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MATHEMATICS COMPETITIONS

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The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the President

The *International Congress on Mathematical Education* (ICME) that will take place in Monterrey, Mexico, from 6–13 July 2008, is an important event for the *World Federation of National Mathematics Competitions* (WFNMC). The Congress in Monterrey is the eleventh in a row and is called ICME–11. The first one was organized in Lyon, France, in 1969. It was followed by the Congress in Exeter (1972) and, since then, ICME has taken place every four years: Karlsruhe (1976), Berkeley (1980), Adelaide (1984), Budapest (1988), Québec (1992), Sevilla (1996), Tokyo/Makuhari (2000) and Copenhagen (2004). All ICMEs are conducted under the umbrella of the *International Commission on Mathematical Instruction* (ICMI) which appoints the *International Program Committee* (IPC) and supervises the organization and the administration of the Congress. Since 1984 the Federation is an *ICMI Affiliated Study Group* and its members participate in ICMEs both as individuals and as members of an organization. It is time to shape our participation in this next event. The aim of this letter is to provide some information that could be of help when planning the participation in ICME–11. More information will be available in September when the ICME–11 Official Announcement will come out.

The program of ICME–11, according to its website located at http://www.icme11.org.mx/icme11/, contains the following major components:

– *Plenary Lectures* and/or panels on topics of interest for the international community engaged with mathematics education (on invitation from the IPC);

– *Reports of Survey Teams* working on new perspectives and emerging issues (on invitation from the IPC);

– *Regular Lectures*(on invitation from the IPC);

– *Topic Study Groups*(designed to gather participants interested in a particular topic of mathematics education; organizers to be appointed by IPC);

– *Discussion Groups* (designed to discuss challenging and/or controversial issues and dilemmas in mathematics education; organizers to be appointed by IPC);

- *National Presentations* (presentation of trends in mathematics education in a certain country or a region);
- *Posters and Round Tables* (Round Tables will address a group of posters developed on the same theme).

The Federation is usually involved with the *Topic Study Groups* and *Discussion Groups* focused on mathematics competitions and on the work with gifted students. The ICME–11 program will provide opportunities in this direction too. In a circular letter to Chairs of Affiliated Study Groups dated 6 March 2007 Prof. Bernard Hodgson, Secretary General of ICMI, gave additional information about the current stage of Congress Program development. Among other interesting topics one finds in his letter, the following items are of interest for WFNMC:

- Activities and programs for gifted students;
- The nature and roles of international cooperation in mathematics education;
- Promoting creativity for all students in mathematics education;
- Public perceptions and understanding of mathematics and mathematics education;
- The role of mathematical competitions and other challenging contexts in the teaching and learning of mathematics.

Maria Falk de Losada, Vice President of WFNMC and Member of IPC for ICME–11, has agreed to design and coordinate the activities of WFNMC during ICME–11. If you have any suggestions in this respect, please, do not hesitate to write to her at

mariadel@venus.uanarino.edu.co

Petar S. Kenderov
President of WFNMC
May 2007

From the Editor

Welcome to *Mathematics Competitions* Vol. 20, No 1.

At first I would like to thank the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:
The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

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Jaroslav Švrček,
June 2007

Polish Mathematical Olympiad For Teenagers (13–16)

Jacek Dymel & Michał Niedźwiedź

Jacek Dymel and Michał Niedźwiedź (below) got their M.Sc. degrees in mathematics from the Jagiellonian University in 1993. They are teachers in Kraków schools and also work with students of mathematics at the Jagiellonian University. In the early nineties of the XXth century so-called mathematical circles for school pupils and students of different ages were organized at the Jagiellonian University.

Dymel and Niedźwiedź were among the creators of these cirles and for 15 years they have prepared pupils to mathematical competitions of different levels. Among Dymel's and Niedźwiedź's pupils there are many winners of the Polish Mathematical Olympiad and some students who were awarded medals in the International Mathematical Olympiad, including a winner of the Gold Medal. They are among the main organizers of the Polish Mathematical Olympiad for Teenagers.

1 School system in Poland

In Poland, mathematics as well as teaching mathematics always have been of great importance. It goes back to the golden period of Polish mathematics of the 1930s (Banach, Kuratowski, Sierpiński, Borsuk, Tarski and many others). During the 1950s, when Poland struggled to rebuild its economy after the ravages of World War II, a saying: "Poland's main exporting goods are coal and mathematical theorems", became popular in academic circles. During the next half-century several important changes were made in the methods of teaching mathematics. These changes more or less followed world wide trends. However, the most important structural change, namely a change of how the school system was organized, occurred during the late 1990s. Before the change, eight years of elementary education were followed by four years of high school. Subsequently, these 12 years of schooling were divided into four stages, each of them taking three years: primary school, elementary school, junior high school (pol. gimnazjum), and senior high school (pol. liceum). Teenagers between 13 and 16 years of age were now grouped together in junior high. It is a very difficult period of life for everybody, creating many challenges both for the young people and for their parents and teachers.

A wave of criticism started shortly after the new system was introduced in 1999. It soon became clear that although the new system had its merits, it had also many shortcomings. One of the shortcomings was the division of school education into too many stages; the other, possibly the most important, was that the young people struggling with problems of adolescence were assembled together, and isolated from other age groups.

Nevertheless, this novel approach created an opportunity for organizing a high level mathematical competition, different from competitions existing in many countries, namely the Olympiad for Students in Junior High, i.e. for youth of age 13 to 16. It started in 2005 as a 'younger sister' of the national contest for students in Senior High (liceum). The purpose of this new competition has been the popularization of mathematics through wide participation of junior high school students from all regions of Poland. Such an approach makes it different from several similar European competitions for teenagers (e.g. Balkan Mathematical Olympiad) where national representations participate. Due to the school system, it makes such a competition unique, at least in Europe.

Let us first present the mathematical curriculum in Junior High in Poland. During each of the three school years mathematics is taught for 4 hours weekly and covers the following topics:

- Basic operations on rational numbers
- Percentage calculations
- Powers with integer exponent
- Roots, examples of irrational numbers
- Algebraic expressions
- Linear functions
- Systems of linear equations and inequalities
- Polygons, circles, angles in circles
- Congruence and similarity in geometry
- Inscribed and circumscribed circles of a triangle
- Geometric transformations (examples)
- Pythagorean theorem
- Prisms, pyramids
- Solids of revolution: cylinder, cone, sphere.

The competition consist of three stages: homework, semifinal and final. Below we present problems from all stages of the 2005/2006 competition.

2 Problems From The First Phase

864 pupils (mainly from the southern Poland) participated in the first phase of the Olympiad. Each of them was presented with 7 problems to solve. This phase was homework and participants could also get some broader help in the form of mathematical reference, internet and other people.

Regardless of the difficulty of the problem pupils could obtain the same amount of points for solving each of them. Assessment was done in the following way: 6 or 5 points were given for a good and proper solution, 2 points for significant attempts and 0 for a wrong solution or lack of it.

Problems	1	2	3	4	5	6	7	Total
6 points	555	168	135	65	691	199	487	2300
5 points	99	105	104	179	118	99	219	923
2 points	42	42	116	307	36	121	83	747
0 points	168	549	509	313	19	445	75	2078
$\overline{x}$	4.52	1.87	1.81	2.20	5.56	2.23	4.84	23.04
σ	2.39	2.60	2.43	2.12	1.19	2.59	1.89	9.29
p	0.24	0.68	0.72	0.72	0.06	0.66	0.18	0.47
ϱ	0.63	0.67	0.62	0.65	0.27	0.67	0.58	

Table 1: Presentation of the number of pupils that managed to obtain certain points for each of the problems and statistical quantities for them. Here $\overline{x}$ is arithmetic mean, σ standard deviation, p item difficulty (proportion of pupils who did not give satisfactory solution, 0 or 2 points, to the total number of participants) and ϱ item discrimination.

Results obtained by participants together with corresponding statistical description are summarized in Table 1. In particular, quantity ϱ, item discrimination, requires additional comments. There are many ways how to measure the effectiveness of a problem to separate students who vary in their degree of knowledge of the material. Here we define item discrimination as the correlation coefficient between an item score and the total test score. It is a measure of how well an item separates, or differentiates, between those that answer the item correctly or incorrectly and have a high or low total test score respectively. It should also be noted that this degree of separation is also related to the item difficulty. In our case problem 5 has a very low discrimination coefficient—it did not serve as a proper tool to differentiate competitors.

The maximum possible score of 42 points was achieved by 5 pupils (among them 1 girl). The average score amounted to 23.04 points. Tests rated at 0 points total were submitted by 3 persons. A total of 409 participants qualified for the second phase, the threshold score was set to 24 points (slightly above average).

Most participants (63 %) were boys. Their average score of 24.05 points was a little higher than the average for the girls (21.36 points).

Problem 1.1. Prove that

$$\sqrt{3-\sqrt{8}}+\sqrt{5-\sqrt{24}}+\sqrt{7-\sqrt{48}}=1$$

Problem 1.2. Given a convex quadrilateral with the following properties:

- a circle may be inscribed in the quadrilateral,
- the diagonals of the quadrilateral are othogonal.

Prove that one of the diagonals bisects the other.

Problem 1.3. 99 points belonging to a circle with a radius of 10 were selected. Prove that there is a point inside the circle that is distant more than 1 from each of the selected points.

Problem 1.4. Determine all solutions of the following system of equations

$$\begin{aligned} 25x^2+9y^2 &= 12yz, \\ 9y^2+4z^2 &= 20xz, \\ 4z^2+25x^2 &= 30xy. \end{aligned}$$

Problem 1.5. A gardener puts 121 apples into 15 buckets in such a way that there is at least one apple in each of the buckets. Is it possible that each bucket contains a different number of apples?

Problem 1.6. It is known that a genuine coin weighs 10 grams and a false one weighs 9 grams. You have 5 coins weighing 48 grams in total and electronic scales. You may put any number of coins on the scales and read their total weight. Can you determine which coins are false, and which are genuine, doing no more then 3 measurements?

Problem 1.7. There are four points: A, B, C and D in a plane. B is the midpoint of segment AC. Further let $AB =$

$BC = BD = 17$ and $AD = 16$. Determine the length of segment CD.

3 Problems From The Second Phase

409 pupils qualified for the second phase and 349 pupils actually participated in the contest, which took place in eleven Polish cities on 28 January 2006. The participants were given a set of 5 problems to solve within 3 hours. The results and corresponding statistical data are summarized in Table 2.

Problems	1	2	3	4	5	Total
6 points	241	90	142	57	42	572
5 points	28	21	40	17	23	129
2 points	16	11	31	126	75	259
0 points	64	227	136	149	209	785
$\overline{x}$	4.64	1.91	3.19	1.95	1.48	13.17
σ	2.36	2.70	2.78	2.18	2.16	8.11
p	0.23	0.68	0.48	0.79	0.81	0.60
ϱ	0.57	0.75	0.65	0.68	0.68	

Table 2: Results of the second stage (cf. Table 1).

