group of fewer than two competitors is a clique.) The number of members of a clique is called its *size*.

Given that, in this competition, the largest size of a clique is even, prove that the competitors can be arranged in two rooms such that the largest size of a clique contained in one room is the same as the largest size of a clique contained in the other room.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points

Second Day

4. In triangle ABC the bisector of angle BCA intersects the circumcircle again at R, the perpendicular bisector of BC at P, and the perpendicular bisector of AC at Q. The midpoint of BC is K and the midpoint of AC is L.

Prove that the triangles RPK and RQL have the same area.

5. Let a and b be positive integers. Show that if $4ab - 1$ divides $(4a^2 - 1)^2$, then $a = b$.

6. Let n be a positive integer. Consider

$$S = \{(x, y, z) : x, y, z \in \{0, 1, \ldots, n\}, x + y + z > 0\}$$

as a set of $(n + 1)^3 - 1$ points in three-dimensional space.

Determine the smallest number of planes, the union of which contains S but does not include $(0, 0, 0)$.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points

2 Results

Some Country Scores		
Rank	Country	Score
1	Russia	184
2	China	181
3	Vietnam	168
3	South Korea	168
5	USA	155
6	Ukraine	154
6	Japan	154
8	North Korea	151
9	Bulgaria	149
9	Taiwan	149
11	Romania	146
12	Hong Kong	143
12	Iran	143
14	Thailand	133
15	Germany	132

Some Country Scores		
Rank	Country	Score
16	Hungary	129
17	Turkey	124
18	Poland	122
19	Belarus	119
20	Moldova	118
21	Italy	116
22	Australia	110
23	Serbia	107
24	Brazil	106
25	India	103
26	Georgia	102
27	Canada	98
28	Kazakhstan	95
28	UK	95
30	Colombia	93

The medal cuts were set at 29 for gold, 21 for silver and 14 for bronze.

Distribution of Awards at the 2007 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Albania	59	0	0	1	5
Argentina	75	0	1	1	3
Armenia	73	0	1	1	4
Australia	110	0	1	4	1
Austria	80	0	1	3	1
Azerbaijan	69	0	0	3	1
Bangladesh (5 members)	31	0	0	0	3
Belarus	119	1	1	4	0
Belgium	78	0	0	3	2
Bolivia (2 members)	2	0	0	0	0
Bosnia & Herzegovina	69	0	1	0	5
Brazil	106	0	2	3	1
Bulgaria	149	2	3	1	0

Distribution of Awards at the 2007 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Cambodia (4 members)	26	0	0	0	3
Canada	98	0	1	3	1
Chile (4 members)	4	0	0	0	0
China	181	4	2	0	0
Colombia	93	0	1	3	1
Costa Rica (5 members)	36	0	0	1	1
Croatia	76	0	0	2	4
Cuba (1 member)	16	0	0	1	0
Cyprus	41	0	0	0	2
Czech Republic	82	0	0	5	1
Denmark	50	0	0	1	3
Ecuador	34	0	0	1	2
El Salvador (4 members)	34	0	0	0	3
Estonia	64	0	0	1	4
Finland	55	0	1	0	2
France	79	1	0	2	0
Georgia	102	1	1	1	3
Germany	132	1	3	1	1
Greece	89	0	1	3	2
Hong Kong	143	0	5	1	0
Hungary	129	0	5	0	1
Iceland	35	0	0	0	1
India	103	0	3	0	3
Indonesia	69	0	1	0	4
Iran	143	1	3	2	0
Ireland	51	0	0	1	3
Israel	71	0	0	3	3
Italy	116	1	1	3	1
Japan	154	2	4	0	0
Kazakhstan	95	0	1	3	2
Kyrgyzstan (5 members)	43	0	0	1	3
Latvia	58	0	0	0	4
Liechtenstein (2 members)	14	0	0	1	0
Lithuania	92	1	0	1	2
Luxembourg (3 members)	34	0	0	1	2

