VOLUME 17 NUMBER 2 2004

MATHEMATICS COMPETITIONS

JOURNAL OF THE
WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

MATHEMATICS COMPETITIONS

JOURNAL OF THE WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

(ISSN 1031 – 7503)

is published biannually by

AMT PUBLISHING
UNIVERSITY OF CANBERRA ACT 2601
AUSTRALIA

Articles (in English) are welcome.
Please send articles to:

The Editor, Mathematics Competitions
Australian Mathematics Trust
University of Canberra ACT 2601
Australia

Fax:+61-2-6201-5052

or

Dr Jaroslav Švrček
Dept. of Algebra and Geometry
Palacký University of Olomouc
Tomkova 40
779 00 Olomouc
Czech Republic

Email:svrcek@inf.upol.cz

Mathematics Competitions Vol 17 No 2 2004

TABLE OF CONTENTS

Contents Page

World Federation of National Mathematics Competitions

Executive

President: Professor Petar Kenderov
Institute of Mathematics
Acad. G. Bonchev Str. bl. 8
1113 Sofia
BULGARIA

Vice Presidents: Dr Tony Gardiner (Senior)
School of Mathematics & Statistics
The University of Birmingham
Birmingham B15 2TT
UNITED KINGDOM

Professor Maria de Losada
Universidad Antonio Narino
Carrera 55 # 45-45
Bogota
COLOMBIA

Professor Peter Crippin
Faculty of Mathematics
University of Waterloo
Waterloo N2L 3G1 Ontario
CANADA

Publications Officer: Dr Jaroslav Švrček
Dept. of Algebra and Geometry
Palacký University, Olomouc
CZECH REPUBLIC

The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the Editor

Welcome to Mathematics Competitions Vol. 17, No 2.

This is the first issue for which I am the editor. First of all I would like to thank the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:

The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

The Editor, *Mathematics Competitions*
Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA

or to

Dr Jaroslav Švrček
Dept. of Algebra and Geometry
Palacky University of Olomouc
Tomkova 40
779 00 OLOMOUC
CZECH REPUBLIC

svrcek@inf.upol.cz

Jaroslav Švrček,
December 2004

20$^{\text{th}}$ Anniversary of the WFNMC

The World Federation of National Mathematics Competitions (WFNMC) was founded in 1984 during the Fifth International Congress on Mathematical Education (ICME 5) in Adelaide, Australia. It was created through the inspiration of Professor Peter O'Halloran from Australia. Here is what Ron G. Dunkley, one of the former Presidents of WFNMC, wrote of this event in his report about the activities of the organization for the period 1992–1996 in ICMI Bulletin No 40: "...While others assisted in the formation, it was the vision and leadership of Professor Peter O'Halloran of Canberra, Australia, that led directly to the Federation's being".

Professor Peter Taylor, the Past President of WFNMC, wrote in ICMI Bulletin No 49: "The founder of WFNMC was Peter O'Halloran, who was President until his death in 1994. He conceived the idea of such an organization in which mathematicians from different countries could compare their experiences and hopefully improve their activities as a result."

A significant part of what are now the major activities of the Federation were initiated by Professor Peter O'Halloran. He recognised the importance of communication and began the publication of *Newsletter of WFNMC* later to become known as the journal *Mathematics Competitions*. He established the Awards of the Federation intended to recognize people with significant achievements in developing Mathematical Enrichment Programs. He understood the importance of regular meetings for an international organization and, in 1990, the first Federation Conference was held in Waterloo, Canada. They are now held every four years with subsequent Conferences having been held in Pravetz, Bulgaria (1994), Zhong Shan, P. R. China (1998) and Melbourne, Australia (2002).

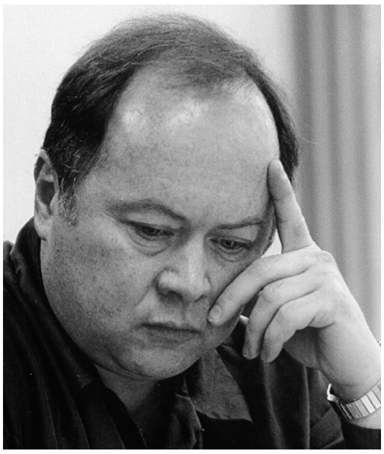
WFNMC has a permanent time-slot in the Programme of every International Congress on Mathematical Education (ICME). The latter takes place every even year between the Conferences of WFNMC and is organized by the International Commission on Mathematical Instruction (ICMI). In 1994 WFNMC became an Affiliated Study Group of ICMI, a status enjoyed by only three other organizations in the world. All of the members of WFNMC are part of this success and can take pride in that. Two things however, should be kept in mind. First, we must remember that the solid base for the rise and development of the Federation is the support of the *Australian Mathematics Trust*. Without the support, devotion and understanding of our Australian colleagues, the WFNMC would have hardly had the position it enjoys today. Second, is that it always makes sense to critically and permanently assess separate aspects of what has been achieved till now with the intention to extend the scope of the Federation and to present its proper face to society in general and to the mathematical community in particular. Only in this way can the future success of the organization be assured. A special role in this direction is played by the journal *Mathematics Competitions* that presents "state of the art" while keeping traditions and provides an encouraging place for discussions. It also disseminates the new and fruitful ideas coming from different parts of the world. It is a proper moment to most cordially thank Warren Atkins for his remarkable (20 years long!) work as Editor of the highly appreciated journal *Mathematics Competitions*. Warren Atkins' efforts created an excellent reputation for this publication of WFNMC and turned it into an important tool for enhancement of all activities associated with mathematics competitions. Thank you Warren!

At the meeting of WFNMC during the International Congress of Mathematical Education in Copenhagen (July 4–11, 2004), at the request of Warren Atkins, the role of Editor of *Mathematics Competitions* was passed to Jaroslav Švrček from Palacký University in Olomouc, Czech Republik. On behalf of the Executive Committee of WFNMC, I would like to congratulate our colleague Jaroslav Švrček on this occasion and to wish him successful editorial work for the wellbeing of the Journal and the WFNMC.

Petar S. Kenderov
President of WFNMC
December, 2004, Sofia, Bulgaria

A Journey from Ramsey Theory to Mathematical Olympiad to Finite Projective Planes[1]

Alexander Soifer[2]



Alexander Soifer is a professor at the University of Colorado, who has just completed his two-year service as a visiting researcher at Princeton University and Rutgers University (2003–2004). He is Chair and founder of Colorado Mathematical Olympiad, *which is in its twenty-first year, a member of* USA Mathematics Olympiad Subcommittee *(1996–2005), Secretary of the* World Federation of National Mathematics Competitions *(1996–2008), and Editor and Publisher of the research Quarterly* Geombinatorics *(1991–present).*

More at http://www.uccs.edu/ asoifer

New Olympiad problems occur to us in mysterious ways. This problem came to me one summer morning of 2003 as I was reading unpublished 1980s manuscript of a Ramsey Theory monograph, while sitting by a mountain lake in the Bavarian Alps. It all started with my finding a hole in a lemma, which prompted a construction of a counterexample (part b of the present problem). Problem (a) is a corrected particular case of that lemma, translated, of course, into a language of a nice "real" story of a chess tournament. I found three distinct striking solutions of (a) and an even more special solution of (b). As a result, this problem became the most beautiful Olympiad problem I have ever created. What is more, the journey that led me from Ramsey Theory to problems of mathematical Olympiads, continued to finite projective planes!

[1] A version of this essay was presented at the X International Congress on Mathematical Education, Copenhagen, 2004.

[2] Thanks to DIMACS for a Long Term Visiting Scholar appointment and Princeton University for a Visiting Fellowship.

Chess 7×7

(21st Colorado Mathematical Olympiad, April 16, 2004, A. Soifer.)

(a) Each member of two 7-member chess teams is to play once against each member of the opposing team. Prove that as soon as 22 games have been played, we can choose 4 players and seat them at a round table so that each pair of neighbors has already played.

(b) Prove that 22 is the best possible; i.e. after 21 games the result of (a) cannot be guaranteed.

(a) Solution I

This solution exploits an algebraic description of convexity. Given an array of real numbers $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ of arithmetic mean $\bar{x}$, a well known inequality (that can be derived from arithmetic-geometric mean inequality) states that

$$\sqrt{\frac{\sum_{i=1}^{7} x_i^2}{7}} \geq \bar{x}.$$

Thus

$$\sum_{i=1}^{7} x_i^2 \geq 7\bar{x}^2.$$

This inequality defines "convexity" of the function $f(x) = x^2$, which easily implies convexity of a binomial function $\binom{x}{2} = \frac{1}{2}x(x-1)$, i.e.

$$\sum_{i=1}^{7} \binom{x_i}{2} \geq 7\binom{\bar{x}}{2}. \tag{1}$$

Observe that above we defined the *binomial function* $\binom{x}{2}$ for all real x (not just for positive integers). Also, in a certain informality of notations, for integral x we would use $\binom{x}{2}$ not only as a number, but also as a set of all 2-element subsets of the set $\{1, 2, \ldots, x\}$.

Let us call players in each team positive integers $1, 2, \ldots, 7$. A game between player i of the first team with a player j of the second team can conveniently be denoted by an ordered pair (i, j). Assume that the set G of 22 games has been played.

Denote by $S(j)$ the number of games played by the player j of the second team: $S(j) = |\{i\colon (i, j) \in G\}|$. Obviously, $\sum_{i=1}^{7} S(j) = 22$.

For a pair (i_1, i_2) of first team players denoted by $C(i_1, i_2)$ the number of second team players j, who played with *both* of this pair's first team players: $C(i_1, i_2) = |\{j\colon (i_1, j) \in G \wedge (i_2, j) \in G\}|$. Adding all $C(i_1, i_2)$ together $T = \sum_{(i_1,i_2)\in\binom{7}{2}} C(i_1, i_2)$ counts the number of triples $(i_1, i_2; j)$ such that each of the first team's players i_1, i_2 has played with the same player j of the second team. This number T can be alternatively calculated as follows: $T = \sum_{i=1}^{7} \binom{S(j)}{2}$. Therefore, we get the equality $\sum_{(i_1,i_2)\in\binom{7}{2}} C(i_1, i_2) = \sum_{i=1}^{7} \binom{S(j)}{2}$. In view of the convexity inequality (1), we finally get

$$\sum_{(i_1,i_2)\in\binom{7}{2}} C(i_1, i_2) = \sum_{i=1}^{7} \binom{S(j)}{2} \geq 7\binom{\frac{\sum_{i=1}^{7} S(j)}{7}}{2} = 7\binom{\frac{22}{7}}{2} >$$
$$> 7\binom{3}{2} = \binom{7}{2},$$

i.e.

$$\sum_{(i_1,i_2)\in\binom{7}{2}} C(i_1, i_2) > \binom{7}{2}.$$

We got the sum of $\binom{7}{2}$ non-negative integers to be greater than $\binom{7}{2}$, therefore, at least one of the summands $C(i_1, i_2) \geq 2$. In our notations this means precisely that the pair of first team players i_1, i_2 played with the same two (or more) players j_1, j_2 of the second team. Surely, you can seat these four players at a round table in accordance with the problem's requirements!

(a) Solution II

This solution harnesses the power of combinatorics. In the selection and editing process, Dr. Col. Bob Ewell suggested using a 7×7 table to record the games played. We number the players in both teams. For each player of the first team we allocate a row of the table, and for each player of the second team a column. We place a checker in the table in location (i, j) if the player i of the first team played the player j of the second team (Fig. 1). If the required four players are found, this would

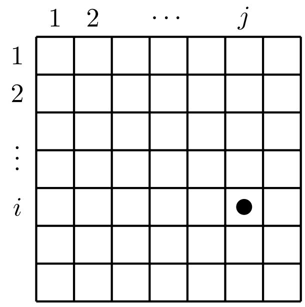


Figure 1

manifest itself in the table as a rectangle formed by four checkers (a checkered rectangle)! The problem thus translates into the new language as follows:

A 7×7 *table with* 22 *checkers must contain a checkered rectangle.*

Assume that a table has 22 checkers but does not contain a checkered rectangle. Since 22 checkers are contained in 7 rows, by Pigeonhole Principle, there is a row with at least 4 checkers in it. Observe that interchanging rows or columns does not affect the property of the table to have or not have a checkered rectangle. By interchanging rows we make the row with at least 4 checkers first. By interchanging columns we make all checkers appear consecutively from the left of the first column. We consider two cases.

1. Top row contains exactly 4 checkers (Figure 2).

Draw a bold vertical line L after the first 4 columns. To the left of L, the top row contains 4 checkers, and all other rows contain at most 1 checker each, for otherwise we would have a checkered rectangle (that includes the top row). Therefore, to the left of L we

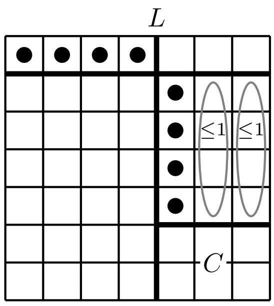


Figure 2

have at most $4 + 6 = 10$ checkers. This leaves at least 12 checkers to the right of L, thus at least one of the three columns to the right of L contains at least 4 checkers; by interchanging columns and rows we put them in the position shown in Figure 2. Then each of the two right columns contains at most 1 checker in total in the rows 2 through 5, for otherwise we would have a checkered rectangle. We thus have at most $4 + 1 + 1 = 6$ checkers to the right of L in rows 2 through 5 combined. Therefore, in the lower right 2×3 part C of the table we have at least $22 - 10 - 6 = 6$ checkers—thus C is completely filled with checkers and we get a checkered rectangle in C in contradiction with our assumption.

2. Top row contains at least 5 checkers (Figure 3).

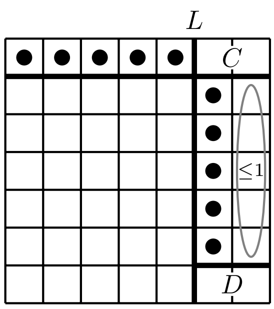


Figure 3

Draw a bold vertical line L after the first 5 columns. To the left of L, the top row contains 5 checkers, and all other rows contain at most 1 checker each, for otherwise we would have a checkered rectangle (that includes the top row). Therefore, to the left of L we have at most $5+6 = 11$ checkers. This leaves at least 11 checkers to the right of L, thus at least one of the two columns to the right of L contains at least 6 checkers; by interchanging columns and rows we put 5 of these 6 checkers in the position shown in Figure 3. Then the last column contains at most 1 checker in total in the rows 2 through 6, for otherwise we would have a checkered rectangle. We thus have at most $5 + 1 = 6$ checkers to the right of L in rows 2 through 6 combined. Therefore, the upper right 1×2 part C of the table plus the lower right 1×2 part D of the table have together have at least $22 - 11 - 6 = 5$ checkers—but they only have 4 cells, and we thus get a contradiction.