12 participants, including three girls, scored the possible maximum of 30 points. On the other hand, 22 participants (again 3 girls) scored 0 points. The majority of pupils (43%) scored between 6 and 14 points, which generally meant they solved 1 or 2 problems. The average score was 13.17 points.

The participation of girls in the second phase was smaller than in the first phase—they amounted to 31% of participants in the second phase. The average score for girls was 12.03 points, and for boys 13.67 points.

Problem 2.1. A prism has twice as many vertices as a pyramid. Which one of those polyhedrons has more faces and by how many?

Problem 2.2. 111 positive whole numbers are given. Prove that 11 numbers can be selected from them in such a way, that their total is divisible by 11.

Problem 2.3. Let ABC be an acute triangle, in which $\angle BAC = 45°$. The altitudes of the triangle intersect at point H. Prove that $|AH| = |BC|$.

Problem 2.4. Determine all positive integers n such that $14^n - 9$ is a prime number.

Problem 2.5. Let $ABCDEF$ be a convex hexagon with internal angles at vertices A, B, C and D equal to $90°$, $128°$, $142°$ and $90°$ respectively. Prove that the area of the hexagon is less than $\frac{1}{2}|AD|^2$.

4 Problems From The Finals

119 pupils participated in the finals. To qualify for the finals one had to score at least 16 points (over 50%) in the second phase. The contest took place in two cities: Bielsko-Biała (south of the country) and Toruń (north) on 25 March 2006. Similar to the second phase, there were 5 problems to be solved in 3 hours.

Problems	1	2	3	4	5	Total
6 points	53	35	21	4	7	120
5 points	21	3	3	3	2	32
2 points	28	5	1	4	1	39
0 points	17	76	94	108	109	404
$\overline{x}$	4.03	1.97	1.20	0.39	0.45	8.05
σ	2.29	2.75	2.37	1.35	1.54	6.19
p	0.38	0.68	0.80	0.94	0.92	0.74
ϱ	0.51	0.70	0.63	0.45	0.64	

Table 3: Results of the finals (cf. Table 1).

The set of problems proved to be much more difficult than in the previous phases. No participant scored the maximum of 30 points. Two pupils scored 29 points (which means they solved all 5 problems) and received first-class awards. Three pupils solved 4 problems each, scoring 24 points and receiving second-class awards. Awards were also given to 12 participants who solved 3 problems each (scoring 18 or 17 points).

Among all 17 award winners were three girls. The average score in the finals was 8.05 points. In each subsequent phase of the contest, the participation of girls was smaller—they totalled 24% of participants in the finals.

Problem 3.1. How many fours (a, b, c, d) of positive integers are there, such that $ab + bc + cd + da = 55$? Justify your answer.

Problem 3.2. Let $ABCD$ be a parallelogram. Point E belongs to the side AB, and point F belongs to the side AD. Line EF intersects line CB at point P and intersects line CD at point Q. Prove that the area of triangle CEF is equal to the area of triangle APQ.

Problem 3.3. There are n points in space ($n \geq 4$) such that no four of them belong to the same plane. Each pair of those points were connected by a blue or red segment. Prove that one of the colors may be chosen in such a way that each pair of points is connected by a segment or a continuous set of segments in that color.

Problem 3.4. Let there be a tetrahedron such that each dihedral angle formed by its adjacent faces is right or acute. Vertices of the tetrahedron lie on a sphere with the center S. Can S lie outside of the tetrahedron? Substantiate your answer.

Problem 3.5. Let p and q be two different prime numbers and let a and b be positive integers such that the number aq divided by p gives 1 as a remainder and the number

bp divided by q gives 1 as a remainder. Prove that $\frac{a}{p} + \frac{b}{q} > 1$ holds.

5 Problem 1.6—solution and generalization

Pupils interested in mathematics can use some competition problems and try to generalize them. We concentrate on Problem 1.6.

Solution To Problem 1.6.
False coins can be found. We know that exactly 3 of 5 coins are genuine. Let us mark all coins with letters A, B, C, D, E.

In the first measurement we weigh coins A and B. Three cases are possible:

1. Both coins A and B are false. Then all the other coins C, D and E are genuine (because only two coins can be false).

2. Both coins A and B are genuine. Then exactly one of the other coins C, D and E is genuine. We can determine it in two measurements, weighing one of the other coins (first C, then D) in each. Either one of them happens to be genuine or both are false and then E is genuine.

3. Only one of the coins A and B is genuine. Then exactly one of the other coins C, D and E is false. In second measurement let us weigh coins A and C together. Again three cases are possible:

   a) Both coins A and C are genuine. Then we know that B is false and one of the coins D, E is false (one measurement will suffice to determine which one of them).

   b) Both coins A and C are false. Then all the other coins B, D, E are genuine.

   c) Only one of the coins A, C is genuine. If A was genuine, then B, C would be false and D, E would be genuine. If C was genuine, then A would be false so B would have to be genuine. Summing up, in this case the set of genuine coins is one of the following: (A, D, E), (B, C, D), (B, C, E). Weighing together coins A and D will suffice to determine which set is genuine.

Comment on Problem 1.6.

This problem may be an inspiration to setting the following problem:

It is known that a genuine coin weighs 10 grams and a false one weighs 9 grams. We know that among n coins there are k coins weighing 10 grams. Using electronic scales we can weigh any number of selected coins in each measurement and read their total weight. How many measurements must be made at least to determine which coins are false, and which are genuine?

A Polish mathematician, Hugo Steinhaus, discussed the following problem in his book *Mathematical Snapshots*[1]:

There are 13 coins, of which one (and only one) is false, and it is not known whether it is heavier or lighter than the genuine ones. Prove that it can be found in three measurements using pan scales by putting any amount of coins on each scale pan.

The problem has many generalizations. Recently a Polish high school student Marcel Kołodziejczyk discussed 48 variations of the original problem and obtained interesting results, which seem to be new.[2]

6 Problem 3.2—solution and extension

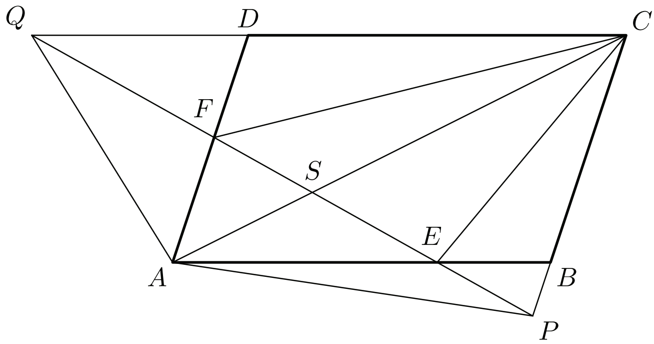
Solution To Problem 3.2.

Let S be the intersection point of lines PQ and AC; S_{AEC} and S_{AEQ} be areas of triangles AEC and AEQ respectively. Note that $S_{AEC} = S_{AEQ}$, because they have common base AE and altitude of the same length (it is the distance between lines AB and CD). Therefore $S_{ASQ} = S_{SEC}$. Analogously it can be shown that $S_{ASP} = S_{FSC}$.

From the above we get: $S_{APQ} = S_{ASQ} + S_{ASP} = S_{SEC} + S_{FSC}$, which was to be proved. (See the picture on the next page.)

[1] H. Steinhaus, Mathematical Snapshots; Oxford University Press 1983.

[2] The paper is available in Polish under http://math.uni.lodz.pl/ andkom/Marcel/Kule.pdf



Extension on Problem 3.2.

Let there be a parallelepiped $ABCDEFGH$, where $\overrightarrow{AB} = \overrightarrow{EF} = \overrightarrow{HG} = \overrightarrow{DC}$. Point R belongs to the edge AE, point P belongs to the edge AB, point Q belongs to the edge AD. Line PR intersects line EF at point X, line PQ intersects line BC at point Y, line RQ intersects line HD at point Z. Calculate the ratio of the volume of tetrahedron $RQPG$ to the volume of tetrahedron $AXYZ$.

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The Art of Problem Solving

David Patrick

David Patrick is Vice President of AoPS Incorporated. He is the author of Introduction to Counting & Probability, *a discrete math textbook for middle and high school students, and is currently working on the sequel,* Intermediate Counting & Probability. *He was a USA Mathematical Olympiad winner in 1988, earned his Ph.D. in Mathematics from MIT in 1997, and did research in noncommutative algebra.*

1 History

The Art of Problem Solving (AoPS) website,[1] established in 2003, has grown to over 29,000 members.[2] We believe that it is the largest website of its kind in the English-speaking world, with mathematics resources developed specifically for high-ability middle and high school students. AoPS has been called "a revolution in mathematics training for the top high school students." [3]

2 Forum, Blogs, and Wiki

The heart of AoPS is the AoPS Forum, which has over 800,000 posts on a variety of topics, mathematical and otherwise. The AoPS Forum is for students, parents, and teachers to discuss various math problems

[1] `http://www.artofproblemsolving.com`
[2] All statistics cited in this article are as of 4 May 2007.

and other topics of interest to people interested in math. The Forum is free for everyone, and its members are students and teachers from all different ages, locations, and abilities. In the late afternoon of May 4, 2007, the list of recent discussions include the following (note that these are the actual discussion titles as they appear on the AoPS Forum, so the titles may include misspellings):

- Maximum of minimum of

$$\frac{a_i}{1+\sum_{i=1}^{n} i}$$

  (in the Olympiad Inequalities forum; the forum is LaTeX-compatible to allow for mathematical discussion)
- “Another TC problem, whose solution I don’t understand” (in the Computer Science and Informatics forum)
- “baseball” (in the Middle School forum; the problem under discussion was a middle-school level problem about baseball)
- “Nysml” (in the New York local forum; the discussion was about the recently-concluded New York State Mathematics League contest)
- “Displaying the power of a matrix” (in the College Linear Algebra forum)
- “perfect square in different bases” (in the High School Basics forum)

All of the posts listed (and many others not listed) occurred in a span of under 20 minutes.

Any AoPS member can set up his or her own personal blog, which is a personal web page on which the owner can post anything he or she wishes. Although the subjects on the blog are unrestricted, many of the blog entries discuss mathematics and problem solving. Currently, over 750 AoPS users have started blogs.

Finally, AoPS has started a wiki about mathematics and problem solving. A wiki is an online encyclopedia that can be edited by anyone. The AoPS wiki has over 2,000 articles on a variety of topics: some are related to mathematical ideas and concepts, others are related to problems and/or problem solving technique, and still others point to other resources. To give a flavor for the types of articles in the AoPS wiki, a list of some of the most recently created or edited articles includes:

- *Fermat's Last Theorem*
- Pascal's Triangle Related Problems
- 2007 USAMO Problems
- *The Art and Craft of Problem Solving*[3]
- Asymptote[4]
- 1997 AIME Problems

3 Online Classes

AoPS runs a number of online classes specifically designed for strong students in grades 7–12. All of the classes are conducted in AoPS's "virtual classroom," an online, moderated chatroom that is LaTeX-compatible and graphics-enabled, to permit mathematical discussion.