Distribution of Awards at the 2007 IMO

Country	Total	Gold	Silver	Bronze	H.M.
Macau	73	0	1	1	4
Macedonia	68	0	0	3	1
Malaysia	34	0	0	1	2
Mexico	86	0	0	4	2
Moldova	118	0	3	2	0
Mongolia	88	0	2	1	3
Montenegro (3 members)	17	0	0	0	1
Morocco	28	0	0	0	2
Netherlands	65	0	0	1	3
New Zealand	71	0	0	3	2
Nigeria	20	0	0	0	1
North Korea	151	1	4	0	1
Norway	79	0	1	1	1
Pakistan	32	0	0	1	1
Paraguay (4 members)	32	0	0	0	3
Peru	91	0	1	2	3
Philippines	21	0	0	0	1
Poland	122	1	2	2	0
Portugal	52	0	0	1	1
Puerto Rico (3 members)	7	0	0	0	0
Romania	146	1	4	1	0
Russia	184	5	1	0	0
Saudi Arabia (4 members)	5	0	0	0	0
Serbia	107	1	0	4	0
Singapore	87	0	0	5	0
Slovakia	86	0	0	4	2
Slovenia	85	0	0	5	1
South Africa	42	0	0	0	4
South Korea	168	2	4	0	0
Spain	48	0	0	2	1
Sri Lanka	25	0	0	0	1
Sweden	81	0	0	4	2
Switzerland	59	0	0	1	3
Taiwan	149	2	3	1	0
Tajikistan	37	0	0	1	2

Distribution of Awards at the 2007 IMO					
Country	Total	Gold	Silver	Bronze	H.M.
Thailand	133	1	3	2	0
Trinidad & Tobago	39	0	0	0	4
Turkey	124	1	2	2	1
Turkmenistan	51	0	0	0	5
Ukraine	154	3	1	2	0
United Kingdom	95	1	0	3	2
United States of America	155	2	3	1	0
Uzbekistan	88	0	1	3	2
Venezuela (3 members)	14	0	0	0	1
Vietnam	168	3	3	0	0
Total (520 contestants)		**39**	**83**	**131**	**149**

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1st Middle European Mathematical Olympiad 20–26 September 2007 Eisenstadt, Austria

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In response to organizing committee of Austrian Mathematical Olympiad (ÖMO) delegations of Middle European countries (Switzerland, Austria, Germany, Slovenia, Croatia, Czech Republic, Slovakia, Poland and Hungary) were informed about the Austrian suggestion to create a new competition for mathematically gifted secondary school students of these countries at the 47th International Mathematical Olympiad (IMO). The initiators of the new competition want to enable more students of Middle European countries to compare their knowledge of mathematics in the international arena. The aims and rules of the new international mathematical competition were set in advance by this meeting. The initiators used the rules of the bilateral international mathematical competition of secondary students "Poland – Austria" which had existed for 29 years till 2006. There are many other regional mathematical competitions (Baltic MO, Baltic Way, Mediterranean MO, Iberoamerican MO etc.) in the world and especially in Europe, where competitors take part every year.

The first Middle European Mathematical Olympiad was held in Austrian Eisenstadt—the capital city of the federal state Burgenland in September 2007 (20th–26th). Seven (out of nine) Middle European countries (Germany and Hungary want to join the competition in its second year) took part in its first year.

Each country could be represented by 6 contestants who did not take part in the previous IMO in Vietnam and in the school year 2007/08 are still students of secondary schools. Finally, 40 individuals from seven countries took part in the introductory year of the competition (Slovenian team arrived with four contestants only to Eisenstadt).

The competition itself took place in two days—on Saturday, 22nd September, competition of individuals, and competition of teams on Sunday, 23rd September. On both days individuals, or teams were solving four problems. Each day the competitors had five hours to solve those problems. Each problem was evaluated (according to approved system of evaluation of each problem) by integers in the range from 0 to 8 points.

As at the IMO individual countries had the possibility to send suggestions of their problems for the competition to the organization team in advance. Out of these the international jury chose two groups of four problems, one for the competition of individuals and one for the teams.

The difficulty of problems was comparable with problems from similar international competitions (including IMO). Competitors from individual countries had the chance to gain international experience needed which they can use at the next (49th) IMO in 2008. To compare their difficulty (and trends in creating new mathematical tasks) we present both sets of competition problems. The country which suggested it to the competition is cited in parenthesis.

1 Individual Competition

1. Let a, b, c, d be positive real numbers with $a+b+c+d=4$. Prove that
$$a^2bc+b^2cd+c^2da+d^2ab \le 4\,.$$

(*Switzerland*)

2. A set of balls contains n balls which are labeled with numbers $1, 2, \ldots, n$. We are given $k > 1$ such sets. We want to colour the balls with two colours, black and white, in such a way that

a) the balls labeled with the same number are of the same colour,

b) any subset of $(k+1)$ balls with labels $a_1, a_2, \ldots, a_{k+1}$ (not necessary different) satisfying the condition $a_1 + a_2 + \cdots + a_k = a_{k+1}$, contains at least one ball of each colour.