(a) Solution III

This solution is the shortest of the three. It also "explains" the meaning of the number 22 in the problem:

$$22 = \binom{7}{2} + 1.$$

Given a placement P of 22 checkers on the 7×7 board, we pick one row; let this row have k checkers in total in it. We compute the number of 2-element subsets of a k-element set; this number is denoted by $\binom{k}{2}$ and is equal $\binom{k}{2} = \frac{1}{2}k(k-1)$. Now we can define a function $C(P)$ as the sum of 7 such summands $\binom{k}{2}$, one per each row. Given a placement P of 22 checkers on a 7×7 board. If there is a row R with r checkers, where $r = 0, 1$ or 2, then there is a row S with s checkers, where $s = 4, 5, 6$ or 7 (for the average number of checkers in a row is $\frac{22}{7}$). We notice that $s - r - 1 \geq 0$, and observe that moving one checker from row S to any open cell of row R would produce a placement P_1 with reduced $C(P) > C(P_1)$ because

$$\binom{r}{2} + \binom{s}{2} - \binom{r+1}{2} - \binom{s-1}{2} = s - r - 1 \geq 0.$$

By moving one checker at a time, we will end up with the final placement P_k, where each row has 3 checkers except one, which has 4. For the final placement $C(P_k)$ can be easily computed as $6\binom{3}{2}+\binom{4}{2}=24$. Thus, for the original placement P, $C(P)$ is at least 24. On the other hand, the total number of 2-element subsets in a 7-element set is $\binom{7}{2}=21$. Since $24>21$, there are two identical 2-element subsets (see Figure 4) among the 24 counted by the function $C(P)$. But the checkers that form these two identical pairs form a desired checkered rectangle!

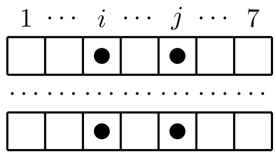


Figure 4

Solution of (b)

Glue a cylinder (!) out of the board 7×7, and put 21 checkers on all squares of the 1^{st}, 2^{nd}, and 4^{th} diagonals (Fig. 5 shows the cylinder with one checkered diagonal; Fig. 6 shows, in a flat representation, the cylinder with all three cylinder diagonals).

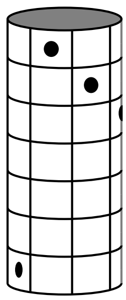


Figure 5

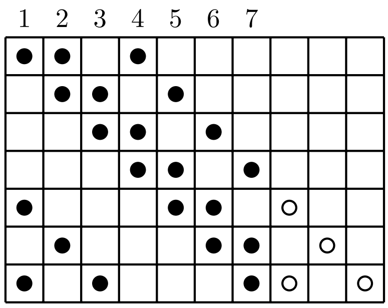


Figure 6

Assume that 4 checkers form a rectangle on our 7×7 board. Since these four checkers lie on 3 diagonals, by Pigeonhole Principle two checkers lie on the same (checkers-covered) diagonal D of the cylinder. But this means that *on the cylinder* our 4 checkers form a square! Two other (opposite) checkers a and b thus must be symmetric to each other with respect to D, which implies that the diagonals of the cylinder that contain a and b must be symmetric with respect to D—but no 2 checker-covered diagonals in our checker placement are symmetric with respect to D. (To see it, observe Fig. 7 which shows the top rim of the cylinder with bold dots for checkered diagonals: square distances between the checkered diagonals, clockwise, are 1, 2, and 4.) This contradiction implies that there are no checkered rectangles in our placement. Done!

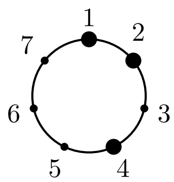


Figure 7

Remark on Problem (b)

Obviously, any solution of problem (b) can be presented in a form of 21 checkers on a 7×7 board (shown in the left 7×7 part with 21 black checkers in Fig. 6). It is less obvious, that the solution is *unique*: by a series of interchanges of rows and columns, any solution of this problem can be brought to precisely the one I presented! Of course, such interchanges mean merely renumbering the players of the same team. The uniqueness of the solution of problem (b) is precisely another way of stating the uniqueness of the projective plane[3] of order 2, the so called "Fano Plane"[4] denoted by $PG(2,2)$. The Fano Plane is an

[3] A finite projective plane of order n is defined as a set of n^2+n+1 points with the properties that:

1. Any two points determine a line,

2. Any two lines determine a point,

3. Every point has $n+1$ lines through it,

4. Every line contains $n+1$ points.

[4] Named after Gino Fano (1871-1952), the Italian geometer who pioneered the study of finite geometries.

abstract construction, with symmetry between points and lines: it has 7 points and 7 lines (think of rows and columns of our 7×7 board as lines and points respectively!), with 3 points on every line and 3 lines through every point (Fig. 8).

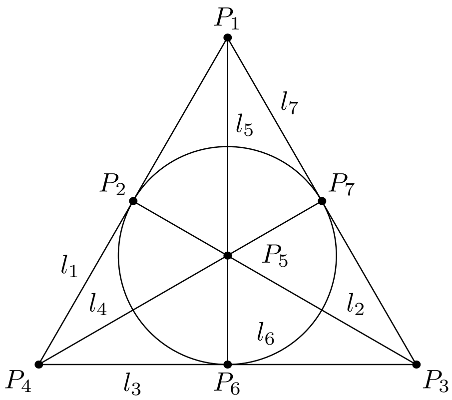


Figure 8

Observe that if in our 7×7 board (left side in Figure 6) we replace checkers by 1 and the rest of the squares by zeroes, we would get the incidence matrix of the Fano Plane.

Alexander Soifer

Princeton University	*DIMACS*	*University of Colorado*
Mathematics	*Rutgers University*	*at Colorado Springs*
Fine Hall	*Piscataway NJ*	*USA*
Princeton, NJ 08544	*USA*	*Email: asoifer@uccs.edu*
USA	*http://www.uccs.edu/˜asoifer*	

Email: asoifer@princeton.edu

About an Unexpected Transition from Algebra to Geometry

Svetoslav Jordanov Bilchev

Svetoslav Bilchev is Professor and Head of Department of Algebra and Geometry at the University of Rousse, Bulgaria. He studied applications of differential geometry to the system of partial differential equations for his Ph.D. and for many years has been interested in oscillations of differential equations with delay, game theory, mathematical models in economics, transformations theory, inequalities, geometry, mathematics education and etc. He took part in two IMO and for 35 years has been involved with the teaching and encouragement of talented and gifted students in countries including Bulgaria, Greece, Russia, UK and Italy. He is the author of more than 150 articles and books in his fields of interest. He has been awarded many national and international prizes for his strong and enthusiastic work as a mathematician and as a lecturer.

Abstract. In this paper an unexpected transition from algebra to geometry is considered. It was achieved by the help of special kind of positive symmetrical sums of three real positive variables.

The result of this transition is a chain of comparable to sharpness geometric inequalities, connecting the semiperimeter, circumradius and inradius of an arbitrary triangle.

Some of the geometric inequalities in the chain are new. Some of the obtained symmetrical sums of three real positive variables as functions of the semiperimeter, circumradius and inradius of an arbitrary triangle are new as well.

The presented idea is very productive for obtaining new results in

geometry and for stressing creative attention and stimulating the creativity of gifted students in mathematics.

Forty years ago when I was a student in a high school I took part in the Sixth International Mathematical Olympiad (IMO) held in Moskow, former USSR, 1964. The competition was in some of the nicest halls in Moskow State University. Almost all the participants on this IMO were surprised by one of the competition problems—the following “nice looking” but very hard (for that time) geometric inequality:

Problem 1.

For an arbitrary triangle ABC with sides a, b, c prove the inequality

$$\sum a^2(b+c-a) \le 3abc, \tag{1}$$

where the sum $\sum$ in (1) is the well-known cyclic sum over the sides a, b, c of the triangle, i.e.

$$\sum a^2(b+c-a) = a^2(b+c-a) + b^2(c+a-b) + c^2(a+b-c) \tag{2}$$

(we will use some analogical cyclic sums and further in the present paper).

The surprise was really very big (by the Problem 1) because only a small number of participants in this IMO proved the inequality (1) successfully.

My own solution was presented in the book [1] as one of the best solutions given at the Sixth IMO. The main idea was to transform (1) with some equivalent procedures to one of the well-known inequalities or to some obvious one. After some smart equivalent algebraic manipulations as follows:

$$\begin{aligned}
0 &\le \sum \big[abc - a^2(b+c-a)\big],\\
0 &\le \sum a(bc-ab-ac+a^2),\\
0 &\le \sum a\big[c(b-a) - a(b-a)\big],\\
0 &\le \sum a(b-a)(c-a),
\end{aligned}$$

$$
\begin{aligned}
&0 \le 2\sum a(b-a)(c-a),\\
&0 \le \sum\big[b(c-b)(a-b) + c(a-c)(b-c)\big],\\
&0 \le \sum(b-c)\big[c(a-c) - b(a-b)\big],\\
&0 \le \sum(b-c)\big[(b-c)(b+c) - a(b-c)\big].
\end{aligned}
$$

I succeeding in obtaining the following equivalent obvious inequality

$$0 \le \sum(b-c)^2(b+c-a), \tag{3}$$

because for every triangle: $b+c-a>0$, $c+a-b>0$, $a+b-c>0$.

Really, it was very big pleasure for me to create this smart solution because at this time I did not know too much about geometric inequalities and almost nothing about general methods of proof.

Later on I read about a very effective general method for proving geometric inequalities for an arbitrary triangle with sides a, b, c, semiperimeter s and radiuses r and R of the incircle and circumcircle respectively. The main idea of this method is to transform the given geometric inequality:

$$f(a,b,c) \ge 0 \tag{4}$$

equivalently to the following one:

$$F(\textstyle\sum a, \sum bc, abc, \sum a^2, \sum a^3, \dots) \ge 0, \tag{5}$$

where the variables of the function F are equal to:

$$
\begin{aligned}
\sum a &= a+b+c = 2s,\\
\sum bc &= bc+ca+ab = s^2+r^2+4Rr,\\
abc &= 4Rrs,\\
\sum a^2 &= 2(s^2-r^2-4Rr),\\
\sum a^3 &= 2s(s^2-3r^2-6Rr).
\end{aligned} \tag{6}
$$

The identities (6) are well-known (see [2]). So, from (5) and (6) we obtain an inequality of the type

$$g(s, r, R) \geq 0, \tag{7}$$

which is necessary to be proved. And, if (7) is true, then the inequality (4) is done.

For example, the inequality can be transformed equivalently:

$$\begin{aligned} \sum a^2\big[(b+c+a)-2a\big] &\leq 3abc, \\ \sum a \sum a^2 - 2\sum a^3 &\leq 3abc, \\ \sum a \sum a^2 &\leq 2\sum a^3 + 3abc. \end{aligned} \tag{8}$$

Then, let us put (6) in (8):

$$2s \cdot 2(s^2 - r^2 - 4Rr) \leq 2 \cdot 2s(s^2 - 3r^2 - 6Rr) + 3 \cdot 4sRr,$$

i.e.

$$2r^2 \leq Rr,$$

or

$$2r \leq R. \tag{9}$$

The last inequality is the well-known Euler's inequality which we know is true.

So, we proved that the inequalities (1), (3), (8) and (9) are equivalent to each other and the famous (9) proves (1) and (8). Reversibly, if it is necessary to prove the Euler's inequality (9) then is enough to consider the equivalent to it inequality (3) which is obvious.

Hence, we found a *nice idea and a good technique (algorithm)* for creating equivalent to (3) inequalities and by this means to get a proof for these inequalities.

All of this I said before is excellent but every time I want to see "what is behind the curtains", what is the nature of the considered method, where its sources are, what is possible to be "at the beginning", what will be there if I start to think reversibly? But what have to be "at the beginning" here and how to think reversibly?

Of course, "the beginning" here, in Problem 1, is the obvious inequality (3).

So, in this sense, we can construct many different homogeneous algebraic inequalities of three real positive variables a, b, c, which are done if a, b, c are the sides of an arbitrary triangle (see for example (3)). After that we transform the considered homogeneous inequality to an inequality of the type (5) with symmetrical sums of a, b, c as variables. Then, by putting (6) in the last inequality, after some, often very hard, calculations, we get a geometric inequality of the type (7) which is done because the initial inequality—the inequality "at the beginning"—is obvious.

The given algorithm above which was created for obtaining geometric inequalities from homogeneous algebraic inequalities was named "an unexpected transition from algebra to geometry" by us. We will give some additional commentaries about the proposed method and algorithm after some characteristic and useful examples of this method.

Example 2.

Let us consider the homogeneous algebraic inequality

$$\sum a(b-c)^2(b+c-a) \geq 0, \tag{10}$$

which is obvious if a, b, c are the sides of an arbitrary triangle.

After some calculation we get consequently:

$$\sum a(b-c)^2(b+c) - \sum a^2(b-c)^2 \geq 0,$$

$$\sum a(b^2-c^2)(b-c) - \sum a^2\big[(b^2+c^2+a^2) - a^2 - 2bc\big] \geq 0,$$

$$\sum a\big[(b^3+c^3+a^3) - a^3 - bc(b+c)\big] - \\ - \Big(\sum a^2\Big)^2 + \sum a^4 + 2abc\sum a \geq 0,$$

$$\sum a \sum a^3 - \sum a^4 - 2abc \sum a - \left(\sum a^2\right)^2 + \sum a^4 + 2abc \sum a \geq 0,$$
$$\sum a \sum a^3 - \left(\sum a^2\right)^2 \geq 0. \tag{11}$$

Analogously, it is not so difficult to obtain from (10) the following equivalent inequalities:

$$\sum a(b^3 + c^3) + 4abc \sum a \geq 2\left(\sum ab\right)^2, \tag{12}$$
$$\sum a \left(\sum a^3 + 4abc\right) \geq \sum a^4 + 2\left(\sum ab\right)^2, \tag{13}$$

Finally, if we put (6) in (11) we get the following inequality of the type (7):

$$s^2 \geq \frac{r(r+4R)}{2R-r}. \tag{14}$$

The inequalities (10)–(14) are equivalent to each other.