AoPS offers three different types of classes. First are subject classes in traditional secondary school math topics: algebra, counting & probability, geometry, number theory, and trigonometry. These classes tend to be similar in content to a traditional in-school class, but with a much greater emphasis on difficult problem solving. Second are classes that are designed as preparation for one of the major US mathematics competitions, such as MATHCOUNTS and the various American Mathematics Competitions (AMC) contests (the AMC contests are those that eventually lead to the selection of the United States IMO team). Finally, AoPS offers a year-long Worldwide Online Olympiad Training (WOOT) program, designed for the very best students whose ambition is to do problem-solving at the IMO level. To give some indication of the quality of the WOOT students, in 2005-06 over 90% of the US students in WOOT qualified for the 2006 USA Mathematical Olympiad, which is very selective: of the approximately 230,000 students in 2006 who wrote one of the initial AMC contests, only 430 qualified for the USAMO. [1]

[3] This is a popular problem-solving textbook, authored by former IMO participant Paul Zeitz.

[4] Asymptote is a LaTeX plug-in for creating high-quality diagrams. The Asymptote wiki pages on AoPS are considered the "official" wiki pages of the Asymptote project.

4 AoPS Foundation and USA Math Talent Search

Separate from the main AoPS website is the Art of Problem Solving Foundation[5]. The AoPS Foundation's mission is to promote problem solving education for middle and high school students in the United States. The Foundation supports two major endeavors.

USA Mathematical Talent Search

The USAMTS was founded in 1989 by George Berzsenyi at the Rose-Hulman Institute of Technology. It has run annually every year since, and after a number of years of being managed by the US National Security Agency, the management of the contest was passed to the AoPS Foundation in 2004.

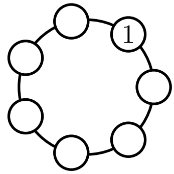
The USAMTS runs during the USA school year (roughly September through April), and consists of 4 rounds of 5 questions each. The USAMTS is a "take-home" contest and is run entirely via the `http://www.usamts.org` website. Students have at least one full month to work on each round of problems, and must write and submit full solutions including proofs. Students are permitted to use any available resource to solve the problems, including books, calculators, and computers, but may not consult with teachers or other students. Another unique feature of the USAMTS is that students not only receive numeric scores on their solutions, but also receive written feedback on both the correctness and the writing style of their submitted work.

The problems on Round 1 of the 2006-07 USAMTS were:

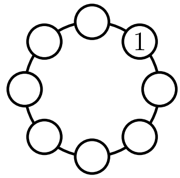
1. When we perform a 'digit slide' on a number, we move its units digit to the front of the number. For example, the result of a 'digit slide' on 6471 is 1647. What is the smallest positive integer with 4 as its units digit such that the result of a 'digit slide' on the number equals 4 times the number?

[5] `http://www.artofproblemsolving.org`

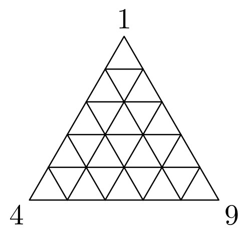
2. (a) In how many different ways can the six empty circles in the diagram at right be filled in with the numbers 2 through 7 such that each number is used once, and each number is either greater than both its neighbors, or less than both its neighbors?



(b) In how many different ways can the seven empty circles in the diagram at right be filled in with the numbers 2 through 8 such that each number is used once, and each number is either greater than both its neighbors, or less than both its neighbors?



3. (a) An equilateral triangle is divided into 25 congruent smaller equilateral triangles, as shown. Each of the 21 vertices is labeled with a number such that for any three consecutive vertices on a line segment, their labels form an arithmetic sequence. The vertices of the original equilateral triangle are labeled 1, 4, and 9. Find the sum of the 21 labels.



(b) Generalize part (a) by finding the sum of the labels when there are n^2 smaller congruent equilateral triangles, and the labels of the original equilateral triangle are a, b, and c.

4. Every point in the plane is colored either red, green, or blue. Prove that there exists a rectangle in the plane such that all four of its vertices are the same color.

5. $ABCD$ is a tetrahedron such that $AB = 6$, $BC = 8$, $AC = AD = 10$, and $BD = CD = 12$. Plane $\mathcal{P}$ is parallel to face ABC and divides the tetrahedron into two pieces of equal volume. Plane $\mathcal{Q}$ is parallel to face DBC and also divides $ABCD$ into two pieces of equal volume. Line ℓ is the intersection of planes $\mathcal{P}$ and $\mathcal{Q}$. Find the length of the portion of ℓ that is inside $ABCD$.

Local Programs

The AoPS Foundation also supports a number of local programs devoted to mathematics and problem solving education in specific communities around the United States. These include a number of Math Circles, including those in San Diego, Stanford, Boulder (Colorado), Charlotte, San Jose, Orange County (California), and Albany (New York). The AoPS Foundation also supports The Teachers' Circle, a new math circle designed specifically for teachers in the San Francisco Bay area. The Teachers' Circle is described in greater detail in [2].

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The Mathematical Content of Junior Contests: Latvian approach

Inese Bērziņa & Dace Bonka & Gunta Lāce

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Dace Bonka is a Ph.D. student at the University of Latvia, lecturer at the University of Latvia and an educational director at A. Liepas Correspondence Mathematics School. While at school Dace Bonka was inspired by math olympiads. Now she herself has been a member of the orgcommittee and jury of Latvian mathematics olympiads for 10 years. Her research area and main activities are correspondence mathematics contests for junior students in Latvia.

Gunta Lāce is a PhD student at the University of Latvia and a teacher at Valmiera State Gymnasium. Her research interests are combinatorics and advanced math education in middle and high school. She has been awarded the prestigious Atis Kronvalds prize in education for successes in conducting the research work of high school students.

1 Introduction

Mathematical contests have become an essential part of the mathematical education in Latvia.

- They provide high standards which have not reduced despite many educational reforms that started with good intentions but resulted in a cheerless decline. In some sense they have become the unofficial standards for advanced education in mathematics.
- Another important aspect is the stability of mathematical contests. Teachers know that the contests will be held, and they can organize their activities and encourage their students to work additionally for a clear and inspiring aim. It is not surprising that the number of participants in the Open Mathematical Olympiad has increased significantly over the last few years because the official curriculum of mathematics has been reduced.

The problems of mathematical Olympiads and correspondence contests are used by many teachers also in everyday's work with strong students.

2 The contest system for junior students

There are a lot of state-scale competitions run in Latvia either as sat or by correspondence for more than 30 years.

Mathematical olympiads

All mathematical olympiads for junior students in Latvia are open to everybody who wants to participate.

- *School olympiads.* They are usually held in November. The olympiad is supported by A. Liepas Correspondence mathematics school with a problem set. However, schools don't have to use this set of problems, they can compose their own. At some schools this olympiad takes place within the framework of a mathematical week. It is not compulsory to organise this round.
- *Regional olympiads* are held in February. This olympiad takes place in district centres, the largest towns and in Riga—altogether in 39 different places. The olympiad is organized by The Centre for Curriculum Development and Examinations and A. Liepas Correspondence mathematics school. All participants of each grade solve the same problems, composed by a jury of the State mathematical olympiad.
- *Open Math Olympiad* (since 1974) is held on the last Sunday of April. This competition is a very large one; $\approx$ 3000 participants (5^{th}–12^{th} Grade students) arrive in Riga. This olympiad is organized by A. Liepas Correspondence mathematics school. It could not be organized without the significant help of volunteers. Every year about 300 people (teachers, students, parents, etc.) are involved in securing the procedure and checking the students' papers.

Along with these olympiads, at some schools mathematical weeks or afternoons are also organized, during which pupils take part in quizzes, team competitions and similar activities.

Correspondence contests

There are many students who need more than 4–5 hours (usually allowed during math olympiads) to go deep enough into the problem. For such students a system of correspondence contests has been developed.

- *Contest of Young Mathematicians* (since 1993) is for students up to 7$^{\text{th}}$ Grade, originally developed for students in Latgale, the eastern region of Latvia. Initially the problems were published only in some regional newspapers in Latgale. Since 1999 the problems have also been published on the internet. Today this contest has become popular all over Latvia.
- *Club of Professor Littledigit* was developed in 1974 for students up to the 9$^{\text{th}}$ Grade. There are six rounds each year, each containing six relatively easy and six harder problems. The problems are published in the newspaper *Latvijas Avīze*, and since 1999 on the internet. The problems are harder than those in the Contest of Young Mathematicians.

Mixed way contest

All the above mentioned contests are oriented to the students from the 5$^{\text{th}}$ Grade. In 2004, due to the great interest from 4$^{\text{th}}$ Grade students, the contest *So much or...how much?* was developed for them by teachers and parents. It is organised jointly with colleagues from Lithuania and consist of four rounds each school year. The first three rounds take place at each school, whose students wish to take part, and are organized by teachers at the school. These three rounds are open to everybody who wants to participate. In the last round only the winners of previous rounds take part. The fourth round takes place simultaneously in Lithuania and Latvia in 13 different places. Each invited student goes to the nearest venue.

3 Problem sets: underlying principles

The contest problem sets are developed in such a way that they are balanced accordingly to several parameters:

- It should cover the main areas of school mathematics: algebra, geometry, number theory and combinatorics. Combinatorics is understood in a broad sense including not only counting but also existence and non-existence of combinatorial objects. Particularly, the general combinatorial methods (mathematical induction, invariants, mean value, extremal element, interpretation) must be reflected. For more details see [1].

- "Discrete" mathematics and "continuous" mathematics are both to be represented.
- The problem set should contain both problems of deductive nature and problems of algorithmical nature.
- There should be problems of "prove it!" type and "construct it!" type along with problems in which the answer must be found by the solver.
- There should be problems of different levels of complexity: at least one easy problem that is solvable for every reasonable student and some hard and very hard problems for really strong students.

Of course, no single set (usually consisting of 5 but no more than 12 problems) can be made well-balanced according to all these parameters. So we try to achieve an (approximate) balance at least through an academic year.

A detailed *classification of problems* and methods of their solution is elaborated, and each published teaching aid is supplied with the index of problems according to this classification (one problem may well correspond to several "boxes"). For more details, see [2].

Problems can be classified in two different ways:
- according to the content of the problem
- according to the methods of solution.

We further mention the topics which are most suitable for junior students:

1. Algebra
  - algebraic conversions and expressions
  - equations
  - systems of equations
  - inequalities
  - sequences
  - functions
  - general methods (see above)

2. Geometry
- classical geometry: areas, triangle geometry, convex figures, constructions
- geometrical inequalities
- systems of points, lines, circles
- splitting and compositions of figures
- general methods (see above)

3. Number theory
- modules and congruencies
- prime decomposition
- Diophantine equations
- grouping
- decimal numeration
- general methods (see above)

4. Combinatorics
- graphs
- counting
- combinatorial systems
- general methods (see above)

5. Algorithms
- mathematical games
- inference of algorithm
- the analysis of a (given) algorithm
- the development of an algorithm
- optimization of an algorithm
- problems of logical content

For more details about algorithmic problems see [3].