Find depending on k, the greatest possible number n which admits such a colouring.

(*Slovenia*)

3. Let k be a circle and k_1, k_2, k_3 and k_4 be four smaller circles with their centres O_1, O_2, O_3 and O_4, respectively, on k. For $i = 1, 2, 3, 4$ and $k_5 = k_1$ the circles k_i and k_{i+1} meet at A_i a B_i such that A_i lie on k. The points O_1, A_1, O_2, A_2, O_3, A_3, O_4, A_4 lie in that order on k and are pairwise different. Prove that $B_1B_2B_3B_4$ is a rectangle.

(*Switzerland*)

4. Determine all pairs (x, y) of positive integers satisfying the equation
$$x! + y! = x^y\,.$$

(*Czech Republic*)

2 Team Competition

1. Let a, b, c, d be arbitrary real numbers with $\frac{1}{2} \leq a, b, c, d \leq 2$, satisfying $abcd = 1$. Find the maximal value of

$$\left(a + \frac{1}{b}\right)\left(b + \frac{1}{c}\right)\left(c + \frac{1}{d}\right)\left(d + \frac{1}{a}\right).$$

(*Czech Republic*)

2. For a set P of five points in the plane in general position, we denote the number of acute-angled triangles with vertices in P by $a(\mathsf{P})$. (A set of five points is said to be in general position if no three of the points lie on a line.) Determine the maximal value of a $a(\mathsf{P})$ over all possible sets P.

(*Switzerland*)

3. Let a MEMO-Tetrahedron be a tetrahedron with the properties that six length of their edges are pairwise different positive integers, where one of them is 2 and another one of them is 3. Let $s(T)$ denote the sum of the lengths of the edges of a MEMO-Tetrahedron T.

(a) Find all positive integers n for which there exists a MEMO-Tetrahedron T with $s(T) = n$.

(b) How many pairwise non-congruent MEMO-Tetrahedra T with $s(T) = 2\,007$ exist?

Two tetrahedra are said to be non-congruent if one cannot be transformed by composition of reflections with respect to plane, translations and rotations into the other

(It is not necessary to prove that tetrahedra are not degenerated, i.e., have positive volume.)

(*Austria*)

4. Determine all positive integers k with the following property: there exists an integer a such that $(a + k)^3 - a^3$ is a multiple of $2\,007$.

(*Austria*)

The formulations of the problems were introduced to contestants (in both competitions) in their mother tongue similarly as at IMO. The contestants could hand in their solutions in their mother tongue, too. All contestants could write relevant questions during the beginning 45 minutes, they were answered by the head of appropriate delegation (with the approval of the international jury)—similarly as at IMO.

The following two days (23rd–24th September) were reserved for coordination of students' solutions. It went the same way as at IMO. The discussion while coordinating the problems was held in English and in German. The limits for gold, silver and bronze medals were set at the final meeting of the jury. The definitive order of countries in the team competition was confirmed. Quite low limits for gaining medals in the individual competition shows evidence that the problems of the first year of the competition were quite demanding. The point range of 23–32 was set for the gold medal, 13–22 points for silver medal and 8–12 points for the bronze medal. The best result in the individual competition was gained by *Joanna Bogdanowicz* from Poland who gained 26 points. Altogether 2 gold medals, 8 silver ones and 10 bronze medals were given.

The organizers prepared two trips for the contestants and other participants of the 1st Middle European MO for the last two days—to Neusiedler See and to Vienna where the participants could visit all the sights of the Austrian capital city.

Final ceremony of the competition took place in the presence of representatives of political life of Burgenland and the Ministry of Education of Austria in congress hall of hotel Ohr in Eisenstadt. The chairman of the organizing committee of the 1st Middle European Olympiad *Prof. Gerd Baron* gave medals to all awarded and thanked *Mag. Thomas Mühlgassner* who was responsible for the course and organization of the 1st Middle European MO in Eisenstadt. The conditions for the competition itself in the competition site were rightly considered above standard by the leaders of all delegations.