Further, in the following examples, we will present only equivalent inequalities. The idea of proving its equivalency is evident.

Example 3.

The inequalities:

$$\sum (a-b)^2(b+c-a)(c+a-b) \geq 0, \tag{15}$$
$$\left(\sum a^2\right)^2 + abc \sum a \leq 2 \sum ab(a^2+b^2), \tag{16}$$
$$2\sum a^2 \sum ab \geq \left(\sum a^2\right)^2 + 3abc \sum a, \tag{17}$$
$$\left(\sum a\right)^2 \sum a^2 - 2\sum a \sum a^3 + 2\sum a^2 \sum ab - 4\left(\sum ab\right)^2 + {} + 3abc \sum a \geq 0, \tag{18}$$
$$s^2 \geq \frac{r(r+4R)^2}{R+r} \tag{19}$$

are equivalent and true when a, b, c are the sides of an arbitrary triangle.

Example 4.

The inequalities:

$$\sum c(a-b)^2(b+c-a)(c+a-b) \geq 0, \tag{20}$$

$$\sum a^2 \left(\sum a^3 + 6abc\right) \geq \sum a \sum a^4 + 6abc \sum ab, \tag{21}$$

$$s^2 \geq \frac{r}{2R+r}(r+4R)(8R-r) \tag{22}$$

are equivalent and true when a, b, c are the sides of an arbitrary triangle.

Example 5.

The inequalities:

$$\sum (b-c)^2(c-a)^2(a+b-c) \geq 0, \tag{23}$$

$$\sum a \left[7 \sum a^4 + \left(\sum ab\right)^2 + 2abc \sum a\right] \geq 6 \sum a^5 + \\ + 2\left(\sum a\right)^2 \sum a^3 + 12abc \sum a^2, \tag{24}$$

$$\left[s^2 - (14Rr - r^2)\right]^2 \geq \left[2r(R-2r)\right]^2, \tag{25}$$

$$\left[s^2 - (16Rr - 5r^2)\right]\left[s^2 - (12Rr + 3r^2)\right] \geq 0,$$

$$s^2 \geq 16Rr - 5r^2 \tag{26}$$

are equivalent and true when a, b, c are the sides of an arbitrary triangle, because from (22) we get easily

$$s^2 \geq \frac{r}{2R+r}(r+4R)(8R-r) \geq 12Rr + 3r^2 = 3r(r+4R).$$

Here, it is necessary to point out that for obtaining (22) and (26) we constructed the following identities:

$$\sum a^4 = 2s^4 - 4s^2 r(4R+3r) + 2r^2(r+4R)^2, \tag{27}$$

$$\sum a^5 = 2s^5 - 20s^3 r(R+r) + 10sr^2(r+2R)(r+4R), \tag{28}$$

which are new as far as we know.

Example 6.

It is very surprising that the following "heavy" looking homogeneous algebraic inequality

$$\sum a^2(b-c)^2(b+c-a) \geq 0 \tag{29}$$

is true for every real triple (a, b, c) because (29) is equivalent to the following well-known inequalities:

$$3\sum a^2 \geq \left(\sum a\right)^2, \tag{30}$$

$$\sum (a-b)^2 \geq 0, \tag{31}$$

but if a, b, c are the sides of an arbitrary triangle then (29)–(31) are equivalent to the following inequality:

$$s^2 \geq 3r(r+4R) \tag{32}$$

which we proved and used already in Example 5.

Hence, with the help of the so named by us "unexpected transition from algebra to geometry" we obtained the following chain of comparable to sharpness geometric inequalities, connecting the semiperimeter, circumradius and inradius of an arbitrary triangle:

$$\begin{aligned} s^2 &\geq 16Rr - 5r^2 \geq \frac{r}{2R+r}(r+4R)(8R-r) \geq \\ &\geq \frac{r}{R+r}(r+4R)^2 \geq 3r(r+4R) \geq \frac{r}{2R-r}(r+4R)^2 \end{aligned} \tag{33}$$

or

$$\frac{s^2}{r(r+4R)} \geq \frac{16R-5r}{r+4r} \geq \frac{8R-r}{2R+r} \geq \frac{r+4R}{R+r} \geq 3 \geq \frac{r+4R}{2R-r}. \tag{34}$$

Some of the inequalities in (33) and (34) are new. The rest of them are obtained by different methods one of which is based on the computing of the distances between some remarkable points in the triangle (see [4], p. 21–22; [5], p. 174–177; [3], p. 36–37). It is possible to find a longer chain of the type (33) in [6], p. 75; [7], p. 36.

Of course, with the given algorithm above it is possible to obtain more new and old geometric inequalities of the types (4), (5) and (7) by starting, for example, from the following symmetric algebraic inequalities:

$$\sum c(b-c)^2(c-a)^2(a+b-c) \geq 0, \tag{35}$$

$$\sum (b-c)^2(c-a)^2(a+b-c)(b+c-a) \geq 0, \tag{36}$$

$$\sum c(b-c)^2(c-a)^2(a+b-c)(b+c-a) \geq 0, \tag{37}$$

which are obvious when a, b, c are the sides of an arbitrary triangle.

For the inequalities (35)–(37) it is necessary to know the following new identities:

$$\begin{aligned} a^6 &= \frac{1}{2}\sum a^2\left[3\sum a^4 - \left(\sum a^2\right)^2\right] + 2(abc)^2 = \\ &= 2s^6 - 6r(4R+5r)s^4 + 6r^2(24r^2+24Rr+5r^2)s^2 - \\ &\qquad - 2(r^2+4Rr)^3, \end{aligned} \tag{38}$$

$$\begin{aligned} a^7 &= \sum a^3\left[2abc\sum a - \left(\sum bc\right)^2\right] + \sum a^2\sum a^5 + (abc)^2\sum a = \\ &= 2s^7 - 14r(2R+3r)s^5 + 14r^2(16r^2+20Rr+5r^2)s^3 - \\ &\qquad - 14r^3(2R+r)(r+4R)^2 s, \end{aligned} \tag{39}$$

$$\begin{aligned} \sum (b-c)^2(c-a)^2 &= 3\left(\sum bc\right)^2 - 4\sum a^2\sum bc + \\ &\qquad + 2\sum a\left(\sum a^3 - abc\right) - \sum a^4 = \\ &= \left[s^2 - 3r(r+4R)\right]^2 \end{aligned} \tag{40}$$

As we pointed out before, the symmetrical sums (27), (28), (38)–(40) of the sides a, b, c of an arbitrary triangle presented here as functions of the semiperimeter s, circumradius R and inradius r are new identities—created with the purpose of realizing the proposed new method of "an unexpected transition from algebra to geometry".

Finally, it is necessary to say that the presented idea is very productive for obtaining new results in geometry and for stressing the creative attention of gifted students in mathematics. Of course, it is possible to use some of the given inequalities above in mathematics competitions of a high level.

References

[1] Budurov, Stoyan: *Mathematical Olympiads*, People's Education, Sofia, 1972.

[2] Soltan, V.P. & Mejdman, S.I.: *Tojdestva i Neravenstva v Treugolnike*, Shtiinca, Kishinev, 1982.

[3] Bilchev, S.J. & Hristo, Lesov: *About an Identity and Connected with it Geometric Inequalities*, Education of Mathematics, Sofia, No. 4, 1986, 33–42.

[4] Bilchev S.J. & Hristo, Lesov: *Remarkable Points in a Triangle and the Distances Between Them*, Jubilee Scientific Session "25 Years Section" UBM—Old Zagora, June 25–26, 1987, 20–23.

[5] Bilchev, S.J., Emiliya, A.V. & Hristo, Lesov: *Spieker and Crelle-Brocard Points and Their Crucial Role for the Theory of Geometric Inequalities*, Mathematics and Education Mathematics, Proceedings of the Twenty-Second Spring Conference of the Union of Bulgarian Mathematicians, Sofia, April 2–5, 1993, 173–80.

[6] Bilchev, S.J. & Emiliya, A.V.: *On a Certain Triangle Transformation and Some of Its Applications*, Elemente Der Mathematic, Basel, Schweiz, Vol. 45, No. 3, 1990, 70–75.

[7] Bilchev, S.J. & Kontogiannis, D.G.: *A Short and Elementary Proof of Mushkarov–Simeonov's Inequality*, Hrvatska Akademija Znanosti I Umjetnosti, Rada 456, Matematičke znanosti, Svezak 10, Zagreb, 1991, 33–37.

Svetoslav Jordanov Bilchev, Ph.D., Associate Professor, Chairman
Department of Algebra and Geometry
Center of Applied Mathematics and Informatics
Faculty of Education, University of ROUSSE
8 Studentska Str., 7017 ROUSSE
BULGARIA
Cell phone: +359-886 735 536, Fax: +359-82-845 708
E-mail: slavy@ami.ru.acad.bg, slavy_bilchev@yahoo.com

Mathematical Competitions in Latvia: E Pluribus Unum[1]

Dace Bonka & Andrejs Cibulis

Dace Bonka is a Ph.D. student at the University of Latvia and educational director at A. Liepas Correspondence Mathematics School that is a centre of advanced mathematics education in Latvia. Her research area and main activities are correspondence mathematics contests for junior pupils.

Andrejs Cibulis graduated from the University of Latvia in 1978, and has a Ph.D. from the Institute of Applied Mathematics and Mechanics in Donetsk, Ukraine in 1984. He is an associate professor at the University of Latvia. His interests cover mathematical analysis, optimization, recreational mathematics, teaching and mathematical toys. In 1993 he was awarded the Atis Kronvalds prize (a prestigious award in Latvia). He has published approximately 150 articles and has invented a number of mathematical toys.

[1]This paper was prepared partially with the support of the state-investment project "Latvian Education Informatization System".

ABSTRACT. In this paper we consider the system of math competitions in Latvia developed over approximately 35 years. It includes:

– current competitions—mainly math olympiads organized at various levels;
– correspondence contests for pupils up to the 9th Grade and contests in solving unsolved problems;
– system of math circles in schools, universities etc. for primary and secondary school pupils;
– teacher courses on advanced problem solving;
– lecture courses on advanced problem solving and similar topics included in the curricula of university students, the future math teachers;
– publishing of teaching aids (including those with free access on the Internet).

1 Introduction

Developing and improving problem solving skills are some of the main goals of any education, as all our life is continuous problem solving—from the elementary ones to the world-scale problems of a moral, political, scientific, technical etc. nature. Math contests are among the best tools to bring children and teenagers into the world of creative mental activity.

In this paper we consider the system of math competitions in Latvia developed over approximately 35 years.

2 The background

Latvia is a small country with approximately $2,300,000$ inhabitants and without any significant natural resources. The independence of Latvia was proclaimed in 1919. In 1940 Latvia was occupied by the Soviet Union. In 1991 Latvia restored her independence. So each talented person is of immense significance for us, and considerable efforts are made not to pass any gifted child. Among other activities, various types of competitions are developed corresponding to different types of mental activities favoured by different children.

It is especially important due to changes that have occurred in the system of education. During various reforms the number of lessons in mathematics has decreased significantly; most advanced topics have also been eliminated from the official curricula. So, as mentioned in [1],

advanced mathematical education in schools faced the danger of becoming occasional and disintegrated. In this situation mathematical olympiads appeared to be a very strong consolidating factor. Olympiad "curricula" was not changed; it was developed further essentially. The standards elaborated in the olympiad movement over many years became the unofficial standards (in some sense) for advanced education in mathematics. There are a lot of topics not included in the official school program whatsoever, nevertheless they are discussed regularly with all pupils interested in mathematics because they occur in some of mathematical olympiads (preparatory, regional, open).

The other positive feature of mathematical olympiads is their stability. Teachers are sure that olympiads will be held to encourage their pupils to work additionally for a clear and inspiring aim. The number of participants in the Open Mathematical Olympiad has increased significantly in recent years and it is now more than $3,000$.

The competitive factor is still extremely important; the educational factor has also become very significant.

The competitions and other activities are organized in a system to cover various needs of different groups of pupils and teachers and to keep them busy during the whole academic year.

3 Current competitions

These are mainly math olympiads organized at various levels:

- School level, grades 5–12. Problems and solutions are prepared at the University of Latvia, but each school is allowed to change or replace any problem. Approximately 30 % of all pupils take part in them.
- Regional level, grades 5–12. Problems and solutions are prepared at the University of Latvia and sent out to 39 places. Problems cannot be changed nor replaced. Approximately $12,000$ pupils take part in them.

- State level, grades 9–12. Problems are prepared at the University of Latvia, the olympiad is held there and grading is also done there. This round consists of two consecutive days; only those who have done well on the first day can participate on the second one. Each region, each major city and each district of Riga has its own number of participants (at least 4) at the national level. Each first prizewinner of the previous year can come, too. Altogether, there are approximately 330 participants.
- Selection round for IMO—April, two days. Those who have done well at the national level and those who are known for many years to be very strong are invited. There are approximately 15 participants there.
- Latvian Open Mathematical Olympiad, grades 5–12. The Olympiad is held on the last Sunday of April. This year there were approximately 3,300 participants. Problems are prepared at the University of Latvia where the olympiad is held and the grading is done. This olympiad has been organized since 1974, and foreign teams also take part.

The contest papers include problems from various branches (algebra, geometry, number theory, combinatorics), of various types (proofs, calculations, constructions, algorithms), at least one simple and at least one hard problem. At the school level, regional level and in the Open Olympiad for each grade one problem is always taken from one of the corresponding olympiads of the two previous years.

School olympiads, regional olympiads and Open Olympiads are "open" to all pupils. Only winners of regional olympiads and some individually invited pupils take part in the national-level olympiad.

Other current competitions are organized at schools and at summer camps and the like.

4 Corresponding contests

There are many pupils who need more than some 4–5 hours (the time usually given for math olympiads) to go deeply enough into the problem. For such children a system of correspondence contests has been developed:

- "Club of Professor Littledigit" (CPL) for pupils up to the 9th Grade. There are 6 rounds each year, each containing 6 relatively easy and 6 harder problems. Problems are published in the newspaper of Latvia "Latvijas Avize" (which has the largest circulation), and on the Internet. There are approximately 200 contestants each year.
- "Contest of Young Mathematicians" for pupils up to 7th Grade, originally developed for weaker pupils than the participants of CPL, especially in Latgale, the eastern region of Latvia. The problems are published in regional newspapers and on the Internet, and today it has become popular all over Latvia. There are approximately 260 contestants each year.