Of course, all basic preparation for the competitions should be done by the student himself, and his teacher should be his best assistant. The assistance from the University consists mainly of organising a series of lectures (also summer camps) and creating teaching aids. These are of two types:

- the sets of problems with extended solutions
- methodical materials on important methods.

They are published in a paper version and also placed on the internet at [4] and [5]. Some of them are translated into English (e.g. [6] and [7]). Those interested can contact us at mailto:nms@lu.lv.

4 Some characteristic examples

Algebra

Problem. There are four numbers written on the blackboard. Andris enlarges one of them by 10%, the second one—by 20%. The third one he reduces by 10% and the last one he reduces by 20%. Is it possible that the all four initial numbers are obtained?

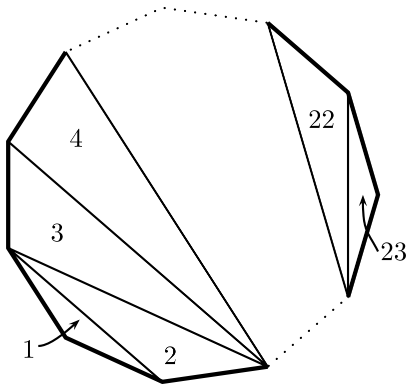
Solution. The answer *no, it is not possible* reposes on an invariant—product of all four numbers. The product of four obtained numbers must be equal to the product of initial numbers; contradiction.

Geometry

Problem. A regular a) 7-gon, b) 25-gon is divided into triangles by drawing non-intersecting diagonals. Prove that at least three of these triangles are isosceles.

Solution. The first part can be dealt with by a case analysis, which is clearly impossible for the second part. The main idea of setting this problem is to demonstrate to students that the method of solution can change significantly if only minor quantitative changes are brought into the text. The second part is solved by applying the Dirichlet principle and the concept of "discrete continuity".

At the beginning, applying the Dirichlet principle, we ascertain the following fact: there are at least two triangles, two sides of which are also the sides of the 25-gon (not diagonals); these triangles are isosceles. If another such triangle exists, the problem is solved.



Let's presume that there exist exactly two such triangles. We consider the special sequence of triangles from one of these isosceles triangles to the second one, such that each two consecutive triangles have a common side. Then we draw a conclusion, based on the concept of "discrete continuity", considering the number of the sides of the polygon lying "at the left" of the current triangle.

Number theory

Problem. Does there exist a 10-digit number consisting only of digits 2 and 3 and divisible by 1024?

Solution. Components of mathematical induction are applied here.

The construction of such number is done by adding digits one by one to the beginning of the number and ensuring that n-digit number is dividing by 2^n. The residues modulo 2^n are used in the n-th step.

Combinatorics

Problem. Gods created the island of Trios which was inhabited by 2005 knights, 2006 dragons and 2007 princesses. Dragons eat princesses, knights kill dragons and princesses bring knights to ruination. Under the rules, drawn up by the gods, it is impossible to excommunicate of somebody who has killed an odd number of other inhabitants. Only one living being is left on the island of Trios today. Who is it?

Solution is based on the method of extremal element.

By considering all presumable answers—dragon, knight or princess—only the last one can be possible. Really, suppose that some knight survives. It follows that all the princesses were eaten. As there were an odd number of them, some particular dragon has eaten an odd number of princesses. So this dragon couldn't be killed by any knight—a contradiction. The case of the surviving dragon is treated in a similar way.

A very important feature of this problem is that an example showing the third possibility to be realisable is required.

It can happen this way: at first one dragon finishes up with 2006 princesses, then one knight kills 2006 dragons and finally the remaining princess, in her inability to decide, brings all 2005 knights to ruination.

Algorithms (Analysis of an algorithm)

Problem. There is a tight hair-grass in the spring. A column consisting of 3 red ants is moving along the hair-grass from the place A to the place B; a column consisting of 3 black ants is moving along the hair-grass from the place B to the place A. The velocities of all ants are the same; the distances between them can be different. Whenever two ants meet each other they immediately turn around and continue the movement. How much meetings will take place on the hair-grass?

Solution. This problem can be solved by interpretation, modelling the initial algorithm with another one. Regarding the method of interpretations see [8].

Let's consider another process when the ants don't turn around but pass each other. It is clear that for any place P and for any moment t there is an ant at P at the moment t during the original process if and only if it takes place during the introduced one. Therefore the number of meetings will be the same for both processes. But it is clear that there will be $3 \cdot 3 = 9$ meetings during the second process.

5 Some conclusions

The problems accumulated over many years are also included in advanced level textbooks for students aged 13–15, and the mathematics behind them has been reflected in now state-accepted standards developed for the period starting with 2008.

The most important educational issues collected based on our years of experience are as follows:

- There are approximately 30% of all students in early grades who do well in the high level math competitions.
- Regular extension work in the classroom is effective, e.g. giving individual problems to the better students once they have completed the obligatory tasks.
- Grade 7–8 are crucial in the development of young talent in mathematics.
- An early success in contests does not only reflect mathematical abilities; it can well be achieved on the basis of developed language skills.
- Combinatorial problems should make up the greatest part in contests for early grades, later reducing to approximately 40% of all problems.

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Auxiliary Constructions in Algebra: Learning to Think Creatively

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The "divide et impera" or "divide and conquer" principle is ascribed to Alexander of Macedonia. It can also be used in science and is of particular importance when a large problem should be decomposed into a series of minor ones. However, inverse transformation is no less crucial in mathematics, i.e. when a "part" should be complemented to achieve a "whole". The students often apply this technique in geometry, making successful complementary constructions. The same technique in algebra is less popular—but not less efficient. We intend to demonstrate it on a number of examples.

Let us begin with a humorous task.

Task 1. A stall keeper sells envelopes. Each standard package contains 100 envelopes. It takes the stall keeper one second to take out an envelope. A buyer wants to buy 75 envelopes. How much time will it take from the stall keeper to sell him 75 envelopes?

Solution. The answer 75 seconds is a possible but not time-saving solution. A stall keeper can count 75 envelopes within a shorter period. The complementation notion acts here as follows: number 75 is complemented to number 100 by 25. The stall keeper will count 25 envelopes within 25 seconds, take them out of the package, and the remaining 75 will be given to the buyer.

The answer is 25 seconds.

Note that folklore-based problems can also become the object of an interesting study.

Task 2. A rich shepherd left 17 camels as an inheritance to his three sons. He bequeathed half of the legacy to his elder son, one third to his middle son, and ninth part to his younger son. After the father had died, the sons began to think: 17 could not be divided by any of the three figures, i.e. 2, 3, and 9. How can they divide the inheritance?

Solution. It seems that the inheritance could not be divided justly. Moreover

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{9}=\frac{17}{18}.$$

The brothers asked a wise man for his advice. The wise man's solution actually struck them with its wisdom: "Take my camel and add him to your father's herd", said the wise man. From here, everything is extremely easy: the elder son took nine out of 18 camels, the middle one, six, and the younger one—two. Everyone was satisfied as they got more than was indicated in the will. And when the brothers took their camels ($9+6+2=17$), the wise man's camel went back to his owner.

It can be easily seen that the wise man's advice was based on the complement principle.

Let us investigate the issue whether the wise man's grandson could use his grandfather's experience in a similar everyday situation, i.e. let $\frac{1}{p}$, $\frac{1}{q}$, $\frac{1}{r}$, $p \le q \le r$ be the devised shares of an inheritance. A rich shepherd's herd has $(N-1)$ camels. From his grandfather's experience, the grandson advises the brothers to complement their father's herd by his camel, and then:

$$\frac{N}{p}+\frac{N}{q}+\frac{N}{r}+1=N. \qquad \text{Or} \qquad \frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\frac{1}{N}=1.$$

Using the exhaustion technique and the $p \le q \le r$ condition, we can obtain all feasible quadruples:

$$A=\left\{\begin{array}{l}(2,3,7,42)\,;(2,3,8,24)\,;(2,3,10,15)\,;(2,3,12,12)\,;\\(2,4,5,20)\,;(2,4,6,12)\,;(2,4,8,8)\,;(2,5,5,10)\,;(2,6,6,6)\,;\\(3,3,4,12)\,;(3,3,6,6)\,;(3,4,4,6)\,;(4,4,4,4)\end{array}\right\}$$

But the $B=\{(2,3,10,15)\,;(3,4,4,6)\}$ quadruples do not suit us: 6 cannot be divided by 4, and 15 cannot be divided by 2. Then, the following 12 versions are possible:

$$C=\left\{\begin{array}{l}(2,3,7,42)\,;(2,3,8,24)\,;(2,3,12,12)\,;\\(2,4,5,20)\,;(2,4,6,12)\,;(2,4,8,8)\,;(2,5,5,10)\,;(2,6,6,6)\,;\\(3,3,4,12)\,;(3,3,6,6)\,;(4,4,4,4)\end{array}\right\}$$

Now let us consider an arbitrary instance of $(2,4,5,20)$

Thus, the father could possess 19 camels. The elder son took no less than the half (10 camels), the second son had at least one fourth (5 camels), and the youngest son took more than the fifth part (4 camels). In total: $10 + 5 + 4 = 19$ camels. The remaining 20th camel is returned to its owner.

Now we can proceed to some other examples that illustrate the *complement to the whole* technique.

Task 3. Factorize the following expressions: 1) $x^4 + 1$; 2) $x^8 + x + 1$.

Solution.