Finally, we offer two different solutions of the first problem of the individual competition for those who are interested in solving problems of Olympic type. It is necessary to point out that this problem was the most difficult one in the individual competition.

Solution to Problem 1 – Individual Competition. Let $p \geq q \geq r \geq s$ be an order of positive real numbers of the set $\{a, b, c, d\}$ with $a+b+c+d = 4$. Using of the rearrangement inequality we have

$$a^2bc + b^2cd + c^2da + d^2ab = a \cdot abc + b \cdot bcd + c \cdot cda + d \cdot dab \leq$$

$$\leq p \cdot pqr + q \cdot pqs + r \cdot prs + s \cdot qrs = (pq + rs)(pr + qs)\,.$$

We can use now (twice) AM–GM inequality for positive real numbers p, q, r, s to obtain

$$(pq + rs)(pr + qs) \leq \left(\frac{pq + rs + pr + qs}{2}\right)^2 = \frac{1}{4}\Big((p + s)(q + r)\Big)^2 \leq$$

$$\leq \frac{1}{4}\left(\frac{(p + s) + (q + r)}{2}\right)^4 = \frac{1}{64}\,(a + b + c + d)^4 = 4\,.$$

This completes the proof.

Another solution. Since the given inequality is cyclic, we may assume without loss of generality $a \geq c$. Further we may assume that $b \leq d$, since for $b < d$ we can use the cyclic shift $(a, b, c, d) \to (d, a, b, c)$ with $d \geq b$ and $a \geq c$, therefore

$$a^2bc + b^2cd + c^2da + d^2ab = ac(ab + cd) + bd(bc + ad)\,. \qquad (1)$$

For $a \geq c$ and $b \geq d$ it holds $(a - c)(b - d) \geq 0$. There is an equivalent form of the following inequality

$$ab + cd \geq bc + ad\,. \qquad (2)$$

Using of (2) and the AM–GM inequality for two positive real numbers we can estimate the right hand side of (1) by the following way

$$ac(ab + cd) + bd(bc + ad) \leq ac(ab + cd) + bd(ab + cd) = (ab + cd)(ac + bd)$$

$$\leq \left(\frac{(ab + cd) + (ac + bd)}{2}\right)^2 = \left(\frac{(a + d)(b + c)}{2}\right)^2$$

$$\leq \left(\frac{\frac{1}{4}\,(a + b + c + d)^2}{2}\right)^2 = 4\,,$$

which concludes the given statement.

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WFNMC Mini-Conference:
Call for papers and registration

WFNMC plans to hold a mini conference on Saturday 05 July 2008 in Monterrey. For those WFNMC members who wish to attend there will be no registration charge. The only extra cost will be that of your hotel and meals for an extra day or so, if you are attending ICME-11 (whose first day is Sunday 06 July).

This will also allow us to use our allotted ICME hours on official business and discussions, a program for which will be released soon.

This mini-conference will allow a day for members to present papers. This is a call for papers for the mini conference. At this stage we do not know how many papers will be given but we are expecting speakers will have 20 minutes to present a paper.

Maria Falk de Losada is coordinating the program for the mini-conference and you should send your proposal to her at rectoria@uan.edu.co by 28 February 2008.

In order to assist with planning we ask anyone intending to attend to advise Maria. Formal registration is not otherwise required.

Maria is also coordinating the WFNMC allocated hours during ICME-11 and a program for this will also be released in due course.

For an update on all this monitor the WFNMC web site www.amt.edu.au/wfnmc.html.

John Oprea: Differential Geometry and Its Applications 2nd edition. Mathematical Association of America, 2007. ISBN 9780883857489

This book is a nice introduction to classical differential geometry, a well-written text for undergraduates, relatively easy to read, with many practical examples outlined in the book. Most of the text concentrates on differential geometry in three dimensions, where plenty of exercises and pictures using MAPLE simplify the understanding, and at the same time prepare the user for further applications. The book might attract even those who do not intend to study math deeply but need some skill in applications as it shows close connections of various areas of mathematics (differential equations, calculus of variations, complex analysis) and their application to engineering, physics etc.

It is always a bit risky, when a textbook includes software. Changes in this direction are very quick, and it means preparing constant up-to-date re-editions. On the other hand, this text is really well prepared, and the author showed the power to go through the process of making necessary changes. This edition of the text is in fact the third one, a good deal longer than the first edition (1st edition Prentice Hall 1996, 2nd edition 2003). Many suggestions from those who have taught from the previous editions of the book has been incorporated. The text makes extensive use of MAPLE (specifically Maple 10) as a computer algebra system. Of course, users of Mathematica might prefer the book by A. Gray. But Mathematica could easily be substituted for Maple throughout.