5 Contests in solving unsolved problems

They are usually organized through various newspapers and at summer camps. The problems proposed there are close to those of recreational mathematics, though the accounts of them can be found in high-level math journals and proceedings of international conferences. Combinatorial geometry problems, particularly those concerning polyforms, are especially popular in Latvia.

Some serious achievements should be mentioned.

- M. Lukjanska's contest work on compatibility of polyiamonds was highly appreciated in the 15th European Union Contest for the Young Scientists, Budapest, Hungary in 2003. She was awarded Honorary Prize and was selected to represent the European Union Contest for Young Scientists at the 46th London International Youth Science Forum, 28 July–11 August, 2004.
- In 2001, Uldis Barbans, a pupil at the University of Latvia succeeded in cracking two very hard nuts. He proved the compatibility of (O_4, U_5) and (O_4, T_5) by constructing two extraordinarily large n-ominoes respectively with $n = 640$ and $n = 1200$, see [2].

A difficult contest problem was offered in the magazine [3]. Contestants were asked to construct a magic square from the given starting position as in Figure 1. The allowed moves are the same as in the famous puzzle-15 of Sam Loyd. Let us recall that the square is *magic* if each row, column, and the two diagonals add up to the same sum. Atis Blumbergs, the winner of the contest came to a conclusion that the problem has no solution; it was done by a computer. Mathematically it means that all magic squares with 16 in the fixed corner are of one and the same parity. It would be nice to find a short mathematical explanation for this interesting fact.

By organizing contests sometimes one succeed in finding the gifted solvers. Thus a difficult problem on magic squares and several others have been solved.

1	2	3	4	
5	6	7	8	
9	10	11	12	
13	14	15	16	

Figure 1

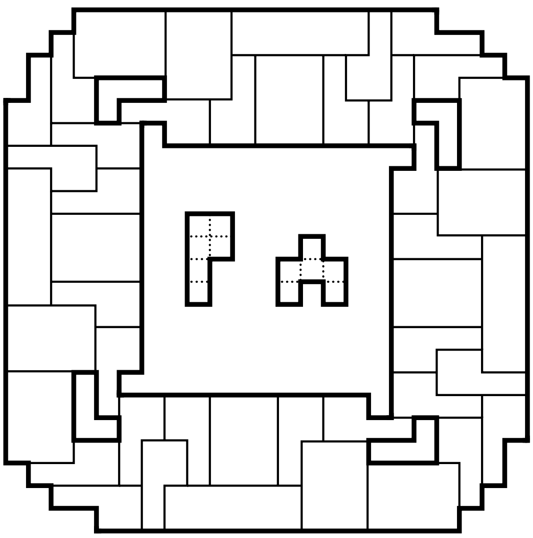


Figure 2

In 2003, V. Plociņš, a first-year student at the University of Latvia, succeeded in finding a 360-mino (see Figure 2) being the common multiple for P- and A-hexominoes shown in the central part of the picture. Let us remember that *polyominoes* are connected plane figures formed by joining unit squares edge to edge. A polyomino A is said to *divide* another figure B if B may be assembled from copies of A. We also say that A is *divisor* of B, B is *divisible* by A, and B is *multiple* of A. If two figures have a common multiple, they are said to be compatible. In 2003 a webpage containing common multiples has appeared [4]. As to hexominoes 195 common multiples are given there. However there are at least 527 compatible pairs of hexominoes as stated by the first author together with V. Plociņš[2].

6 Preparation for the competitions

A broad system of math circles in schools, universities etc. for primary and secondary school pupils has been developed. Teacher courses on advanced problem solving are regular events; lecture courses on this and similar topics are included in the curricula of university students who intending to become math teachers. A series of teaching aids is published regularly, including some with free access on the Internet, see [5]. International co-operation with mathematicians of Estonia, Lithuania, Iceland, Finland and Bulgaria in the area of competitions has been established.

References

[1] Andžāns, A. & Ramāna, L.: *What Problem Set Should Be Called Good for a Mathematical Olympiad*, Matematika ir matematikos destymas, Kaunas, Technologija, 2002, 5–8.

[2] Barbans, U., Cibulis, A., Lee, G., Liu, A. & Wainwright, B.: *Polyomino Number Theory (II)*, Mathematical Properties of Sequences and Other Combinatorial Structures, Kluwer Academic, 2003, 93–100.

[3] *Labirints*, 1996, No. 11, p. 69; 1997, No. 6, p. 66. (in Latvia)

[4] http://www.geocities.com/jorgeluismireles/images/

[5] http://www.liis.lv/NMS/

[2]Recently a webpage [6] containing new remarkable results on compatibility of hexominoes as well as on other polyominoes has been opened.

[6] http://www.imc.pi.cnr.it/resta/Poly/

Dace Bonka, Mg.mat.
University of Latvia
19 Rainis boulevard
Riga, LV-1586
Latvia
Phone: ++371 7034498
Email: dace.bonka@lu.lv

Andrejs Cibulis, Assoc. Prof.
University of Latvia
29 Rainis boulevard
Riga, LV-1459
Latvia
Phone: ++371 7211421
Email: Andrejs.Cibulis@mii.lu.lv

ICME 10
Discussion Group 16 Proceedings

Peter Taylor, Frédéric Gourdeau & Petar Kenderov

This discussion group was run by an organizing team of three persons, Peter Taylor (University of Canberra) and Frédéric Gourdeau (Université Laval, Québec) as co-Chairs with Petar Kenderov (Bulgarian Academy of Sciences) making up the third member of the team. Andre Deledicq (France) had originally been appointed as co-Chair with Peter Taylor, and it should be recorded that he had a role in the design of the program, but he withdrew from this role before ICME and his place was taken by Frédéric Gourdeau. It should also be noted that Titu Andreescu (USA) had been appointed as a member of the organizing team, but did not take part in any of the planning or discussion.

The program had two two-hour sessions and a third session of a single hour. The first session, with invited contributions from Andrejs Cibulis and Dace Bonka (Latvia) and Peter Crippin (University of Waterloo), focused on the range of competitions and related activities which are available. The second session, with an invited introduction from Andy Liu (University of Alberta), discussed the relation between competitions (and related activities) and the teaching and learning process. The last session summarized the previous proceedings with a view to writing a final report.

It should be noted that the discussion group was well-attended, not only by regular participants in World Federation of National Mathematics Competitions activities but also by many different people, mainly teachers and educators from countries in Europe. About 40 people attended each of the first two sessions and about 12 attended the last session. It is estimated that between 60 and 70 people attended at least one of the sessions. The following report on competitions has been prepared by the organizing team based on the discussions and after providing all participants who left their email address a chance to comment.

1 What are Competitions?

The discussion group noted that in recent years the meaning of the word "competition" has become much more general than the traditional meaning of either a national Olympiad, or more broadly based multiple choice question exams which have become popular in a number of countries. The World Federation of National Mathematics Competitions, the principal international body comprising mathematics academics and teachers who administer competitions, has itself formally defined competitions as also comprising such activities as enrichment courses and activities in mathematics, mathematics Clubs or "Circles", Mathematics Days, Mathematics Camps, including live-in programs in which students solve open-ended or research-style problems over a period of days.

It was also noted that the publication of journals for students and teachers containing problem sections, book reviews, review articles on historic and contemporary issues in mathematics and support for teachers who desire and/or require extra resources in dealing with talented students were also important activities related to competitions.

It was noted that competitions themselves come in a number of categories, the elite national Olympiads, the broader and popular inclusive competitions usually involving (regretfully) multiple choice questions, and special themed competitions, which sometimes involve teams rather than individuals. In some cases, these teams are composed of whole classes, giving a very different feel to the competition.

In particular, special note was made of project, or research based activities, in which students have a longer time frame to solve problems than normally permitted in an exam-based environment.

These activities all have in common the values of creativity and enrichment beyond the normal syllabus, opportunities for students to experience problem solving situations and the provision of challenge for the student. Competitions give students the opportunity to be drawn by their own interest to experience some mathematics beyond their normal classroom experience.

It was noted that competitions are usually administered by teachers on a voluntary basis beyond their required duties and that the bodies which administer competitions are usually independent of the normal curriculum and assessment bodies.

2 How Competitions contribute positively to the Teaching and Learning Process

Competitions provide for example a focus on problem solving, sometimes giving students an opportunity to be associated with a cutting edge area of mathematics in which new methods may evolve and old methods may be revived.

Competitions provide opportunity for creativity, as students often use different than envisaged solution styles in solving problems. The success of competitions over the years, particularly the resurgence in the last 50 years, indicates that these are events in which students enjoy mathematics. Different students derive different experiences, and it is exciting for students when they see a solution to a problem can be reached by two quite different techniques.

Since competitions give students an opportunity to discover a talent which they may not normally demonstrate, they provide a stimulus for improving learning. There was also a feeling that competitions allow independent thinking by students.

There was a discussion about various attitudes towards competitions. Some present preferred individual competitions, others said it was positive for students to develop a competitive attitude. There was also strong support for team competitions and those which involve interactivity.

There was some discussion about the creation of problems and the importance of creating problems with good structure which can capture the imagination.

It was noted that Paul Erdös had commented on competitions, noting the most important thing was the enthusiasm they generated. It was noted that for most participants in popular competitions, the aim was not to win, but to take part, taking up the challenge provided. Olympiads provided higher mountains for the more able students to climb.

3 Assessing Negative Images of Competitions

A number of criticisms are often made of competitions. These include claims that competitions are only for the elite, they involve pressure, widen the knowledge gap, are a negative experience for many students, and favour boys over girls.

The discussion group did not engage in a detailed discussion of these criticisms for each of the competitions represented. Competitions are varied and have different objectives: for instance while some favour broad-based competitions with a high level of success, others aim to support more gifted individuals.

Some participants argued that for competitions to have a positive impact, the teachers must see the progress made by their students. In this view, the role of competitions is to develop a critical body of children who can do problem solving: in a sense, this role is to get people interested. This suggests that a different look at competitions may be needed.

For some, the suggestion that doing mathematical competitions had a negative impact on many students was not born out at all by their experience of broad-based mathematical competitions.

However, it is noted that International Mathematical Olympiad teams contained predominantly more boys than girls. (Apparently, evidence shows that average scores of boys and girls are similar and that boys show a greater standard deviation.) This could be an important subject to research and better understand. Certainly evidence from large, broad-based competitions indicated at least equal participation by girls, at least up to the age of about 15.

With relation to the other points, it was noted that entry in competitions was usually voluntary; they did not normally affect the student's normal assessment, and often gave the student an opportunity to discover talent (as argued in the previous section). One teacher noted that in their experience, elite students in mathematics do not act elitely and that mathematics was an area in which there was much less social pressure than, for instance, in sport.

4 Collaboration and Support for Teachers

Finally there was much discussion on this item. It had been noted in the invited talk by Peter Crippin and elsewhere that competition organizers are now focusing increased attention on support for teachers. This takes place in various forms.

The competitions themselves, often available in well graded and classified form, provide vast resources for classroom discussion. It was noted that the material available to the teachers should not just include problems and solutions, but also that such information should be well structured, with good information on practical use. Some competitions even provide didactical notes for the teachers so that they can know what type of solutions to expect and how to use these in their teaching. Many organizations which run competitions are now running more structured seminars and workshops for teachers.

Mathematics Competitions and Mathematics Education

Andy Liu

We seem to be always complaining about how poorly prepared high school graduates are when they darken our hallowed halls of higher learning, perhaps more urgently now than ever. Certainly, the same complaints are being made in the staff rooms of high schools regarding their new recruits, and so on down the line. Of course, we need to examine carefully how we teach, but it is evident that the system as a whole is not serving a large number of our students very well.

In undergraduate mathematics classes, the first thing we tend to notice is that the students' writing is atrocious. They have a very difficult time putting together a few coherent sentences. When we finally figure out what they want to say, we discover that their logical reasoning is wanting. Most have difficulty handling quantifiers such as *any*, and have no idea what is the difference between a necessary and a sufficient condition, or between a conditional statement and its converse.

Many of them are quite capable of handling routine single-step problems. However, when a problem consists of two very simple steps put together, it suddenly becomes an impossible task. Their skill in analysis and synthesis is very weak, and they lack the initiative to pursue a problem when they cannot see a simple solution immediately. All of these are poor work habits built up over years.

All these criticisms do not seem to apply to students who have had some experience in mathematics competitions. This tends to speak well of the competitions, in that the participants become well-trained in the *process* of mathematics. They have acquired the basic skill in problem-solving, and the difficulty encountered by other students appears trivial. They are able to tackle problems independently, and they do not give up easily.

Are mathematics competitions the answer to the improvement of mathematics education? Despite mass participation mathematics competitions that are held nowadays in many countries, the proportion of students reached is still very small, and the impact too fleeting on too many of them. What is needed is a genuine grass-roots approach to education reform.

That being said, mathematics competitions are still valuable activities that complement classroom learning. Very often, the *content* of mathematics is revealed to the contestants in completely new angles, even those parts that are in the standard curriculum. More importantly, the students gain exposure to areas of mathematics that they do not encounter in school.

As an aside, teachers can benefit from mathematics competitions because they are a fertile ground for hunting down good problems as classroom examples or homework assignments. For those teachers who are involved in the preparation of competitions and who are contemplating introducing new material into the curriculum, they can use the competitions to test them out.

Perhaps the greatest value of mathematics competitions is their benefit on the students' *psyche*. Brought up on a diet of easy gratification, the students need to learn at some point that guaranteed success is no success, just as everyday sale is no sale. They must conquer the fear of failure. Without setback, there is no progress.

Prof. Andy Liu,
Department of Mathematics,
University of Alberta,
Edmonton,
CANADA, T6G 2G1.
Email: aliu@vega.math.ualberta.ca

45^{th} International Mathematical Olympiad 4–18 July 2004 Athens, Greece

The 45^{th} International Mathematical Olympiad (IMO) took place in Athens during the period 9–18 July 2004. Some 486 High School students from 85 different countries representing 80 percent of the world's population arrived to apply their mathematical problem solving skills to six challenging questions.

Just a few days earlier the Team Leaders from each of the participating countries had arrived in the delightful town of Delphi. There they had worked on setting the two exam papers. Their task was to pick six questions from a short list of thirty problems that had been submitted by various countries from around the world. Their aim was to create a balanced paper of attractive problems covering a range of areas and difficulties. The final product contained one question from each of Estonia, Iran, Poland and Romania and two from Korea.