1) In the first example, we will complement the expression to complete the square:

$$\begin{aligned} x^4 + 1 &= x^4 + 2x^2 + 1 - 2x^2 = \left(x^2 + 1\right)^2 - 2x^2 \\ &= \left(x^2 + 1 - x\sqrt{2}\right)\left(x^2 + 1 + x\sqrt{2}\right) \end{aligned}$$

2) In the second example, the complement is plotted “ladder-like”. It may look rather sophisticated but is easily understood:

$$\begin{aligned} x^8 + x + 1 &= \left(x^8 + x^7 + x^6\right) - \left(x^7 + x^6 + x^5\right) + \left(x^5 + x^4 + x^3\right) \\ &\qquad - \left(x^4 + x^3 + x^2\right) + \left(x^2 + x + 1\right) \\ &= \left(x^2 + x + 1\right)\left(x^6 - x^5 + x^3 - x^2 + 1\right) \end{aligned}$$

Task 4. (The Orange 2004 Olympiad, for the 11–12 grades.) Prove that

$$\frac{1}{1003} + \frac{3}{1004} + \cdots + \frac{2003}{2004} = \frac{2003}{2} - \frac{2002}{3} + \frac{2001}{4} - \cdots - \frac{2}{2003} + \frac{1}{2004}.$$

Solution. Here, the auxiliary construction is as follows: add number S to both parts of the assumed equality

$$S = \frac{2003}{2} + \frac{2002}{3} + \frac{2001}{4} + \cdots + \frac{1002}{1003} + \frac{1001}{1004} + \cdots + \frac{2}{2003} + \frac{1}{2004}.$$

We have

$$\left(\frac{1}{1003}+\frac{3}{1004}+\cdots+\frac{2003}{2004}\right)+S=$$
$$=\left(\frac{2003}{2}-\frac{2002}{3}+\frac{2001}{4}-\cdots-\frac{2}{2003}+\frac{1}{2004}\right)+S.$$

a) We will examine the left-hand part of the assumed equality

$$\begin{aligned}
&\left(\frac{1}{1003}+\frac{3}{1004}+\cdots+\frac{2003}{2004}\right)+S\\
&=\left(\frac{1}{1003}+\frac{3}{1004}+\cdots+\frac{2001}{2003}+\frac{2003}{2004}\right)\\
&+\left(\frac{2003}{2}+\frac{2002}{3}+\frac{2001}{4}+\cdots+\frac{1002}{1003}+\frac{1001}{1004}+\cdots+\frac{2}{2003}+\frac{1}{2004}\right)\\
&=\left(\frac{2003}{2004}+\frac{1}{2004}\right)+\left(\frac{2001}{2003}+\frac{2}{2003}\right)+\cdots+\left(\frac{3}{1004}+\frac{1001}{1004}\right)\\
&+\left(\frac{1}{1003}+\frac{1002}{1003}\right)+\frac{2003}{2}+\frac{2002}{3}+\frac{2001}{4}+\cdots+\frac{1003}{1002}\\
&=\underbrace{1+\cdots+1}_{1002}+\left(\frac{2003}{2}+\frac{2002}{3}+\frac{2001}{4}+\cdots+\frac{1003}{1002}\right)\\
&=1002+\left(\frac{2003}{2}+\frac{2002}{3}+\frac{2001}{4}+\cdots+\frac{1003}{1002}\right).
\end{aligned}$$

b) We will now examine the right-hand part of the assumed equality

$$\begin{aligned}
&\left(\frac{2003}{2}-\frac{2002}{3}+\frac{2001}{4}-\cdots-\frac{2}{2003}+\frac{1}{2004}\right)\\
&+\frac{2003}{2}+\frac{2002}{3}+\frac{2001}{4}+\cdots+\frac{1002}{1003}+\frac{1001}{1004}+\cdots+\frac{2}{2003}+\frac{1}{2004}\\
&=2\left(\frac{2003}{2}+\frac{2001}{4}+\frac{1999}{6}+\cdots+\frac{1003}{1002}+\frac{1001}{1004}+\cdots+\frac{3}{2002}+\frac{1}{2004}\right)
\end{aligned}$$

$$= 2003 + \frac{2001}{2} + \frac{1999}{3} + \cdots + \frac{1003}{501} + \frac{1001}{502} + \cdots + \frac{3}{1001} + \frac{1}{1002}.$$

It remains to show that the following equality is valid

$$\begin{aligned} 1002 + \left(\frac{2003}{2} + \frac{2002}{3} + \frac{2001}{4} + \cdots + \frac{1004}{1001} + \frac{1003}{1002}\right) \\ = 2003 + \frac{2001}{2} + \frac{1999}{3} + \cdots + \frac{3}{1001} + \frac{1}{1002}. \end{aligned}$$

Simplifying the equidenominator fractions, we have identity

$$1002 + \left(\frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \cdots + \frac{1002}{1002}\right) = 2003, \qquad 2003 = 2003.$$

The equality is proved.

Task 5. (Generalization of Task 2). Formulate and prove the initial task in general.

Solution. Having carefully examined the construction rule for the assumed equality

$$\frac{1}{1003} + \frac{3}{1004} + \cdots + \frac{2003}{2004} = \frac{2003}{2} - \frac{2002}{3} + \frac{2001}{4} - \cdots - \frac{2}{2003} + \frac{1}{2004},$$

we can generalize the task for any integer n

$$\begin{aligned} \frac{1}{2n+1} + \frac{3}{2n+2} + \frac{5}{2n+3} + \cdots + \frac{4n-1}{4n} \\ = \frac{4n-1}{2} - \frac{4n-2}{3} + \frac{4n-3}{4} - \cdots - \frac{2}{4n-1} + \frac{1}{4n} \end{aligned} \qquad (*)$$

For instance, for $n = 2$, we obtain the equality

$$\frac{1}{5} + \frac{3}{6} + \frac{5}{7} + \frac{7}{8} = \frac{7}{2} - \frac{6}{3} + \frac{5}{4} - \frac{4}{5} + \frac{3}{6} - \frac{2}{7} + \frac{1}{8}$$

and for $n = 501$, we obtain the desired task.

Certainly, the (*) equality can be proved by induction as well. But we will perform it generally, by means of the complement technique.

We will add S to both sides of the assumed equality

$$S = \frac{4n-1}{2} + \frac{4n-2}{3} + \frac{4n-3}{4} + \cdots + \frac{2}{4n-1} + \frac{1}{4n}.$$

We have

$$\left(\frac{1}{2n+1} + \frac{3}{2n+2} + \frac{5}{2n+3} + \cdots + \frac{4n-1}{4n}\right) + S = \left(\frac{4n-1}{2} - \frac{4n-2}{3} + \frac{4n-3}{4} - \cdots - \frac{2}{4n-1} + \frac{1}{4n}\right) + S$$

a) We will examine the left-hand side of the assumed equality

$$\begin{aligned}
&\left(\frac{1}{2n+1} + \frac{3}{2n+2} + \frac{5}{2n+3} + \cdots + \frac{4n-1}{4n}\right) + S \\
&= \left(\frac{1}{2n+1} + \frac{3}{2n+2} + \frac{5}{2n+3} + \cdots + \frac{2n-1}{3n}\right. \\
&\qquad \left. + \frac{2n+1}{3n+1} + \cdots + \frac{4n-5}{4n-2} + \frac{4n-3}{4n-1} + \frac{4n-1}{4n}\right) \\
&\qquad + \left(\frac{4n-1}{2} + \frac{4n-2}{3} + \frac{4n-3}{4} + \cdots + \frac{2n+1}{2n}\right. \\
&\qquad \left. + \frac{2n}{2n+1} + \frac{2n-1}{2n+2} + \cdots + \frac{3}{4n-2} + \frac{2}{4n-1} + \frac{1}{4n}\right) \\
&= \left(\frac{4n-1}{4n} + \frac{1}{4n}\right) + \left(\frac{4n-3}{4n-1} + \frac{2}{4n-1}\right) + \cdots + \left(\frac{3}{2n+2} + \frac{2n-1}{2n+2}\right) \\
&\quad + \left(\frac{1}{2n+1} + \frac{2n}{2n+1}\right) + \frac{4n-1}{2} + \frac{4n-2}{3} + \frac{4n-3}{4} + \cdots + \frac{2n+1}{2n} \\
&= \underbrace{1 + \cdots + 1}_{2n} + \left(\frac{4n-1}{2} + \frac{4n-2}{3} + \frac{4n-3}{4} + \cdots + \frac{2n+1}{2n}\right) \\
&= 2n + \left(\frac{4n-1}{2} + \frac{4n-2}{3} + \frac{4n-3}{4} + \cdots + \frac{2n+1}{2n}\right)
\end{aligned}$$

b) We will examine the right-hand side of the assumed equality

$$\begin{aligned}
&\left(\frac{4n-1}{2}-\frac{4n-2}{3}+\frac{4n-3}{4}-\cdots-\frac{2}{4n-1}+\frac{1}{4n}\right)+S\\
&=\left(\frac{4n-1}{2}+\frac{4n-2}{3}+\frac{4n-3}{4}+\cdots+\frac{2}{4n-1}+\frac{1}{4n}\right)\\
&=2\left(\frac{4n-1}{2}+\frac{4n-3}{4}+\frac{4n-5}{6}+\cdots+\frac{3}{4n-2}+\frac{1}{4n}\right)\\
&=(4n-1)+\frac{4n-3}{2}+\frac{4n-5}{3}+\cdots+\frac{3}{2n-1}+\frac{1}{2n}
\end{aligned}$$

It remains to show that

$$\begin{aligned}
2n+\left(\frac{4n-1}{2}+\frac{4n-2}{3}+\frac{4n-3}{4}+\cdots+\frac{2n+2}{2n-1}+\frac{2n+1}{2n}\right)&\\
=(4n-1)+\frac{4n-3}{2}+\frac{4n-5}{3}+\cdots+\frac{3}{2n-1}+\frac{1}{2n}&
\end{aligned}$$

Grouping the equidenominator fractions, we have

$$\begin{aligned}
&\left(\frac{4n-1}{2}-\frac{4n-3}{2}\right)+\left(\frac{4n-2}{3}-\frac{4n-5}{3}\right)+\cdots\\
&\qquad\cdots+\left(\frac{2n+2}{2n-1}-\frac{3}{2n-1}\right)+\left(\frac{2n+1}{2n}-\frac{1}{2n}\right)=(2n-1)
\end{aligned}$$

$$2n-1=2n-1$$

Thus, the equality is proved for any integer n.

It remains to explain how one can determine the type of relevant supplementary expression

$$S=\frac{4n-1}{2}+\frac{4n-2}{3}+\frac{4n-3}{4}+\cdots+\frac{2}{4n-1}+\frac{1}{4n}$$

It is evident that no general recommendations can be given. The authors determined the expression S by the following line of reasoning: to supplement the left-hand side of the known general formula

$$\begin{aligned}\frac{1}{2n+1}+\frac{3}{2n+2}+\frac{5}{2n+3}+\cdots+\frac{4n-1}{4n}&\\ =\frac{4n-1}{2}-\frac{4n-2}{3}+\frac{4n-3}{4}-\cdots-\frac{2}{4n-1}+\frac{1}{4n}.&\end{aligned}\qquad(*)$$

Task 6. Prove the inequality

$$\frac{200\cdot198}{199\cdot197}+\frac{200\cdot198\cdot196\cdot194}{199\cdot197\cdot195\cdot193}+\frac{200\cdot198\cdot196\cdot194\cdot192\cdot190}{199\cdot197\cdot195\cdot193\cdot191\cdot189}+\cdots$$
$$\cdots+\frac{200\cdot198\cdot196\cdot194\cdot\ \cdots\ \cdot4\cdot2}{199\cdot197\cdot195\cdot193\cdot\ \cdots\ \cdot3\cdot1}>100$$

(each fraction has an even number of factors in the numerator and the same even number—from 2 till 100—in the denominator).