The book can be strongly recomended to teachers of basic courses of Differential Geometry and to their students. It is suitable for either a one-quater or one-semester course as well as a full-year course.

Reviewed by: Alena Vanžurová

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To attain an appropriate level of achievement in mathematics, students require talent in combination with commitment and self-discipline. The following books have been published by the AMT to provide a guide for mathematically dedicated students and teachers.

Australian Mathematics Competition (AMC) Solutions and Statistics
Edited by DG Pederson

This book provides, each year, a record of the AMC questions and solutions, and details of medallists and prize winners. It also provides a unique source of information for teachers and students alike, with items such as levels of Australian response rates and analyses including discriminatory powers and difficulty factors.

Australian Mathematics Competition Book 1 (1978-1984)

Australian Mathematics Competition Book 2 (1985-1991)

Australian Mathematics Competition Book 3 (1992-1998)

Book 3 also available on CD (for PCs only).

Australian Mathematics Competition Book 4 (1999-2005)

Excellent training and learning resources, each of these extremely popular and useful books contains over 750 past AMC questions, answers and full solutions. The questions are grouped into topics and ranked in order of difficulty.

Problem Solving Via the AMC
Edited by Warren Atkins

This 210 page book consists of a development of techniques for solving approximately 150 problems that have been set in the Australian Mathematics Competition. These problems have been selected from topics such as Geometry, Motion, Diophantine Equations and Counting Techniques.

Methods of Problem Solving, Book 1
Edited by JB Tabov, PJ Taylor

This introduces the student aspiring to Olympiad competition to particular mathematical problem solving techniques. The book contains formal treatments of methods which may be familiar or introduce the student to new, sometimes powerful techniques.

Methods of Problem Solving, Book 2
JB Tabov & PJ Taylor
After the success of Book 1, the authors have written Book 2 with the same format but five new topics. These are the Pigeon-Hole Principle, Discrete Optimisation, Homothety, the AM-GM Inequality and the Extremal Element Principle.

Mathematical Toolchest
Edited by AW Plank & N Williams

This 120 page book is intended for talented or interested secondary school students, who are keen to develop their mathematical knowledge and to acquire new skills. Most of the topics are enrichment material outside the normal school syllabus, and are accessible to enthusiastic year 10 students.

International Mathematics — Tournament of Towns (1980-1984)

International Mathematics — Tournament of Towns (1984-1989)

International Mathematics — Tournament of Towns (1989-1993)

International Mathematics — Tournament of Towns (1993-1997)

International Mathematics — Tournament of Towns (1997-2002)
Edited by PJ Taylor

The International Mathematics Tournament of Towns is a problem solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. Each book contains problems and solutions from past papers.

Challenge! 1991 – 1998 Book 1
Edited by JB Henry, J Dowsey, A Edwards, L Mottershead, A Nakos, G Vardaro & PJ Taylor

This book is a major reprint of the original Challenge! (1991-1995) published by the Trust in 1997. It contains the problems and full solutions to all Junior and Intermediate problems set in the Mathematics Challenge for Young Australians, exactly as they were proposed at the time. It is expanded to cover the years up to 1998, has more advanced typography and makes use of colour. It is highly recommended as a resource book for classes from Years 7 to 10 and also for students who wish to develop their problem solving skills. Most of the problems are graded within to allow students to access an easier idea before developing through a few levels.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

Australian Mathematical Olympiads 1979 – 1995
H Lausch & PJ Taylor

This book is a complete collection of all Australian Mathematical Olympiad papers since the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads 1981-1993 and 1993-2001
A Liu

These books contain the papers and solutions of two contests, the Chinese National High School Competition and the Chinese Mathematical Olympiad. China has an outstanding record in the IMO and these books contain the problems that were used in identifying the team candidates and selecting the Chinese teams. The problems are meticulously constructed, many with distinctive flavour. They come in all levels of difficulty, from the relatively basic to the most challenging.