As at the Olympic Games there were opening and closing ceremonies. The contestants paraded on stage in their country groups. Most of the teams were quite content to walk conservatively across the stage, waving to the audience. Some others had surprises, such as the Japanese team who enacted a sword fighting scene, the Columbian team who threw packets of coffee into the audience as the Swiss team did with chocolate. There was a huge applause for the Cuban team consisting of just one person. With only a few weeks until the Games, it was not surprising that in the speeches analogies were made between them and the IMO. Indeed the IMO contestants were referred to as the "athletes of mathematical thinking" by the Alternate Minister of Culture. Following the speeches we were treated to some entertainment which included a presentation of ancient Greek instruments, the organ-like Hydraulis from the 3rd century B.C. (which may be the earliest keyboard instrument) and also the cymbal-like Chalkeophone.

On each of the next two days the contestants applied themselves for four and a half hours to three questions. The IMO is an individual contest, and while $4\frac{1}{2}$ hours may seem a long time for just three questions, they can be extremely challenging and even after a solution is found the student must be very precise in how he presents his work if he wants to gain the full seven points available for each question.

After the exams the contestants were treated to seeing some of Athens including the Acropolis. During this time the Team Leaders, in conjunction with local mathematicians called coordinators, assess the students' work. The whole marking process takes about two days.

The awards were presented at the closing ceremony. There were 243 (50 %) medals awarded of which 120 (24.7 %) were bronze, 78 (16 %) were silver and 45 (9.3 %) were gold. The cutoff scores for medals were set at 16 for bronze, 24 for silver and 32 for gold. Four contestants, one from Canada, one from Hungary and two from Russia managed to achieve the remarkable feat of gaining a perfect score of 42 for the IMO. There were 78 honourable mentions for those students who did not receive a medal but who still did manage to solve one question perfectly. In the ancient Olympic Games the winner was given an olive wreath to wear as a crown and so at the conclusion of the closing ceremony each contestant was presented with an olive wreath. Jozsef Pelikan, President of the IMO Advisory Board, said in his speech to the contestants, "You are all winners!"

Our thanks go to the Hellenic Mathematical Society and the 2004 IMO Organizing committee for hosting a most successful IMO.

Next year the IMO will be held in Mexico.

1 IMO Paper

First Day

1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC. The bisectors of the angles BAC and MON intersect at R.

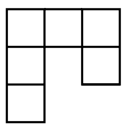
Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC.

2. Find all polynomials $P(x)$ with real coefficients which satisfy the equality

$$P(a-b)+P(b-c)+P(c-a)=2P(a+b+c),$$

for all real numbers a, b, c such that $ab+bc+ca=0$.

3. Define a *hook* to be a figure made up of six unit squares as shown in the diagram or any of the figures obtained by applying rotations



and reflections to this figure.

Determine all $m \times n$ rectangles that can be covered with hooks so that

- the rectangle is covered without gaps and without overlaps
- no part of a hook covers area outside the rectangle.

Second Day

4. Let $n \geq 3$ be an integer. Let t_1, t_2, …, t_n be positive real numbers such that

$$n^2 + 1 > (t_1 + t_2 + \cdots + t_n)\left(\frac{1}{t_1} + \frac{1}{t_2} + \cdots + \frac{1}{t_n}\right).$$

Show that t_i, t_j, t_k are side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$.

5. In a convex quadrilateral $ABCD$ the diagonal BD bisects neither the angle ABC nor the angle CDA. A point lies inside $ABCD$ and satisfies

$$\angle PBC = \angle DBA \quad \text{and} \quad \angle PDC = \angle BDA.$$

Prove that $ABCD$ is cyclic if and only if $AP = CP$.

6. We call a positive integer *alternating* if every two consecutive digits in its decimal representation are of different parity.

Find all positive integers n such that n has a multiple which is alternating.

2 Distribution of Awards

G—Gold medal, S—Silver medal, B—Bronze medal, HM—Honourable mention

Country	Score	G	S	B	HM
Albania	57	-	-	1	1
Argentina	92	1	-	2	1
Armenia	98	-	-	4	1
Australia	125	1	1	2	1
Austria	55	-	-	1	1
Azerbaijan	70	-	1	-	-
Belarus	154	-	4	2	-
Belgium	86	-	1	2	1
Bosnia and Herzegovina	40	-	-	-	2
Brazil	132	-	2	4	-
Bulgaria	194	3	3	-	-
Canada	132	1	-	3	2
China	220	6	-	-	-
Colombia	122	-	2	2	-
Croatia	89	-	-	3	3
Cuba (1 member)	17	-	-	1	-
Cyprus	49	-	-	1	1
Czech Republic	109	-	2	2	-
Denmark	46	-	-	1	-
Ecuador	14	-	-	-	1
Estonia	85	-	-	2	3
Finland	49	-	-	1	-
France	94	-	-	4	1
Georgia	123	-	-	5	1
Germany	130	-	3	1	2
Greece	126	-	2	3	1
Hong Kong	120	-	2	2	2
Hungary	187	2	3	1	-
Iceland	34	-	-	-	1
India	151	-	4	2	-
Indonesia	61	-	-	1	3

Country	Score	G	S	B	HM
Iran	177	1	5	-	-
Ireland	48	-	-	1	-
Israel	147	1	1	4	-
Italy	69	-	-	2	1
Japan	182	2	4	-	-
Kazakhstan	132	2	-	2	-
Korea	166	2	2	2	-
Kuwait	4	-	-	-	-
Kyrgyzstan	63	-	-	1	1
Latvia	63	-	-	1	2
Lithuania	65	-	-	-	4
Luxembourg (3 members)	36	-	1	-	1
Macau	86	-	-	2	2
Macedonia	71	-	-	1	4
Malaysia	34	-	-	1	1
Mexico	96	-	-	3	1
Moldova	135	2	-	4	-
Mongolia	135	-	3	2	-
Morocco	88	-	-	3	3
Mozambique (3 members)	13	-	-	-	-
Netherlands	53	-	-	-	2
New Zealand	56	-	-	2	-
Norway	55	-	-	-	-
Paraguay (3 members)	13	-	-	-	1
Peru (3 members)	49	-	-	2	-
Philippines (5 members)	16	-	-	-	-
Poland	142	2	1	1	1
Portugal	26	-	-	-	2
Puerto Rico (5 members)	43	-	1	-	1
Romania	176	1	4	1	-
Russia	205	4	1	1	-
Saudi Arabia	5	-	-	-	-
Serbia and Montenegro	132	-	2	3	-
Singapore	139	-	3	3	-
Slovakia	119	-	3	-	3
Slovenia	69	-	-	2	2

Country	**Score**	**G**	**S**	**B**	**HM**
South Africa	110	-	3	1	1
Spain	57	-	-	1	3
Sri Lanka	33	-	-	-	2
Sweden	75	-	-	3	1
Switzerland	57	-	-	2	1
Taiwan	190	3	3	-	-
Thailand	99	-	-	4	2
Trinidad and Tobago (5 members)	29	-	-	-	1
Tunisia	31	-	-	-	1
Turkey	118	-	2	3	-
Turkmenistan	52	-	-	2	1
Ukraine	174	1	5	-	-
United Kingdom	133	-	2	4	-
United States of America	212	5	1	-	-
Uruguay	47	-	-	-	-
Uzbekistan	79	-	-	3	1
Venezuela (2 members)	15	-	-	-	1
Vietnam	196	4	2	-	-
Total (486 contestants)		**45**	**78**	**120**	**78**

Angelo Di Pasquale
Australian IMO Team Leader
Department of Mathematics
University of Melbourne
Melbourne
AUSTRALIA
email: pasqua@ms.unimelb.edu.au

Tournament of Towns Corner

Andrei Storozhev

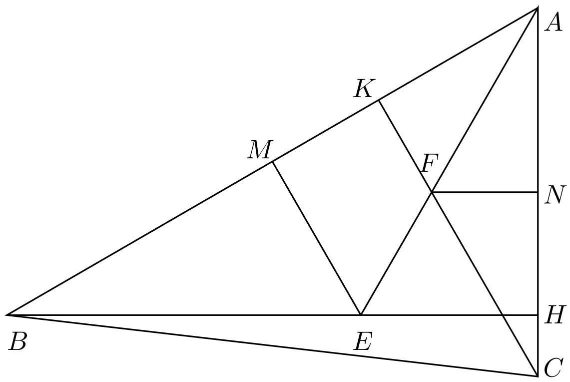
Andrei Storozhev is a Research Officer at the Australian Mathematics Trust. He gained his Ph.D. at Moscow State University specializing in combinatorial group theory. He is a member of the Australian Mathematics Competition Problems Committee, Australian Mathematics Olympiad Committee and one of the editors of the 'Mathematics Contests—The Australian Scene' journal.

1 Selected Problems from the Second Round of Tournament 25

In the second round of Tournament 25, there were five problems in each of Junior and Senior O Level papers, and there were six problems in each of Junior and Senior A Level papers. Selected questions with solutions from this round are presented here.

1. In triangle ABC, the bisector of angle A intersects the perpendicular bisector of AB at E and the perpendicular bisector of AC at F. Prove that if BE is perpendicular to AC, then CF is perpendicular to AB.

Solution. Let M and N be the respective midpoints of AB and AC. Let the extension of BE cut AC at H, and the extension of CF cut AB at K. Note that triangles AEH, AEM and BEM are congruent to one another. Hence $\angle BEM = \angle MEA = \angle AEH = 60°$. It follows that $\angle MAE = \angle EAH = 30°$. Since triangles AFN and CFN are congruent to each other, $\angle FCN = 30°$, so that $\angle CKA = 90°$. Thus CF is indeed perpendicular to AB.



2. There are three identical buckets: the first one with 3 litres of syrup, the second with n litres of water, and the third is empty. We can perform any combination of the following operations:

- We may pour away the entire contents of any bucket.
- We may pour the entire contents of one bucket into another one.
- We may choose two buckets and then pour from the remaining bucket into one of the chosen ones until the chosen buckets contain the same amount.

(a) How can we obtain 10 litres of 30% syrup if $n = 20$?

(b) Determine all values of n for which it is possible to obtain 10 litres of 30% syrup.

Solution.

(a) We describe the process in the following chart.

Action Taken	Amount in Bucket A	Bucket B	Bucket C
Initial State	3	20	0
Pour from B into C until C=A	3	17	3
Pour away C	3	17	0
Pour from B into C until C=A	3	14	3
Pour away C	3	14	0
Pour from B into C until C=A	3	11	3
Pour away C	3	11	0
Pour from B into C until C=A	3	8	3
Pour away C	3	8	0
Pour from B into C until C=A	3	5	3
Pour from A into C until C=B	1	5	5
Pour from B into A until A=C	5	1	5
Pour from C into A	10	1	0

(b) If $n \equiv 0 \pmod 3$, the task is impossible, because the amount of liquid in any bucket at any time will be a multiple of 3, and our target 10 is not. Suppose $n \equiv 2 \pmod 3$. If $n = 2$ or 5, we do not have enough water. If $n = 8$, we can proceed as in (a) from the point where there are 3 L in Bucket A, 8 L in Bucket B and 0 L in Bucket C. If $n \geq 11$, we can reduce the amount 3 litres at a time. Finally, suppose $n \equiv 1$ (mod 3). If $n = 1$ or 4, we do not have enough water. If $n = 7$, we can simply pour everything from bucket B into bucket A. If $n \geq 10$, we can reduce the amount 3 litres at a time. In summary, the task is possible except for $n = 1$, 2, 4, 5 and $n \equiv 0 \pmod 3$.

3. Two 10-digit numbers are called neighbours if they differ only in one digit. For example, the numbers 1234567890 and 1234507890 are neighbours. How many of the numbers from 1000000000 to 9999999999 can we choose so that no two of them are neighbours?

Solution. There are 9×10^9 10-digit numbers. If two of them are non-neighbours, they cannot have the same digits in each of the first nine places. Thus the number of 10-digit numbers we can choose is no more than the number of 9-digit numbers, which is 9×10^8. On the other hand, for each 9-digit number, we can add a unique tenth digit so that the sum of all 10 digits is a multiple of 10. If two of the 10-digit numbers obtained this way differ in only one digit, not both digit sums can be multiples of 10. Hence no two are neighbours among these 9×10^8 10-digit numbers.

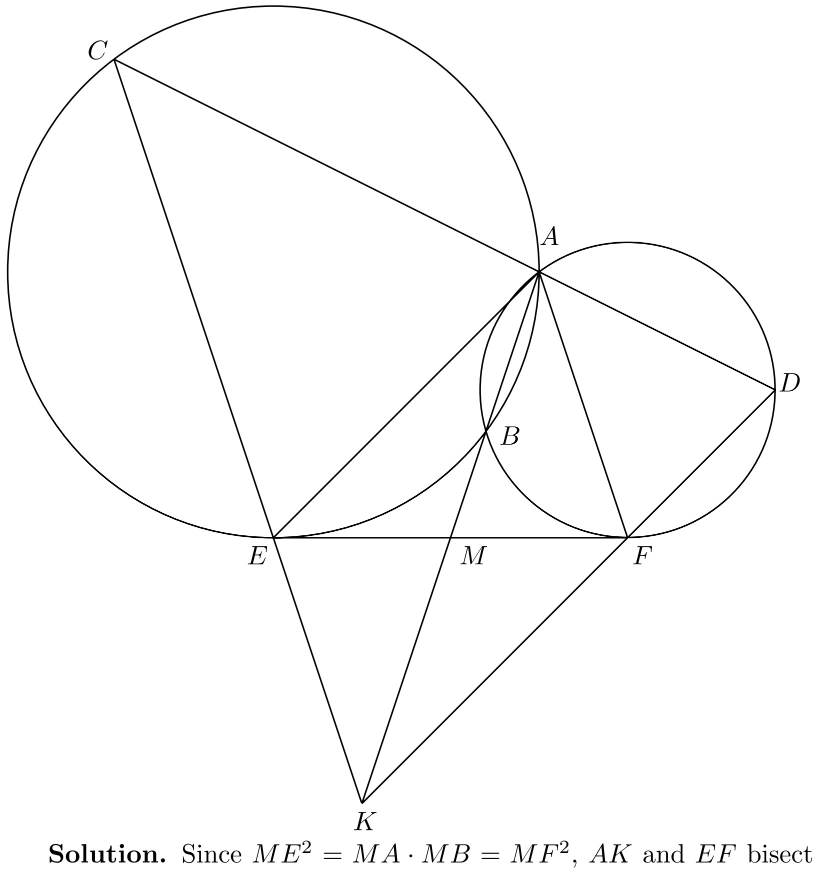
4. What is the maximal number of checkers that can be placed on an $8{\times}8$ checkerboard so that each checker stands on the middle one of three squares in a row diagonally, with exactly one of the other two squares occupied by another checker?