Solution. We will introduce

$$S_1=\frac{200}{199}+\frac{200\cdot198\cdot196}{199\cdot197\cdot195}+\frac{200\cdot198\cdot196\cdot194\cdot192}{199\cdot197\cdot195\cdot193\cdot191}+\cdots$$
$$\cdots+\frac{200\cdot198\cdot196\cdot194\cdot\ \cdots\ \cdot4}{199\cdot197\cdot195\cdot193\cdot\ \cdots\ \cdot3}$$

$$S_2=\frac{200\cdot198}{199\cdot197}+\frac{200\cdot198\cdot196\cdot194}{199\cdot197\cdot195\cdot193}+\frac{200\cdot198\cdot196\cdot194\cdot192\cdot190}{199\cdot197\cdot195\cdot193\cdot191\cdot189}$$
$$+\cdots+\frac{200\cdot198\cdot196\cdot194\cdot\ \cdots\ \cdot4\cdot2}{199\cdot197\cdot195\cdot193\cdot\ \cdots\ \cdot3\cdot1}$$

Then

$$S_1+S_2=\frac{200}{199}+\frac{200\cdot198}{199\cdot197}+\frac{200\cdot198\cdot196}{199\cdot197\cdot195}+\ldots$$
$$+\frac{200\cdot198\cdot196\cdot\ \cdots\ \cdot8}{199\cdot197\cdot195\cdot\ \cdots\ \cdot7}+\frac{200\cdot198\cdot196\cdot\ \cdots\ \cdot8\cdot6}{199\cdot197\cdot195\cdot\ \cdots\ \cdot7\cdot5}$$
$$+\frac{200\cdot198\cdot196\cdot\ \cdots\ \cdot8\cdot6\cdot4}{199\cdot197\cdot195\cdot\ \cdots\ \cdot7\cdot5\cdot3}+\frac{200\cdot198\cdot196\cdot\ \cdots\ \cdot8\cdot6\cdot4\cdot2}{199\cdot197\cdot195\cdot\ \cdots\ \cdot7\cdot5\cdot3\cdot1}$$

Note that if the members of the series are added from the beginning to the end, it "folds" as a telescopic antenna.

Let us demonstrate it.

$$\begin{aligned}
&\frac{200\cdot 198\cdot 196\cdot\dots\cdot 4}{199\cdot 197\cdot 195\cdot\dots\cdot 3}+\frac{200\cdot 198\cdot 196\cdot\dots\cdot 4\cdot 2}{199\cdot 197\cdot 195\cdot\dots\cdot 3\cdot 1}\\
&=\frac{200\cdot 198\cdot 196\cdot\dots\cdot 4}{199\cdot 197\cdot 195\cdot\dots\cdot 3}+\frac{2008\cdot 2006\cdot 2004\cdot\dots\cdot 4}{2007\cdot 2005\cdot 2003\cdot\dots\cdot 3}\cdot 2\\
&=\frac{200\cdot 198\cdot 196\cdot\dots\cdot 6\cdot 4}{199\cdot 197\cdot 195\cdot\dots\cdot 5\cdot 3}\cdot 3=\frac{200\cdot 198\cdot 196\cdot\dots\cdot 6}{199\cdot 197\cdot 195\cdot\dots\cdot 5}\cdot 4
\end{aligned}$$

Now we will add the result to the third from the end member

$$\begin{aligned}
&\frac{200\cdot 198\cdot 196\cdot\dots\cdot 6}{199\cdot 197\cdot 195\cdot\dots\cdot 5}+\frac{200\cdot 198\cdot 196\cdot\dots\cdot 6}{199\cdot 197\cdot 195\cdot\dots\cdot 5}\cdot 4\\
&\qquad=\frac{200\cdot 198\cdot 196\cdot\dots\cdot 6}{199\cdot 197\cdot 195\cdot\dots\cdot 5}\cdot 5=\frac{200\cdot 198\cdot 196\cdot\dots\cdot 8}{199\cdot 197\cdot 195\cdot\dots\cdot 7}\cdot 6.
\end{aligned}$$

Continuing to "devour the number of members" in a similar way, we will come to the series beginning

$$\begin{aligned}
&\frac{200}{199}+\frac{200\cdot 198\cdot 196}{199\cdot 197\cdot 195}+\frac{200\cdot 198\cdot 196}{199\cdot 197\cdot 195}\cdot 194\\
&=\frac{200}{199}+\frac{200\cdot 198}{199\cdot 197}+\frac{200\cdot 198}{199\cdot 197}\cdot 196=\frac{200}{199}+\frac{200}{199}\cdot 198\\
&=\frac{200}{199}\cdot 199=200.
\end{aligned}$$

Thus, $S_1+S_2=200$.

It can be seen easily that each S_1 and S_2 sum has exactly 100 members and so that each S_2 sum member is bigger than the corresponding S_1 sum member. Therefore, $S_2>S_1$. Taking into account that $S_1+S_2=200$, we can conclude that $S_1<100$ or that $S_2>100$.

The readers who appreciate the generalization can easily perform it for any n divisible by 4.

Task 7. (*Quant* Journal, 1985, No. 1.) Prove that

$$N_1 = \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \cdots + \frac{1}{\sqrt{77}+\sqrt{78}} + \frac{1}{\sqrt{79}+\sqrt{80}} > 4.$$

Solution. We will use the complement principle and will add such N_2 number to N_1 that

$$N_2 = \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{5}+\sqrt{4}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \cdots + \frac{1}{\sqrt{79}+\sqrt{78}} + \frac{1}{\sqrt{81}+\sqrt{80}}$$

we'll obtain

$$\begin{aligned} S &= N_1 + N_2 \\ &= \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{79}+\sqrt{80}} + \frac{1}{\sqrt{80}+\sqrt{81}} \\ &= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \cdots + \frac{\sqrt{80}-\sqrt{79}}{80-79} + \frac{\sqrt{81}-\sqrt{80}}{81-80} \\ &= \sqrt{81} - 1 = 8. \end{aligned}$$

It is also easily seen that $N_1 > N_2$. Thus, $8 = S < 2N_1$. In this case $N_1 > 4$.

Task 8. (The Orange 2006 Olympiad, the 12$^{\text{th}}$ Grade, Israel.) Find the first 100 digits of the number

$$N = 3 + \frac{4}{9} + \frac{5}{9\cdot 11} + \frac{6}{9\cdot 11\cdot 13} + \frac{7}{9\cdot 11\cdot 13\cdot 15} + \cdots \cdots + \frac{1002}{9\cdot 11\cdot 13\cdot \cdots \cdot 2005} + \frac{1003}{9\cdot 11\cdot 13\cdot \cdots \cdot 2007}.$$

Solution. We will complement the number N by the fraction

$$\frac{1}{2\cdot 9\cdot 11\cdot 13\cdot \cdots \cdot 2007}.$$

We obtain

$$S = N + \frac{1}{2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2007},$$

$$S = 3 + \frac{4}{9} + \frac{5}{9 \cdot 11} + \frac{6}{9 \cdot 11 \cdot 13} + \cdots + \frac{1002}{9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2005} + \frac{1003}{9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2005 \cdot 2007} + \frac{1}{2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2005 \cdot 2007}.$$

Now, similarly to task 2, we perform addition of the fractions from the end to the beginning. The entire expression will "fold" as a telescopic antenna.

$$\begin{aligned}
S &= 3 + \frac{4}{9} + \frac{5}{9 \cdot 11} + \frac{6}{9 \cdot 11 \cdot 13} + \cdots \\
&\qquad + \frac{1002}{9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2005} + \frac{1}{2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2005} \\
&= 3 + \frac{4}{9} + \frac{5}{9 \cdot 11} + \frac{6}{9 \cdot 11 \cdot 13} + \cdots + \frac{1}{2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2003} \\
&\qquad \cdots \\
&= 3 + \frac{4}{9} + \frac{5}{9 \cdot 11} + \frac{6}{9 \cdot 11 \cdot 13} + \frac{1}{2 \cdot 9 \cdot 11 \cdot 13} \\
&= 3 + \frac{4}{9} + \frac{5}{9 \cdot 11} + \frac{1}{2 \cdot 9 \cdot 11} = 3 + \frac{4}{9} + \frac{1}{2 \cdot 9} = 3.5
\end{aligned}$$

Therefore

$$N = 3.5 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2007} = 3.4999\ldots$$

We took into account that $2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2007 > 10^{997}$. Hence $0 < \frac{1}{2 \cdot 9 \cdot 11 \cdot 13 \cdot \cdots \cdot 2007} < 10^{-997}$. Answer is $3.4\underbrace{9999\ldots9}_{99}\ldots$

The complement principle is also applicable in the field of inequalities.

Task 9. (*Quant* Journal, 2006, No. 5.)
a) Find some natural value of n, for which the following inequality holds

$$\frac{1}{2} \cdot \frac{5}{6} \cdot \frac{9}{10} \cdot \cdots \cdot \frac{4n-3}{4n-2} \cdot \frac{4n+1}{4n+2} < \frac{1}{2005} < \frac{4}{5} \cdot \frac{8}{9} \cdot \frac{12}{13} \cdot \cdots \cdot \frac{4n}{4n+1} \cdot \frac{4n+4}{4n+5}.$$

b) Find some natural value of n, for which the following inequality holds

$$\frac{1}{2}\cdot\frac{5}{6}\cdot\frac{9}{10}\cdot\dots\cdot\frac{2005}{2006}<\sqrt[4]{\frac{1}{4n+5}}<\frac{4}{5}\cdot\frac{8}{9}\cdot\frac{12}{13}\cdot\dots\cdot\frac{2008}{2009}.$$

Solution.

a)

$$x=\frac{1}{2}\cdot\frac{5}{6}\cdot\frac{9}{10}\cdot\dots\cdot\frac{4n-3}{4n-2}\cdot\frac{4n+1}{4n+2},$$
$$y=\frac{2}{3}\cdot\frac{6}{7}\cdot\frac{10}{11}\cdot\dots\cdot\frac{4n-2}{4n-1}\cdot\frac{4n+2}{4n+3},$$
$$z=\frac{3}{4}\cdot\frac{7}{8}\cdot\frac{11}{12}\cdot\dots\cdot\frac{4n-1}{4n}\cdot\frac{4n+3}{4n+4},$$
$$t=\frac{4}{5}\cdot\frac{8}{9}\cdot\frac{12}{13}\cdot\dots\cdot\frac{4n}{4n+1}\cdot\frac{4n+4}{4n+5}.$$

For arbitrary integers $k>n>0$, the inequality $\frac{k}{k+1}>\frac{n}{n+1}$ holds which implies $x<y<z<t$. Thus

$$x^4<x\cdot y\cdot z\cdot t=\frac{1\cdot 2\cdot 3\cdot 4\cdot\dots\cdot(4n+1)\cdot(4n+2)\cdot(4n+3)\cdot(4n+4)}{2\cdot 3\cdot 4\cdot\dots\cdot(4n+2)\cdot(4n+3)\cdot(4n+4)\cdot(4n+5)}$$
$$=\frac{1}{4n+5}<t^4,$$

$$x^4<\frac{1}{4n+5}<t^4\Leftrightarrow x<\sqrt[4]{\frac{1}{4n+5}}<t \qquad (**)$$

if $\sqrt[4]{\frac{1}{4n+5}}=\frac{1}{2005}$, then $4n+5=2005^4\Leftrightarrow n=\frac{2005^4-5}{4}\in\mathbb{N}$.