Asian Pacific Mathematics Olympiads 1989-2000
H Lausch & C Bosch-Giral

With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981-1995
ME Kuczma & E Windischbacher

Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
Edited by T Andreescu & Z Feng

This book contains one hundred and one highly rated problems used in training and testing the USA International Mathematical Olympiad team. These problems are carefully graded, ranging from quite accessible towards quite challenging. The problems have been well developed and are highly recommended to any student aspiring to participate at National or International Mathematical Olympiads.

Hungary Israel Mathematics Competition
S Gueron

This 181 page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions. The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Bulgarian Mathematics Competition 1992-2001
BJ Lazarov, JB Tabov, PJ Taylor, AM Storozhev

The Bulgarian Mathematics Competition has become one of the most difficult and interesting competitions in the world. It is unique in structure, combining mathematics and informatics problems in a multi-choice format. This book covers the first ten years of the competition complete with answers and solutions. Students of average ability and with an interest in the subject should be able to access this book and find a challenge.

Mathematical Contests – Australian Scene
Edited by AM Storozhev, K McAvaney & A Di Pasquale

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

WFNMC — Mathematics Competitions
Edited by Jaroslav Švrček

This is the journal of the World Federation of National Mathematics Competitions (WFNMC). With two issues each of approximately 80-100 pages per year, it consists of articles on all kinds of mathematics competitions from around the world.

Parabola incorporating Function

This Journal is published in association with the School of Mathematics, University of New South Wales. It includes articles on applied mathematics, mathematical modelling, statistics, history and pure mathematics that can contribute to the teaching and learning of mathematics at the senior secondary school level. The Journal's readership consists of mathematics students, teachers and researchers with interests in promoting excellence in senior secondary school mathematics education.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Education, Science and Training) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

Newton Enrichment Student Notes
JB Henry
Recommended for mathematics students of about Year 5 and 6 as extension material. Topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

Dirichlet Enrichment Student Notes
JB Henry
This series has chapters on some problem solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory. It is designed for students in Years 6 or 7.

Euler Enrichment Student Notes
MW Evans and JB Henry
Recommended for mathematics students of about Year 7 as extension material. Topics include elementary number theory and geometry, counting, pigeonhole principle.

Gauss Enrichment Student Notes
MW Evans, JB Henry and AM Storozhev
Recommended for mathematics students of about Year 8 as extension material. Topics include Pythagoras theorem, Diaphantine equations, counting, congruences.

Noether Enrichment Student Notes
AM Storozhev
Recommended for mathematics students of about Year 9 as extension material. Topics include number theory, sequences, inequalities, circle geometry.

Pólya Enrichment Student Notes
G Ball, K Hamann and AM Storozhev
Recommended for mathematics students of about Year 10 as extension material. Topics include polynomials, algebra, inequalities and geometry.

T-SHIRTS

T-shirts celebrating the following mathematicians are made of 100% cotton and are designed and printed in Australia. They come in white, and sizes Medium (Polya only) and XL.

Carl Friedrich Gauss T–shirt

The Carl Friedrich Gauss t-shirt celebrates Gauss' discovery of the construction of a 17-gon by straight edge and compass, depicted by a brightly coloured cartoon.

Emmy Noether T–shirt

The Emmy Noether t-shirt shows a schematic representation of her work on algebraic structures in the form of a brightly coloured cartoon.

George Pólya T–shirt

George Pólya was one of the most significant mathematicians of the 20th century, both as a researcher, where he made many significant discoveries, and as a teacher and inspiration to others. This t-shirt features one of Pólya's most famous theorems, the Necklace Theorem, which he discovered while working on mathematical aspects of chemical structure.

Peter Gustav Lejeune Dirichlet T–shirt

Dirichlet formulated the Pigeonhole Principle, often known as Dirichlet's Principle, which states: "If there are p pigeons placed in h holes and p>h then there must be at least one pigeonhole containing at least 2 pigeons." The t-shirt has a bright cartoon representation of this principle.

Alan Mathison Turing T-shirt

The Alan Mathison Turing t-shirt depicts a colourful design representing Turing's computing machines which were the first computers.

ORDERING

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- the quality, freshness and variety of its questions in the Australian Mathematics Competition, the Mathematics Challenge for Young Australians, and other Trust contests;
- the production of valued, accessible mathematics materials;
- dedication to the concept of solidarity in education;
- credibility and acceptance by educationalists and the community in general whether locally, nationally or internationally; and
- a close association with the Australian Academy of Science and professional bodies.