Solution. Clearly, no checkers can be placed on any of the 28 outside squares. Moreover, in the diagram below, at least one in each set of five squares with the same label must be left vacant.

	2		2	3		3	
		2			3		
	2		2	3		3	
	1		1	4		4	
		1			4		
	1		1	4		4	

Thus the number of checkers that can be placed is at most 32. If we leave vacant the 28 outside squares and the 4 central squares, it is easy to verify that all conditions are satisfied. Thus the maximum is 32.

5. Two circles intersect in points A and B. Their common tangent nearer B touches the circles at points E and F, and intersects the extension of AB at the point M. The point K is chosen on the extension of AM so that $KM = MA$. The line KE intersects the circle containing E again at the point C. The line KF intersects the circle containing F again at the point D. Prove that the points A, C and D are collinear.



Solution. Since $ME^2 = MA \cdot MB = MF^2$, AK and EF bisect each other, so that $AEKF$ is a parallelogram. Moreover, since EF is tangent to the circles,

$$\angle KCA + \angle KDA + \angle CKD = \angle AEF + \angle AFE + \angle EAF = 180^\circ.$$

It follows that C, A and D are collinear.

6. At the beginning of a two-player game, the number 2004! is written on the blackboard. The players move alternately. In each move, a positive integer smaller than the number on the blackboard and divisible by at most 20 different prime numbers is chosen. This is subtracted from the number on the blackboard, which is erased and replaced by the difference. The winner is the player who obtains 0. Does the player who goes first or the one who goes second have a guaranteed win, and how should that be achieved?

Solution. Let P be the product of the first 21 primes. The player who goes second has a winning strategy, by always leaving a multiple of P for the opponent. This guarantees a win since both 2004! and 0 are multiples of P. We first show that the opponent cannot turn the table around. Suppose the first player is left with a multiple n of P. Suppose m is subtracted, leaving a difference d. The only way for d to be a multiple of P is for m to be one also, but this is impossible since m is divisible by at most 20 distinct primes. So the first player must leave behind a difference d which is not a multiple of P. Divide d by P and let the remainder be $r < P$. Since P is the smallest number that is divisible by at least 21 distinct primes, r is divisible by at most 20 distinct primes. Hence the second player can subtract r from d and leave behind another multiple of P.

2 World Wide Web

Information on the Tournament, its rules and how to enter it, available the World Wide Web. Information on the Tournament can be obtained from the Australian Mathematics Trust web site at

http://www.amt.edu.au

3 Books on Tournament Problems

There are four books on problems of the Tournament available. Information on how to order these books may be found in the Trust's advertisement elsewhere in this journal, or directly via the Trust's web page.

Please note the Tournament's postal address in Moscow:

NN Konstantinov
PO Box 68
Moscow 121108
RUSSIA

Andrei Storozhev
Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA.
email: andreis@amt.edu.au

WFNMC Congress 5—Cambridge (UK) 22–26 July 2006

We hereby invite all those who are interested in mathematics competitions to attend the 5th World Federation conference in 2006. The venue has been chosen carefully so that WFNMC 2006 can handle record numbers of participants without losing the intimate character of these meetings.

The conference will be held in the self-contained, modern setting of Robinson College, Cambridge—whose two auditoriums, mid-sized and small teaching rooms, chapel, bar and beautiful grounds provide the ideal venue for the "family" atmosphere of a WFNMC conference, within 5–10 minutes walk of the centre of Cambridge. (For information about the venue see http://www.robinson.cam.ac.uk/) WFNMC 2006 will aim to recapture the practical spirit of the original WFNMC conference in Waterloo (1990). Each day will begin with 10–12 small groups working on creating and improving problems, so that less experienced delegates can engage actively and learn from those with more (or just different) experience, sharing their creative ideas in real time. There will be groups to represent all ages (from primary to upper secondary), and all abilities (from popular multiple choice papers to Olympiad style papers). We also hope to run one group devoted to team events. There will also be sessions focusing on practical aspects of running mathematics competitions (administration, finance, sponsorship, how to organise problem setting groups and marking weekends, etc.), where experiences can be shared and developed. Part of each day will be devoted to parallel sessions for contributed papers, covering all possible aspects of national provision—from setting and marking (multiple choice, Olympiad, or team) competitions, through problem creation, analysis of results, administration, all the way to maths camps, maths journals, provision for teachers, etc. In addition we plan a varied programme of related activities, including mathematical talks by "significant" local mathematicians; a "team competition" for delegates and a mathematical pub quiz; pilgrimages to some of the more obvious "mathematical" locations in Cambridge with informed commentaries or short lectures, and a couple of concerts in the delightfully stunning (modern redbrick and stained glass) college chapel.

Hope to see you there!

Tony Gardiner (Chair, WFNMC 2006 Organising Committee)

If you would like to receive further details and application forms when they are available, please send your name, email or postal address, as soon as possible, to either

Tony Gardiner
School of Mathematics
University of Birmingham
Birmingham B15 2TT
UK
email: a.d.gardiner@bham.ac.uk

or

Howard Groves
RGS Worcester
Upper Tything
Worcester, WR1 1HP
UK
email: hg@rgsw.org.uk

The Erdös Award
Call for Nominations

The Awards Committee of the WFNMC calls for nominations for the Erdös Award.

As described in the formal nomination procedures (see www.amt.edu.au/wfnmcaw.html or page 60 this issue), nominations should be sent to the Chair of the Committee at the address below by 1 May 2005 for consideration and presentation in 2006. Such a nomination must include a description of the nominee's achievements together with the names and addresses of (preferably) four persons who can act as referees.

Committee Chair:

Peter J Taylor
Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA

or directly by email at pjt@olympiad.org

WFNMC International & National Awards

1 David Hilbert International Award

The David Hilbert International Award was established to recognise contributions of mathematicians who have played a significant role over a number of years in the development of mathematical challenges at the international level which have been a stimulus for mathematical learning.

Each recipient of the award is selected by the Executive and Advisory Committee of the World Federation of National Mathematics Competitions on the recommendations of the WFNMC Awards Sub-committee.

Past recipients have been: Arthur Engel (Germany), Edward Barbeau (Canada), Graham Pollard (Australia), Martin Gardner (USA), Murray Klamkin (Canada), Marcin Kuczma (Poland), Maria de Losada (Colombia), Peter O'Halloran (Australia) and Andy Liu (Canada).

2 Paul Erdös National Award

The Paul Erdös National Award was established to recognise contributions of mathematicians who have played a significant role over a number of years in the development of mathematical challenges at the national level and which have been a stimulus for the enrichment of mathematics learning.

Each recipient of the award is selected by the Executive and Advisory Committee of the World Federation of National Mathematics Competitions on the recommendations of the WFNMC Awards Sub-committee.

Past recipients have been: Luis Davidson (Cuba), Nikolay Konstantinov (Russia), John Webb (South Africa), Walter Mientka (USA), Ronald Dunkley (Canada), Peter Taylor (Australia), Sanjmyatav Urjintseren (Mongolia), Qiu Zonghu (China), Jordan Tabov (Bulgaria), George Berzsenyi (USA), Tony Gardiner (UK), Derek Holton (New Zealand), Wolfgang Engel (Germany), Agnis Andžāns (Latvia), Mark Saul (USA), Francisco Bellot Rosado (Spain), János Surányi (Hungary), Istvan Reiman (Hungary), Bogoljub Marinkovich (Yugoslavia), Harold Reiter (USA) and Wen-Hsien Sun (Taiwan).

The general meeting of the WFNMC in Melbourne agreed, from 2003, to merge the above two awards into one award titled *the Paul Erdös Award*.

Requirements for Nominations for the Paul Erdös Award

The following documents and additional information must be writtten in English:

- A one or two page statement which includes the achievements of the nominee and a description of the contribution by the candidate which reflects the objectives of the WFNMC.
- Candidate's present home and business address and telephone/telefax number.

Nominating Authorities

The aspirant to the Award may be proposed through the following authorities:

- The President of the World Federation of National Mathematics Competitions.
- Members of the World Federation of National Mathematics Competitions Executive Committee or Regional Representatives.

The Federation encourages the submission of such nominations from Directors or Presidents of Institutes and Organisations, from Chancellors or Presidents of Colleges and Universities, and others.

ICMI Study 16
Challenging Mathematics in and beyond the Classroom
Discussion Document

From time to time ICMI (International Commission of Mathematical Instruction) mounts studies to investigate in depth and detail particular fields of interest in mathematics education. This paper is the Discussion Document of the forthcoming ICMI Study 16 *Challenging Mathematics in and beyond the Classroom*.

1 Introduction

Mathematics is engaging, useful, and creative. What can we do to make it accessible to more people?

Recent attempts to develop students' mathematical creativity include the use of investigations, problems, reflective logs, and a host of other devices. These can be seen as ways to attract students with material that challenges the mind.

Initiatives taken around the globe have varied in quality and have met with different degrees of success. New technologies have enabled us to refine our efforts and restructure our goals. It is time to assess what has been done, study conditions for success and determine some approaches for the future.

Accordingly, ICMI has embarked on its 16^{th} Study, to examine challenging mathematics in and beyond the classroom, and is planning a Conference to be held in Trondheim, Norway, from 27 June to 03 July 2006 at which an invited group of mathematicians and mathematics educators, drawn from around the world, will analyze this issue in detail and produce a report.

This document will suggest specific issues and invite those who might contribute to the discussion to submit a paper, so that the International Programme Committee can select those attending the Conference.

Finally, using the contributions to this Conference, a book (the Study Volume) will be produced. This book will reflect the state of the art in providing mathematics challenges in and beyond the classroom and suggest directions for future developments in research and practice.

The authors of this Discussion Document are the members of the International Programme Committee (IPC) for this ICMI Study. The committee comprises 13 people from different countries, listed at the end of this document. The structure of this Discussion Document is as follows. In section 2 we define and discuss fundamental terms used in the Study. In section 3 we look at the current context, list examples of current practice, observe changes in recent years and identify problems. In section 4 we pose a number of critical questions leading to the results of the Study. In section 5 we call for contributions and outline the process of the Study.

2 Description

Challenge

What is a mathematical challenge? While this may be the topic of discussions during the Study Conference itself, we offer some preliminary thoughts to provide background to debate.

One answer is that a challenge occurs when people are faced with a problem whose resolution is not apparent and for which there seems to be no standard method of solution. So they are required to engage in some kind of reflection and analysis of the situation, possibly putting together diverse factors. Those meeting challenges have to take the initiative and respond to unforeseen eventualities with flexibility and imagination.

Note that the word "challenge" denotes a relationship between a question or situation and an individual or a group. Finding the dimensions of a rectangle of given perimeter with greatest area is not a challenge for one familiar with the algorithms of the calculus, or with certain inequalities. But it is a challenge for a student who has come upon such a situation for the first time. A challenge has to be calibrated so that the audience is initially puzzled by it but has the resources to see it through. The analysis of a challenging situation may not necessarily be difficult, but it must be interesting and engaging.

We have some evidence that the process of bringing structure to a challenge situation can lead one to develop new, more powerful solution methods. One may or may not succeed in meeting a challenge, but the very process of grappling with its difficulties can result in fuller understanding. The presentation of mathematical challenges may provide the opportunity to experience independent discovery, through which one can acquire new insights and a sense of personal power. Thus, teaching through challenges can increase the level of the student's understanding of and engagement with mathematics.

We do note that there are several terms used to sometimes describe similar things, but which really have quite distinct meanings. These terms include the expressions "challenge", "problem solving" and "enrichment". We have discussed the term "challenge" above. Problem solving would appear to refer to methodology, but problem solving is often associated with a challenging situation. Enrichment would be the process of extending one's mathematical experience beyond the curriculum. This might or might not happen in a challenging context.

How do we provide challenges?

Mathematics can challenge students both inside and outside the classroom. Learning takes place in many contexts. Mathematical circles, clubs, contests, exhibits, recreational materials, or simply conversations with peers can offer opportunities for students to meet challenging situations. It is our responsibility to provide these situations to students, so that they are exposed to challenges both in the classroom and beyond.

In this endeavour, the role of the teacher is critical. It is the teacher who is faced with the difficult task of keeping alive in the classroom the spontaneity and creativity students may exhibit outside the classroom.

We note that many teachers do not select problems for lessons on their own, but just follow what is given in a textbook. In this context the role of good textbooks and books of problems is very important. To provide challenge one needs not only to include challenging problems, but also, which is often more helpful, to construct small groups of problems, leading a student from very simple, basic facts and examples to deeper and challenging ones. By carefully selecting problems and organising the structure of textbooks the authors can very much help teachers in providing challenge. It can happen that a student with a good book may develop an interest in the subject even without any help from a teacher.

The support of the general public is likewise critical. Since children are products of their entire social environment, they need the support of the adults around them in acquiring an understanding and appreciation of mathematics. In supporting the new generation, the engagement of citizens in mathematics will open new opportunities for their own personal growth and the public good.

It is important for us to challenge students of every level of motivation, background or ability. Highly motivated students need challenges so that they don't turn their active minds away from mathematics and towards endeavours they find more appealing. Mathematical challenges can serve to attract students who come to school with less motivation, and such students learn from challenging material more than they can learn from the mastery of algorithms or routine methods.

It is particularly important, albeit difficult, to provide challenges for students who struggle to learn mathematics. It is all too easy for students with learning difficulties to content themselves with competence at, or mastery of, algorithmic mathematics, and not attempt to think more deeply about mathematics. However, some practitioners have found that even the learning of routine material is improved when taking place in a challenging environment.

Particularly valuable are situations that can be used to challenge all students, regardless of their background, or motivational level.

The process of providing students with challenge situations itself presents challenges for educators. Some of these challenges are mathematical. Teachers must have a wide and deep knowledge of the mathematics they teach, in order to support students who are working on non-standard material. Other challenges to the teacher are pedagogical. In expanding the kinds of experiences students have, teachers must likewise expand their knowledge of student learning, and their ability to interpret what students say. It is the responsibility of the mathematics and mathematics education community to support teachers in these aspects of their growth.

Where are challenges found?