Answer: for instance, $n=\frac{2005^4-5}{4}$.

b)

$$\frac{1}{2}\cdot\frac{5}{6}\cdot\frac{9}{10}\cdot\dots\cdot\frac{2005}{2006}<\sqrt[4]{\frac{1}{4n+5}}<\frac{4}{5}\cdot\frac{8}{9}\cdot\frac{12}{13}\cdot\dots\cdot\frac{2008}{2009}$$

From the general solution of the previous inequality (**) we obtain

$$\frac{1}{2}\cdot\frac{5}{6}\cdot\frac{9}{10}\cdot\dots\cdot\frac{4n-3}{4n-2}\cdot\frac{4n+1}{4n+2} < \sqrt[4]{\frac{1}{4n+5}} < \frac{4}{5}\cdot\frac{8}{9}\cdot\frac{12}{13}\cdot\dots\cdot\frac{4n}{4n+1}\cdot\frac{4n+4}{4n+5}.$$

Hence it follows that for $n = 501$ we obtain

$$\sqrt[4]{\frac{1}{4n+5}} = \sqrt[4]{\frac{1}{2009}}.$$

Answer: $n = 501$.

Task 10. (The Orange 2005 Olympiad, the 11–12th grades, Israel) Arrange the A, B, C numbers in ascending order

$$A = 2; \qquad B = \frac{8000}{7999}\cdot\frac{7997}{7996}\cdot\frac{7994}{7993}\cdot\dots\cdot\frac{1004}{1003}\cdot\frac{1001}{1000};$$

$$C = \sqrt[3]{\sqrt{576}-\sqrt{575}+\sqrt{573}-\sqrt{572}+\sqrt{570}-\sqrt{569}+\dots+\sqrt{3}-\sqrt{2}}.$$

Solution.

1. We will examine the following complements

$$B_1 = \frac{7998}{7997}\cdot\frac{7995}{7994}\cdot\frac{7992}{7991}\cdot\dots\cdot\frac{1005}{1004}\cdot\frac{1002}{1001};$$
$$B_2 = \frac{7999}{7998}\cdot\frac{7996}{7995}\cdot\frac{7993}{7992}\cdot\dots\cdot\frac{1006}{1005}\cdot\frac{1003}{1002}.$$

It is evident that $B\cdot B_1\cdot B_2 = 8$. It is also evident that $B > B_1 > B_2$. Therefore, $8 = B\cdot B_1\cdot B_2 < B^3$. Hence, $B > 2$. But then

$$2 = A < B.$$

2. Prove that $C < A$. Denote

$$D = C^3 = \sqrt{576}-\sqrt{575}+\sqrt{573}-\sqrt{572}+\sqrt{570}-\sqrt{569}+\dots+\sqrt{3}-\sqrt{2}.$$

Now we can use the auxiliary construction

$$D_1 = \sqrt{575}-\sqrt{574}+\sqrt{572}-\sqrt{571}+\sqrt{569}-\sqrt{568}+\dots+\sqrt{2}-\sqrt{1},$$
$$D_2 = \sqrt{574}-\sqrt{573}+\sqrt{571}-\sqrt{570}+\sqrt{568}-\sqrt{567}+\dots+\sqrt{1}-\sqrt{0}.$$

It is evident that $D < D_1 < D_2$. It is also evident that $D+D_1+D_2 = 24$. Hence, $D < 8$, which means that $C < 2$.

Task 11. (Tournament of Towns, 2006) Prove that an infinite number of positive integers can be found so that all the digits in the decimal notation of each number are at least 7 and simultaneously the product of each pair of them is also the number where all the digits would be at least 7.

Solution. Let's prove that for any integer $k \geq 0$ a couple of numbers $m = 8\underbrace{9\ldots9}_{k\text{ times}}87$; $n = 8\underbrace{7\ldots7}_{(k+2)\text{ times}}$ satisfy the task conditions.

It is evident that $m = 8\underbrace{9\ldots9}_{k\text{ times}}87 = 9\cdot 10^{k+2} - 13$,

$$\begin{aligned}
n &= 8\underbrace{7\ldots7}_{(k+2)\text{ times}} = 8\cdot 10^{k+2} + \underbrace{7\ldots7}_{(k+2)\text{ times}} \\
&= 8\cdot 10^{k+2} + \left(\underbrace{7\ldots7}_{(k+2)\text{ times}} + \underbrace{2\ldots22}_{(k+2)\text{ times}} + 1\right) - \underbrace{2\ldots22}_{(k+2)\text{ times}} - 1 \\
&= 9\cdot 10^{k+2} - \underbrace{2\ldots22}_{(k+2)\text{ times}} - 1 = 9\cdot 10^{k+2} - \frac{2}{9}\cdot\left(10^{k+2}-1\right) - 1 \\
&= \frac{79}{9}\cdot 10^{k+2} - \frac{7}{9}.
\end{aligned}$$

Therefore, $n = \frac{79}{9}\cdot 10^{k+2} - \frac{7}{9}$ and $m\cdot n = 8\underbrace{9\ldots9}_{k\text{ times}}87\cdot 8\underbrace{7\ldots7}_{(k+2)\text{ times}}$.

$$\begin{aligned}
m\cdot n &= \left(9\cdot 10^{k+2} - 13\right)\left(\frac{79}{9}\cdot 10^{k+2} - \frac{7}{9}\right) \\
&= 79\cdot 10^{2k+4} - 10^{k+2}\cdot\left(\frac{79\cdot 13}{9} + 7\right) + \frac{91}{9} \\
&= 79\cdot 10^{2k+4} - \left(\frac{1090\cdot 10^{k+2} - 91}{9}\right) \\
&= 79\cdot 10^{2k+4} - \left(\frac{1089\cdot 10^{k+2}}{9} + \frac{10^{k+2}-1}{9} - 10\right)
\end{aligned}$$

$$\begin{aligned}
m \cdot n &= 79 \cdot 10^{2k+4} - \left(121 \cdot 10^{k+2} + \underbrace{11\ldots1}_{(k+2)\,\text{times}} - 10\right) \\
&= 79 \cdot 10^{2k+4} - \left(121 \cdot 10^{k+2} + \underbrace{11\ldots101}_{(k+2)\,\text{times}}\right) \\
&= 79 \cdot 10^{2k+4} - 121\underbrace{11\ldots101}_{(k+2)\,\text{times}} = 79 \cdot 10^{k+2} \cdot 10^{k+2} - 121\underbrace{11\ldots101}_{(k+2)\,\text{times}}.
\end{aligned}$$

If $k = 0$, we obtain: $m \cdot n = 79 \cdot 10^2 \cdot 10^2 - 12101 = 777899 = 887 \cdot 877$.

$$\begin{array}{r}
79\ \underbrace{0\ldots000}_{(k+2)\,\text{times}}\ \underbrace{00\ldots000}_{(k+2)\,\text{times}} \\
-\qquad 121\,\underbrace{11\ldots101}_{(k+2)\,\text{times}} \\
\hline
789\ldots9878\,88\ldots899
\end{array}$$

Task 12. What digit will be in the 98th place after the decimal point in the following number $M = \left(\sqrt{2}+1\right)^{500}$?

Solution. Let's prove the following Lemma: For any natural n, the algebraic expression $A(n) = \left(\sqrt{2}+1\right)^{2n} + \left(\sqrt{2}-1\right)^{2n}$ gives only natural values.

Using Newtons' binomial expansion, we have

$$\begin{aligned}
A(n) &= \left(\sqrt{2}+1\right)^{2n} + \left(\sqrt{2}-1\right)^{2n} \\
&= \Big[\left(\sqrt{2}\right)^{2n} + C_{2n}^{1} \cdot \left(\sqrt{2}\right)^{2n-1} + C_{2n}^{2} \cdot \left(\sqrt{2}\right)^{2n-2} + \cdots \\
&\qquad + C_{2n}^{2n-k} \cdot \left(\sqrt{2}\right)^{2n-k} + \cdots + 1\Big] \\
&\quad + \Big[\left(\sqrt{2}\right)^{2n} - C_{2n}^{1} \cdot \left(\sqrt{2}\right)^{2n-1} + C_{2n}^{2} \cdot \left(\sqrt{2}\right)^{2n-2} + \cdots \\
&\qquad + (-1)^k \cdot C_{2n}^{2n-k} \cdot \left(\sqrt{2}\right)^{2n-k} + \cdots + 1\Big]
\end{aligned}$$

$$A(n) = 2\left(2^n + \cdots + C_{2n}^{2n-2k} \cdot 2^{2n-2k} + \cdots + 1\right).$$

Hence the algebraic expression $A(n) = \left(\sqrt{2}+1\right)^{2n} + \left(\sqrt{2}-1\right)^{2n}$ obtains only natural values. Now, we can introduce our initial expression M

$$M = \left(\sqrt{2}+1\right)^{500} = A(250) - \left(\sqrt{2}-1\right)^{500}.$$

We know that $A(250) = N$ and N is an integer. It is clear that $0 < \sqrt{2}-1 < \frac{1}{2}$. Now we have

$$0 < l = \left(\sqrt{2}-1\right)^{500} < \left(\frac{1}{2}\right)^{500} = \left(\frac{1}{16}\right)^{125} < \left(\frac{1}{10}\right)^{125} = 0.\underbrace{00\ldots00}_{124 \text{ digits}},$$

$$M = \left(\sqrt{2}+1\right)^{500} = A(250) - \left(\sqrt{2}-1\right)^{500} = (N-1) + (1-l),$$

$$1 - l = 0.\underbrace{99\ldots99}_{124 \text{ digits}}\ldots$$

Answer: 9

Task 13. (Zota Olympiad 2006, Israel) Every second a computer program chooses two (a and b) of the following numbers: $1, 2, 3, \ldots, 1001$, erases them and writes a new number equal to $(a + b + a \cdot b)$. The program will stop when only one number remains. What are the last seven digits in the last number?

Solution. Let's prove the following Lemma: For given numbers $a_1, a_2, \ldots, a_n$ we'll let $I_n = (1 + a_1) \cdot (1 + a_2) \cdot \cdots \cdot (1 + a_{n-1}) \cdot (1 + a_n)$. If instead of two numbers a_1 and a_2 we will write the number $b_2 = a_1 + a_2 + a_1 \cdot a_2$, then we will have a new sequence $b_2, a_3, a_4, \ldots, a_n$ where the following equivalence exists $I_{n-1} = I_n$.

Proof.

$$\begin{aligned}
I_{n-1} &= (1 + b_2) \cdot (1 + a_3) \cdot \cdots \cdot (1 + a_{n-1}) \cdot (1 + a_n) \\
&= (a_1 + a_2 + a_1 \cdot a_2 + 1) \cdot (1 + a_3) \cdot \cdots \cdot (1 + a_{n-1}) \cdot (1 + a_n) \\
&= (1 + a_1) \cdot (1 + a_2) \cdot \cdots \cdot (1 + a_{n-1}) \cdot (1 + a_n) = I_n.
\end{aligned}$$

According to the Lemma, we can conclude: $I_n = I_{n-1} = I_{n-2} = \cdots = I_1$. In our case we have

$$\begin{aligned} I_n &= (1 + a_1) \cdot (1 + a_2) \cdot \cdots \cdot (1 + a_{n-1}) \cdot (1 + a_n) \\ &= (1 + 1) \cdot (1 + 2) \cdot \cdots \cdot (1 + 1000) \cdot (1 + 1001) = 1002! \end{aligned}$$

If the last number is b_{1001} and $(1 + b_{1001}) = I_1 = I_n = 1002!$, then $b_{1001} = 1002! - 1$. It is clear that last seven digits will be 9999999.

Conclusion

We would like to conclude with a quote from Descartes:

Nobody can assert that I want to suggest here the method that everyone should follow ... I only want to clarify the method I followed myself.

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The Erdös Awarding at the WFNMC5 Congress in Cambridge

Alexander Soifer (right) receives the Paul Erdös Award from Petar Kenderov

Alexander Soifer and Riccardo de Losada in the Library of Trinity College

Tournament of Towns Corner

Andrei Storozhev

Andrei Storozhev is a Research Officer at the Australian Mathematics Trust. He gained his PhD at Moscow State University specializing in combinatorial group theory. He is a member of the Australian Mathematics Competition Problems Committee, Australian Mathematics Olympiad Committee and one of the editors of the 'Mathematics Contests—The Australian Scene' *journal.*

1 Selected Problems from the First Round of Tournament 28

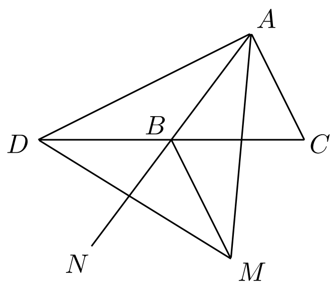
In the first round of Tournament 28, both Junior and Senior O Level papers consisted of five problems, while both Junior and Senior A Level papers were made up of seven problems. Below are selected questions with solutions from the first round of Tournament 28.

1. Three positive integers x, y and z are written on the blackboard. Mary records in her notebook the product of any two of them and reduces the third number on the blackboard by 1. With the new trio of numbers, she repeats the process, and continues until one of the numbers on the blackboard becomes zero. What will be the sum of the numbers in Mary's notebook at that point?

   Solution. We claim that the sum of the numbers in Mary's notebook is equal to the product of the three numbers originally on the blackboard. We use induction on the number n of steps for Mary to reduce one of the numbers to 0. For $n = 1$, one of the three numbers on the blackboard must be equal to 1 and is reduced to 0. In recording the product of the other two numbers, Mary is in fact recording the product of all three numbers. Suppose the claim holds for some $n \geq 1$. Let the original numbers be x, y and z. By symmetry, we may assume that Mary records xy in her notebook and replaces z by $z-1$. By the induction hypothesis, the sum of the remaining numbers in her notebook is equal to $xy(z-1)$, so that the sum of all the numbers in her notebook is equal to $xy + xy(z-1) = xyz$.

2. Given triangle ABC, BC is extended beyond B to the point D such that $BD = BA$. The bisectors of the exterior angles at vertices B and C intersect at the point M. Prove that A, D, M and C are concyclic.

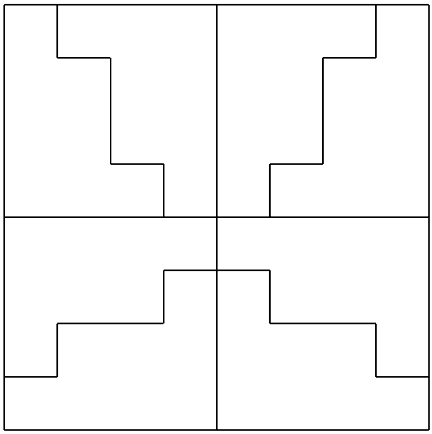
Solution.



Note that $\angle NBD = \angle ABC$ and $\angle NBM = \angle CBM$. Hence $\angle DBM = \angle ABM$. Since we also have $BD = BA$ and $BM = BM$, triangles DBM and ABM are congruent, so that $\angle MDC = \angle MAN$. Now M is an excentre of triangle ABC. Hence $\angle MAN = \angle MAC$. From $\angle MDC = \angle MAC$, we can conclude that A, C, M and D are concyclic.

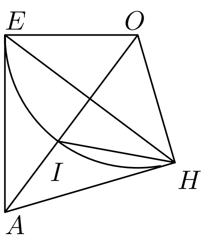
3. A square is dissected into n congruent non-convex polygons whose sides are parallel to the sides of the square, and no two of these polygons are parallel translations of each other. What is the maximum value of n?

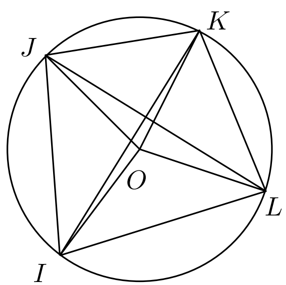
Solution. The maximum value of n is at most 8 because such a polygon can only have 8 possible orientations. We may use each of them once as otherwise we would have two copies which are parallel translations of each other. The maximum value is in fact 8 as it is attained by the polygon in the diagram below.



4. The incircle of the quadrilateral $ABCD$ touches AB, BC, CD and DA at E, F, G and H respectively. Prove that the line joining the incentres of triangles HAE and FCG is perpendicular to the line joining the incentres of triangles EBF and GDH.

Solution.



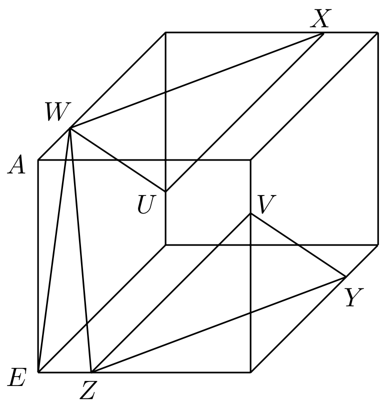


Let O be the incentre of $ABCD$. Let AO intersect the incircle of $ABCD$ at I. Let $\angle AOH = \angle AOE = 2\alpha$. Since $\angle AHO = 90^\circ = \angle AEO$, A, E, O and H are concyclic, so that $\angle AHE = \angle AOE = 2\alpha$. We have $\angle OAH = 180^\circ - \angle AOH - \angle AHO = 90^\circ - 2\alpha$ and since $OH = OI$, $\angle OIH = \frac{1}{2}(180^\circ - \angle IOH) = 90^\circ - \alpha$. It follows that $\angle AHI = \angle OIH - \angle OAH = \alpha = \frac{1}{2}\angle AHE$. Hence I is the incentre of triangle HAE. Similarly, the respective incentres J, K

and L of triangles EBF, FCG and GDH all lie on the incircle of $ABCD$. Let $\angle BOE = \angle BOF = 2\beta$, $\angle COF = \angle COG = 2\gamma$ and $\angle DOG = \angle DOH = 2\delta$. Then $\angle IOJ + \angle KOL = 2\alpha + 2\beta + 2\gamma + 2\delta = 180°$. Now $\angle ILJ + \angle KIL = \frac{1}{2}(\angle IOJ + \angle KOL) = 90°$. Hence IK and JL are perpendicular to each other.

5. Can a regular octahedron be inscribed in a cube in such a way that all vertices of the octahedron are on cube's edges?

Solution. The task is possible. Let the side length of the cube be 4. In the diagram below, each of U, V, W, X, Y and Z is at a distance 1 from the nearest vertex of the cube. Clearly, UWX and VYZ are equilateral triangles with side length $3\sqrt{2}$. Note that $\angle EAW = 90° = \angle WEZ$. Hence $WZ = \sqrt{EZ^2 + EA^2 + WA^2} = 3\sqrt{2}$ also. By symmetry, WV, XV, XY, UY and UZ all have the same length. It follows that $UVWXYZ$ is indeed a regular octahedron.



2 World Wide Web

Information on the Tournament, how to enter it, and its rules are on the World Wide Web. Information on the Tournament can be obtained from the Australian Mathematics Trust web site at

http://www.amt.edu.au

3 Books on Tournament Problems

There are four books on problems of the Tournament available. Information on how to order these books may be found in the Trust's advertisement elsewhere in this journal, or directly via the Trust's web page.

Please note the Tournament's postal address in Moscow:

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RUSSIA

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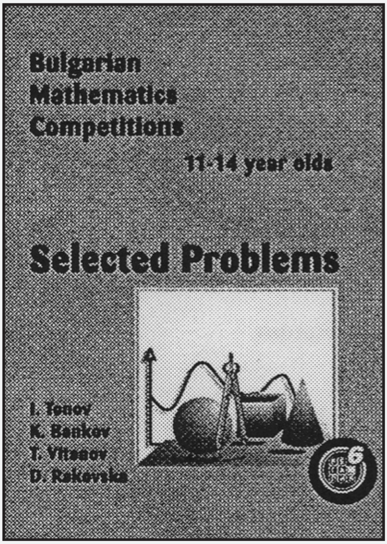

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(11-14 year olds)

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The International Mathematics Tournament of Towns is a problem solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. Each book contains problems and solutions from past papers.

Challenge! 1991 – 1995
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The Mathematics Challenge for Young Australians attracts thousands of entries from Australian High Schools annually and involves solving six in depth problems over a 3 week period. In 1991-95, there were two versions – a Junior version for Year 7 and 8 students and an Intermediate version for Year 9 and 10 students. This book reproduces the problems from both versions which have been set over the first 5 years of the event, together with solutions and extension questions. It is a valuable resource book for the class room and the talented student.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

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This book is a complete collection of all Australian Mathematical Olympiad papers since the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads 1981-1993 and 1993-2001
A Liu

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Asian Pacific Mathematics Olympiads 1989-2000
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With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981-1995
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Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
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Hungary Israel Mathematics Competition
S Gueron

This 181 page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions. The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Bulgarian Mathematics Competition 1992-2001
BJ Lazarov, JB Tabov, PJ Taylor, AM Storozhev

The Bulgarian Mathematics Competition has become one of the most difficult and interesting competitions in the world. It is unique in structure, combining mathematics and informatics problems in a multi-choice format. This book covers the first ten years of the competition complete with answers and solutions. Students of average ability and with an interest in the subject should be able to access this book and find a challenge.

Mathematical Contests – Australian Scene
Edited by AM Storozhev, JB Henry & DC Hunt

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

WFNMC — Mathematics Competitions
Edited by Jaroslav Švrček

This is the journal of the World Federation of National Mathematics Competitions (WFNMC). With two issues each of approximately 80-100 pages per year, it consists of articles on all kinds of mathematics competitions from around the world.

Parabola incorporating Function

This Journal is published in association with the School of Mathematics, University of New South Wales. It includes articles on applied mathematics, mathematical modelling, statistics, history and pure mathematics that can contribute to the teaching and learning of mathematics at the senior secondary school level. The Journal's readership consists of mathematics students, teachers and researchers with interests in promoting excellence in senior secondary school mathematics education.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Education, Science and Training) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

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