- Challenge situations provide an opportunity to do mathematics, and to think mathematically. Some are similar to the activities of professional mathematicians. These include:

  Solving non-routine problems
  Posing problems
  Working on problems without achieving a complete solution
  Individual investigations
  Collaborative investigations in teams
  Projects
  Historical investigations
  Organizing whole-class discussions searching ways to solve a problem, a puzzle or a sophism.

- Other challenges are less like formal mathematics. These attract in a different way, leading into mathematics from other contexts. Some of these are:

  Games
  Puzzles
  Construction of models
  Manipulation of hands-on devices

- Still other challenges connect mathematics with other fields. Some examples are:

  Mathematics and other sciences
  Mathematics and the humanities
  Mathematics and the arts
  Real-world problems

- Challenges can be found in a variety of venues and vehicles, including:

  Classrooms
  Competitions
  Mathematics clubs, circles or houses
  Independent study
  Expository lectures
  Books

Papers
Journals
Web sites
Science centres
Exhibits
Festivals, such as mathematics days
Mathematics camps

3 The Current Context

Practices and Examples

There are many ways that students are currently being challenged. These challenges occur both within and outside school and include students as well as general members of the public. They can also be classified in several categories such as competitions, problem solving, exhibitions, publications, and what may be roughly called "mathematics assemblies". Below we refer to some particular cases where challenge is organised. To illustrate this we have used examples which are familiar to members of the International Programme Committee.

COMPETITIONS

Exclusive and Inclusive Competitions

There are many well known competitions such as the *International Mathematical Olympiad (IMO)* and *Le Kangourou des Mathématiques.* The former involves small groups of students from many countries (an example of an exclusive competition) while the latter involves thousands of students in France and Europe (an example of an inclusive competition). Details of these and many other competitions can be found on their web sites as well as in the World Federation of National Mathematics Competition's journal *Mathematics Competitions.*

The word "competitions" may initially conjure up an image of rivalry between individual students with "winners" and "losers". While this may be so in certain situations it is not always the case. Even in the IMO, where medals and prestige are at stake, there is more cooperation than rivalry outside the competition room. In all competitions, though, students work "against the problem" as much as they work "against each other" and there are situations where completing the questions is the main aim rather than "winning". There are also competitions where the students have to compose questions for other students to solve rather than having the questions imposed by the competition organizers. Below we give examples of two competitions that are different in some way from the traditional competition where students are essentially submitted to an examination.

An exclusive competition of interactive style

The competition *Euromath* is a European cup of mathematics. Each team is composed of 7 people: students from primary school to university and one adult. The six best teams are chosen to participate in the final competition by the results of their work on logical games. In the final, these teams work in front of spectators. To win, a team needs to be quick and to have good mathematical knowledge but the most important thing is "l'esprit d'équipe".

Another model of an inclusive competition

KappAbel is a Nordic competition for 14 year olds in which whole classes participate as a group. The first two rounds consist of problems distributed on the Internet and downloaded by the teacher. Within a 90 minute time limit, the class discusses the problems and decides how to answer each problem. The third round is divided into two parts: a class project with a given theme (that ends with a report, a presentation and an exhibition), and a problem solving session run as a relay where two boys and two girls represent the class. Recent themes have been Mathematics and local handicraft traditions (2000), Mathematics in games and play (2001), Mathematics and sports (2002), Mathematics and technology (2003) and Mathematics and music (2004). The three best teams from the third round meet on the following day for the final, which is a problem solving session. The remaining teams form the audience.

References

A web site for IMO: olympiads.win.tue.nl/imo/
Web site for Le Kangarou: www.mathkang.org/
Mathematics Competitions reference: via WFNMC site www.amt.edu.au/wfnmc.html

CLASSROOM USE OF CHALLENGE

Problem Solving

The words "problem solving" have been used to cover a variety of experiences but here we mean allowing students to work on closed questions that they are not immediately able to solve. Hence they need to apply their mathematical content knowledge as well as ingenuity, intuition and a range of metacognitive skills in order to obtain an answer.

Problem solving is often used in classrooms as a one-off exercise that may or may not be connected to the main mathematical curriculum. It can be seen as a "filler" that many students enjoy but it is not always viewed as central to the mathematics classroom.

Investigations and projects may be extended problem solving exercises where students look into more difficult problems over more than one period of class time. They frequently involve a written report.

Teachers who use problems to develop students' ideas, knowledge and understanding of curriculum material can be considered as taking a "problem solving approach" to the topic. This approach can reflect the creative nature of mathematics and give students some feel for the way that mathematics is developed by research mathematicians. Examples of both problem solving lessons and lessons which take a problem solving approach can be found on the web site www.nzmaths.co.nz.

Challenge in traditional education: An example

A traditional method in Japanese elementary school is to solve a problem through full-class discussion. With a skilful teacher, the children can learn more than the curriculum intends. For example, suppose that they are given the problem of dividing 4/5 by 2/3. One student might observe that 6 is the least common multiple of 2 and 3, and write

$$\begin{aligned}(4/5)/(2/3) &= (4 \times (6/2))/(5 \times (6/3)) \\ &= (4 \times 3)/(5 \times 2) \\ &= 12/10.\end{aligned}$$

The children can come to realise that this method is equivalent to the standard algorithm and can be used with other choices of fractions. From the teacher's point of view, this dynamic is unpredictable, and so the teacher requires deep mathematical understanding and sure skills in order to handle the situation. But when the approach succeeds, the children deepen their mathematical experience.

EXHIBITIONS

Exhibitions, in the sense of gathering material together for people to view or interact with, are becoming increasingly common. These are generally outside the classroom and may be aimed as much at the general public as they are at students. They can also take place in a variety of settings from schools to museums to shopping malls to the open air. We mention several examples.

The idea of a science centre is to present scientific phenomena in a hands-on way. This means that the visitors are challenged by a real experiment and then try to understand it. Some science centres have mathematical experiments, but there are also science centres devoted exclusively to mathematics, for instance the Mathematikum in Germany or Giardino di Archimede in Italy. These permanent centres, best visited with a guide, attract tens and hundreds of thousands of visitors per year.

There are also annual exhibitions, varying in content from year to year. An example of this which attracts tens of thousands of visitors per day is *Le Salon de la Culture Mathématique et des Jeux* in Paris. Further, there are also occasional exhibitions, such as the international exhibition *Experiencing Mathematics*, sponsored by UNESCO and ICMI jointly with other bodies, presented in 2004 at the European Congress of Mathematics and the 10^{th} International Congress on Mathematical Education.

Exhibitions can have a special theme, such as the one at the University of Modena and Reggio Emilia, featuring mathematical machines. These machines are copies of historical instruments that include curve drawing devices, instruments for perspective drawing and instruments for solving problems.

Instruments for museums, laboratories or mathematics centres may be very expensive. For classroom use small cheaper kits may be available with information about possible classroom use.

References

Mathematikum www.mathematikum.de
Giardino di Archimede www.math.unifi.it/archimede/
Le Salon de la Culture Mathematiques et des Jeux
www.cijm.org
Mathematical machines www.mmlab.unimo.it

PUBLICATIONS, INCLUDING INTERNET

Publications cover journals, books, web sites, CDs, games and software. They are generally accessible to a wide audience.

School Mathematics Journals

There are many examples around the world of journals designed to stimulate student interest in mathematics. These journals contain historical articles, articles exposing issues within current research, such as the four colour theorem and Fermat's Last Theorem, and Problem corners, where new problems are posed, other current problems from Olympiads are discussed to which students may submit their own solutions. Examples of such journals in the Eastern Bloc, where the traditions are older, are *Kömal* (Hungary) and *Kvant* (Russia). In the West outstanding examples are *Crux Mathematicorum* (Canada), *Mathematics Magazine* and *Mathematics Spectrum* (UK).

Books

There are many publications which enrich and challenge the student's interest in mathematics. In the English language the Mathematical

Association of America has a massive catalogue and the Australian Mathematics Trust has a significant number of publications. In Russia, there is also a very rich resource, traditionally published through Mir. In the French language the *Kangourou* and other publishers have a prodigious catalogue, as does the Chiu Chang Mathematics Education Foundation in the Chinese language. This just refers to major languages. It is expected to be impossible to try to list individual references in this Study. We expect it will be difficult enough to identify the major publishers.

Internet

There are a number of examples in which people can join a classroom by internet. The "e-classroom" conducted by Noriko Arai is a virtual classroom in which everybody interested in mathematics can join by registration. The classroom is run and supervised by a few mathematicians called moderators. Usually one of them gives a problem such as "characterise a fraction which is a finite decimal". Then, discussions start. A student gives a vague idea to solve the problem, a partial answer or a question, and other students give comments on it or improve former ideas. Moderators encourage the discussions, giving hints if necessary. Usually the discussions end with a complete answer. Sometimes a new problem arises from discussions. Otherwise, another problem will be given by a moderator.

N. Arai developed software so that only students of the classroom can have access to the discussions. In this environment a shy child or an elder person who is not so strong in mathematics may feel more comfortable in joining discussions.

"MATHEMATICS ASSEMBLIES"

These activities are aimed at groups of people who generally assemble together in one place to be educated by an expert or group of experts. We have in mind here such things as mathematics clubs, mathematics days, summer schools, master classes, mathematics camps, mathematics festivals and so on. Five specific examples are given below that refer to mathematics days, research classes and industry classes.

School mathematics days

There are many examples around the world of mathematics days in which teams of students from various schools in a district come together. During the day they will participate in various individual and team events in an enjoyable atmosphere, and there may be expository lectures.

Mathematics Clubs

The world has many examples of mathematics clubs (or circles as they are sometimes known) of students who meet at regular intervals in their town to solve new problems. Often these clubs use a correspondence competition such as the *International Mathematics Tournament of Towns* as a focus for their activity. These clubs are usually coordinated by local academics, research students or teachers who do so in a voluntary capacity.

Mathematics Houses

In Iran, a team of teachers and university staff have established what are called *Mathematics Houses* throughout the country. The Houses are meant to provide opportunities for students and teachers at all levels to experience team work by being involved in a deeper understanding of mathematics through the use of information technology and independent studies. Team competitions, e-competitions, using mathematics in the real world, studies on the history of mathematics, the connections between mathematics and other subjects such as art and science, general expository lectures, exhibitions, workshops, summer camps and annual festivals are some of the non-classic mathematical activities of these Houses.
(see www.mathhouse.org)

Research Classes

For many years in Germany the prize for the winners of a mathematical competition is an invitation to a “Modellierungswoche”. In this, groups of 8 students together with two teachers work on a real application problem posed by local industry. Many of the problems are optimisation problems. The solution normally requires modelling, mathematical

analysis and making a computer program.
(see zfm.mathematik.tu-darmstadt.de/)

As another example, in *Math en jeans* each team works in collaboration with a university researcher who has proposed a problem, ideally connected to his/her research, on which the students work for a long period (often up to a full school year).

Trends

It seems that, with few exceptions, the overall trends are positive. For example, there are many new competitions that cater for a wider range of students than the more traditional Olympiad-style competition and include younger children than before. Many competitions now involve groups of students rather than just individuals.

In recent years too, problem solving has been added to the curricula of a number of countries. However, without some professional development for teachers, it may not appear in the actual curriculum delivered in class.

In the same vein, there appears to be an increasing number of mathematical exhibitions. For a while, mathematical exhibits generally appeared in science centres but now there are more exhibitions devoted solely to mathematics. Instead of being held in museum-like settings, mathematics exhibitions exist that are portable or appear in such unusual settings as shopping centres, subways and the open air.

As for publications, there recently seems to have been an increasing number of books and films of a mathematical nature for the general public. Some of these, such as *Fermat's Last Theorem* and *A Beautiful Mind*, have been extremely successful. On the book side though, there may be a trend away from classical problem books to books that discuss mathematical topics and are meant to be read rather than worked on. These books may attempt to convey deep and complicated mathematics but they do so by creating an impression rather than going into great details.

In recent years the Moscow Centre for Continuous Mathematical Education has published a series of books *The library of mathematical* *education*. These are small books (20–30 pages) written by professional mathematicians and addressed to interested high school students. They include popular explanations of various areas of mathematics, challenging problems for students and history. The small size of the volumes, good illustrations, and popular style of writing attract a lot of readers.

It appears that magazines and newspapers are currently carrying more mathematics, both with stories about contemporary mathematics and with problems or puzzles.

Mathematics can be found in many sites on the internet. These sites range from discussions of specific topics to problem sites, to the history of mathematics, to teacher professional development, to games (including sites that claim to read your mind), to emergency rooms where you can ask for mathematical help. There are even more and varied sites that all help to make mathematics more accessible, if not popular.

Problems Identified

The difficulties that these contexts produce fall into two categories: development and applications. In the former category most new initiatives depend on a small number of people for their success. This makes them fragile. It seems often easier to find money to begin new projects than to find continuing support for them.

By applications we mean applications in schools. It is not clear that much of the new material available is being used successfully by great numbers of teachers in the regular classroom. This may be for a variety of reasons. First, teachers are frequently plagued with time constraints as more material, especially involving new subjects outside of mathematics, enters the school curriculum. These subjects reduce the time available for mathematics. Second, especially in senior secondary school, high stakes examinations force teachers into teaching for the examination rather than developing mathematical ideas. And third, teachers may lack the confidence to deal with the new material that was not part of their undergraduate training. They may also be uncomfortable with the more open pedagogy required for challenging situations which are, by their nature, less structured than the traditional pedagogy.

4 Questions Arising

One goal for the Study Conference will be to get a good picture of what is "the state of the art". Here are some examples of issues that may be considered in the context of this Study.

Impact of teaching and learning in the classroom:

- How do challenges contribute to the learning process?
- How can challenges be used in the classroom?
- How much challenge is provided in current curricula?
- What further opportunities to challenge would enhance teaching and learning in the regular classroom?
- How can teachers be made aware of the existence of the different types of challenges?
- How can we ensure that these challenges are compatible with the mandated syllabus?
- How can time constraints in the classroom be handled?
- How can challenges be evaluated?
- How can students be evaluated in challenges?
- How can the effectiveness of using challenge materials be supported by the grading system?
- What sorts of challenges are appropriate for remedial and struggling students?
- What are the implications for teacher training of challenges which are in the classroom?
- What are the implications for teacher training for challenges which exist outside of the classroom?

- What background do students need to handle challenge material and how can this be introduced into the classroom? This includes familiarity with mathematical notation and conventions, ability to reason and draw conclusions, ability to observe and classify and skill at communication.
- How can 'beyond classroom activities' like competitions, exhibitions, clubs, maths fairs etc influence the classroom activities and learning in such a way that all students in the class are challenged and motivated?
- How can teachers, parents and students be made aware that these kinds of activities and challenges will also strengthen the learning and understanding of basic concepts and skills in mathematics?
- Can experience with competitions, maths fairs etc be part of teacher training and in service teacher education? And will this help to engage teachers in "beyond classroom activities" or implement these kinds of activities in classroom practice?
- How can textbooks be written so that challenge is the philosophy and leading idea behind the textbook, and not only fragmental parts of the content of the book?
- How can technology be used by teachers and students to create challenging environments?

Beyond classroom activities

- What is the effect on visitors to exhibitions, festivals etc where they have only a short meeting with mathematical challenges? How can parents, teachers, students and others be helped to go deeper into the mathematics beyond these short meetings?
- How can one make visible the mathematics behind everyday technological devices, and how can this be put into a context that is accessible and mathematically challenging for different groups of people?

Research

- What research has been done to evaluate the role of challenge?
- What can research into the use of challenge tell us about the teaching and learning of mathematics?
- What questions require further research?

More general questions

- How can the mathematics and mathematics education community be involved in this kind of challenge activity that goes beyond their own research interests?
- Are there some branches of mathematics that are more suitable for producing challenging problems and situations?
- How can different designs of challenge activities, in particular competitions, attract different groups of people (the very able students, gender, cultural differences, different achievement etc)?
- What can be done to identify, stimulate and encourage the mathematically talented students?

5 Call for Contributions

The work of this Study will take place in two parts. The first consists of a Conference to take place in Trondheim, Norway, from 27 June to 03 July 2006. The Conference will be a working one. Every participant will be expected to be active. Participation is by invitation only, based on a submitted contribution. Among the attendees, it is planned to represent a diversity of expertise, experience, nationality and philosophy. Such attendance should be drawn broadly from the mathematics and mathematics education community. It is hoped that the Conference will attract not only long term workers in the field but also newcomers with interesting and refreshing ideas or promising work in progress. In the past, ICMI Study Conferences have included about 80 participants.

The IPC hereby invites individuals or groups to submit contributions on specific questions, problems or issues related to the theme of the Study for consideration by the Committee. Those who would like to participate should prepare
(a) a one-page listing of their current position and contact information, as well as of their past and present publications and activities pertinent to the theme of the Study;
(b) a paper of 6–10 pages addressing matters raised in this document or other issues related to the theme of the Study. Proposals for research that is in progress, or still to be carried out, are also welcome. Research questions should be carefully stated and a sketch of the outcome—actual or hoped for—should be presented, if possible with reference to earlier and related studies.

These documents should be submitted no later than August 31, 2005, to both co-chairs of the Study either by post, by facsimile or (preferably) by e-mail. All such documents will be regarded as input to the planning of the Study Conference and will assist the IPC in issuing invitations no later than January 31, 2006. All submissions must be in English, the language of the Conference.

The contributions of those invited to the Conference will be made available to other participants beforehand as preparation material. Participants should not expect to present their papers orally at the Conference, as the IPC may decide to organize it in other ways that facilitate the Study's effectiveness and productivity.

Unfortunately an invitation to participate in the Conference does not imply financial support from the organizers, and participants should finance their own attendance at the Conference. Funds are being sought to provide partial support to enable participants from non-affluent countries to attend the Conference, but the number of such grants will be limited.

The second part of the Study is a publication which will appear in the ICMI Study Series. This Study Volume will be based on selected contributions submitted as well as on the outcomes of the Conference. The exact format of the Study Volume has not yet been decided but it is expected to be an edited coherent book which it is hoped will be a standard reference in the field for some time.

International Programme Committee

Co-chair: Edward J. Barbeau
University of Toronto, CANADA
Co-chair: Peter J. Taylor
University of Canberra, AUSTRALIA

Chair, Local Organising Committee
Ingvill M. Stedøy
Norwegian University of Science and Technology
Trondheim, NORWAY

Mariolina Bartolini Bussi
Universitỳ di Modena et Reggio Emilia, ITALY
Albrecht Beutelspacher
Mathematisches Institut, Gießen, GERMANY
Patricia Fauring
Buenos Aires, ARGENTINA
Derek Holton
University of Otago, Dunedin, NEW ZEALAND
Martine Janvier
IREM, Le Mans, FRANCE
Vladimir Protasov
Moscow State University, RUSSIA
Ali Rejali
Isfahan University of Technology, IRAN
Mark E. Saul
Gateway Institute, City University of New York, USA
Kenji Ueno
Kyoto University, JAPAN
Bernard R. Hodgson, Secretary-General of ICMI
Université Laval, Québec, CANADA

Advisors from ICMI Executive Committee

Maria Falk de Losada
University Antonio Nariño, Bogota, COLOMBIA
Petar Kenderov
Academy of Sciences, Sofia, BULGARIA

Inquiries

Inquiries on all aspects of the Study, suggestions concerning the content of the Study Conference and submission of contributions should be sent to both co-chairs:

Prof. Edward J. Barbeau
Department of Mathematics
University of Toronto
Toronto M5S 3G3
CANADA

Tel: +1 416 653 1961
Fax: +1 416 978 4107
E-mail: barbeau@math.toronto.edu

Prof. Peter J. Taylor
Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA

Tel: +61 2 6201 2440
Fax: +61 2 6201 5096
E-mail: pjt@olympiad.org

The official website for the Study is

http://www.amt.edu.au/icmis16.html

Subscriptions

Journal of the World Federation of National Mathematics Competitions

2 Issues Annually

Current subscribers will receive a subscription notice after the publication of the second issue each year.

For new subscribers, information can be obtained from:

Australian Mathematics Trust
University of Canberra ACT 2601
AUSTRALIA
Tel: +61 2 6201 5137
Fax:+61 2 6201 5052
Email: publications@amt.edu.au

or from our Web site:
www.amt.edu.au

Useful Problem Solving Books from AMT Publications

These books are a valuable resource for the school library shelf, for students wanting to improve their understanding and competence in mathematics, and for the teacher who is looking for relevant, interesting and challenging questions and enrichment material.

To attain an appropriate level of achievement in mathematics, students require talent in combination with commitment and self-discipline. The following books have been published by the AMT to provide a guide for mathematically dedicated students and teachers.

Australian Mathematics Competition (AMC) Solutions and Statistics
Edited by DG Pederson

This book provides, each year, a record of the AMC questions and solutions, and details of medallists and prize winners. It also provides a unique source of information for teachers and students alike, with items such as levels of Australian response rates and analyses including discriminatory powers and difficulty factors. In 2005 there is a separate Primary book containing the questions, solutions and statistics for the 2004 Primary Divisions of the AMC.

Australian Mathematics Competition Book 1 (1978-1984)

Australian Mathematics Competition Book 2 (1985-1991)

Australian Mathematics Competition Book 3 (1992-1998)

An excellent training and learning resource, each of these extremely popular and useful books contains over 750 past AMC questions, answers and full solutions. The questions are grouped into topics and ranked in order of difficulty. Book 3 also available on CD (for PCs only).

Problem Solving Via the AMC
Edited by Warren Atkins

This 210 page book consists of a development of techniques for solving approximately 150 problems that have been set in the Australian Mathematics Competition. These problems have been selected from topics such as Geometry, Motion, Diophantine Equations and Counting Techniques.

Methods of Problem Solving, Book 1
Edited by JB Tabov, PJ Taylor

This introduces the student aspiring to Olympiad competition to particular mathematical problem solving techniques. The book contains formal treatments of methods which may be familiar or introduce the student to new, sometimes powerful techniques.

Methods of Problem Solving, Book 2
JB Tabov & PJ Taylor
After the success of Book 1, the authors have written Book 2 with the same format but five new topics. These are the Pigeon-Hole Principle, Discrete Optimisation, Homothety, the AM-GM Inequality and the Extremal Element Principle.

Mathematical Toolchest
Edited by AW Plank & N Williams

This 120 page book is intended for talented or interested secondary school students, who are keen to develop their mathematical knowledge and to acquire new skills. Most of the topics are enrichment material outside the normal school syllabus, and are accessible to enthusiastic year 10 students.

International Mathematics — Tournament of Towns (1980-1984)

International Mathematics — Tournament of Towns (1984-1989)

International Mathematics — Tournament of Towns (1989-1993)

International Mathematics — Tournament of Towns (1993-1997)
Edited by PJ Taylor

The International Mathematics Tournament of Towns is a problem solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. Each book contains problems and solutions from past papers.

Challenge! 1991 – 1995
Edited by JB Henry, J Dowsey, A Edwards, L Mottershead, A Nakos, G Vardaro

The Mathematics Challenge for Young Australians attracts thousands of entries from Australian High Schools annually and involves solving six in depth problems over a 3 week period. In 1991-95, there were two versions – a Junior version for Year 7 and 8 students and an Intermediate version for Year 9 and 10 students. This book reproduces the problems from both versions which have been set over the first 5 years of the event, together with solutions and extension questions. It is a valuable resource book for the class room and the talented student.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

Australian Mathematical Olympiads 1979 – 1995
H Lausch & PJ Taylor

This book is a complete collection of all Australian Mathematical Olympiad papers since the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads 1981-1993
A Liu

This book contains the papers and solutions of two contests, the Chinese National High School Competition from 1981-82 to 1992-93, and the Chinese Mathematical Olympiad from 1985-86 to 1992-93. China has an outstanding record in the IMO and this book contains the problems that were used in identifying the team candidates and selecting the Chinese teams. The problems are meticulously constructed, many with distinctive flavour. They come in all levels of difficulty, from the relatively basic to the most challenging.

Asian Pacific Mathematics Olympiads 1989-2000
H Lausch & C Bosch-Giral

With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981-1995
ME Kuczma & E Windischbacher

Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
Edited by T Andreescu & Z Feng

This book contains one hundred and one highly rated problems used in training and testing the USA International Mathematical Olympiad team. These problems are carefully graded, ranging from quite accessible towards quite challenging. The problems have been well developed and are highly recommended to any student aspiring to participate at National or International Mathematical Olympiads.

Hungary Israel Mathematics Competition
S Gueron

The Hungary Israel Mathematics Competition commenced in 1990 when diplomatic relations between the two countries were in their infancy.

This 181 page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions.

The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Mathematical Contests – Australian Scene
Edited by AM Storozhev, JB Henry & DC Hunt

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

WFNMC — Mathematics Competitions
Edited by Warren Atkins

This is the journal of the World Federation of National Mathematics Competitions (WFNMC). With two issues each of approximately 80-100 pages per year, it consists of articles on all kinds of mathematics competitions from around the world.

Parabola

In 2005 Parabola will become Parabola incorporating Function, edited by Bruce Henry at the University of New South Wales. It will be issued three times per year in approximately April, July and November. It includes articles on applied mathematics, mathematical modelling, statistics, and pure mathematics that can contribute to the teaching and learning of mathematics at the senior secondary school level. The Journal's readership consists of mathematics students, teachers and researchers with interests in promoting excellence in senior secondary school mathematics education.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Education, Science and Training) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

Newton Enrichment Student Notes
JB Henry

Recommended for mathematics students of about Year 5 and 6 as extension material. Topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

Dirichlet Enrichment Student Notes
JB Henry

This series has chapters on some problem solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory. It is designed for students in Years 6 or 7.

Euler Enrichment Student Notes
MW Evans and JB Henry

Recommended for mathematics students of about Year 7 as extension material. Topics include elementary number theory and geometry, counting, pigeonhole principle.

Gauss Enrichment Student Notes
MW Evans, JB Henry and AM Storozhev

Recommended for mathematics students of about Year 8 as extension material. Topics include Pythagoras theorem, Diaphantine equations, counting, congruences.

Noether Enrichment Student Notes
AM Storozhev

Recommended for mathematics students of about Year 9 as extension material. Topics include number theory, sequences, inequalities, circle geometry.

Pólya Enrichment Student Notes
G Ball, K Hamann and AM Storozhev

Recommended for mathematics students of about Year 10 as extension material. Topics include polynomials, algebra, inequalities and geometry.

T-SHIRTS

T-shirts celebrating the following mathematicians are made of 100% cotton and are designed and printed in Australia. They come in white, and sizes Medium (Polya only) and XL.

Carl Friedrich Gauss T-shirt

The Carl Friedrich Gauss t-shirt celebrates Gauss' discovery of the construction of a 17-gon by straight edge and compass, depicted by a brightly coloured cartoon.

Emmy Noether T-shirt

The Emmy Noether t-shirt shows a schematic representation of her work on algebraic structures in the form of a brightly coloured cartoon.

George Pólya T–shirt

George Pólya was one of the most significant mathematicians of the 20th century, both as a researcher, where he made many significant discoveries, and as a teacher and inspiration to others. This t-shirt features one of Pólya's most famous theorems, the Necklace Theorem, which he discovered while working on mathematical aspects of chemical structure.

Peter Gustav Lejeune Dirichlet T–shirt

Dirichlet formulated the Pigeonhole Principle, often known as Dirichlet's Principle, which states: "If there are p pigeons placed in h holes and p>h then there must be at least one pigeonhole containing at least 2 pigeons." The t-shirt has a bright cartoon representation of this principle.

Alan Mathison Turing T-shirt

The Alan Mathison Turing t-shirt depicts a colourful design representing Turing's computing machines which were the first computers.

ORDERING

All the above publications are available from AMT Publishing and can be purchased on-line at:

www.amt.edu.au/amtpub.html

or contact the following:

AMT Publishing
Australian Mathematics Trust
University of Canberra ACT 2601
Australia

Tel: +61 2 6201 5137

Fax: +61 2 6201 5052

Email: mail@amt.edu.au

The Australian Mathematics Trust

The Trust, of which the University of Canberra is Trustee, is a non-profit organisation whose mission is to enable students to achieve their full intellectual potential in mathematics. Its strengths are based upon its:

- network of dedicated mathematicians and teachers who work in a voluntary capacity supporting the activities of the Trust;
- quality, freshness and variety of its questions in the Australian Mathematics Competition, the Mathematics Challenge for Young Australians, and other Trust contests;
- production of valued, accessible mathematics materials;
- dedication to the concept of solidarity in education;
- credibility and acceptance by educationalists and the community in general whether locally, nationally or internationally; and
- close association with the Australian Academy of Science and professional bodies.