ICMI Study: Mathematical Challenges

Why should we teach mathematics? Traditionalists and politicians tend to answer this question in a practical way: to provide people with skills needed for life and to prepare students for careers. However, many educators decry such a narrow vision. They feel that their appreciation for their discipline can be more widely shared. Even accepting technical proficiency as an important goal, teachers realize that simple exposition and practice are often insufficient to foster utility, depth and fluency; students must somehow be engaged. While this idea is not new in theory or in practice, only recently has it attained widespread currency in facilities for teacher training, classrooms and public presentations around the world. This is due to several factors:

1. the ferment in mathematical education brought on by the need to appeal to a wide variety of students in the modern school and the realization that schools have frequently failed to achieve mathematical literacy
2. research developments in mathematics and its pedagogy
3. technological advances that have engendered access to information, communication among widely dispersed individuals and the ability to explore mathematics to some depth
4. a need to better inform the general public about a subject that (whether it realizes it) plays a large role in its wellbeing.

Into this environment comes the Study Volume for ICMI Study 16, *Challenging Mathematics in and Beyond the Classroom*, commissioned by the International Commission on Mathematics Instruction in 2002. The first fifteen studies, appearing about once a year, have embraced a range of issues, from the different areas of the curriculum to the impact of history, psychology and culture. This one is broadly based, touching on many areas of mathematics, as well as different styles of pedagogy and cultural milieux.

Peter Taylor of the Australian Mathematics Trust and Ed Barbeau were invited to chair the Study and were consulted in the appointment of the International Programme Committee. The IPC met in Modena, Italy to determine the scope of the Study and to issue an invitation for

participation in the Study Conference held in Trondheim, Norway in July, 2006. In all, forty-five scholars from around the world gathered there to lay the foundation for the Study Volume. Over the next two years, teams of authors produced the individual chapters, and the book, entitled *Challenging Mathematics In and Beyond the Classroom*, is due to appear at the very end of 2008. The reader is invited to visit the site for the Study, www.amt.edu.au/icmis16.html, where the papers submitted by the participants in advance of the Study Conference along with a list of participants and the discussion document (the last in four languages) can be found.

To many, the words "mathematical challenge" evoke competitions. Indeed, the two chairmen were concerned that submissions from those in this area would predominate. The discussion paper was designed to make sure that the Study would attract a broader range of participants. A challenge arises in any situation in which "people are faced with a problem whose resolution is not apparent and for which there seems to be no standard method of solution." The committee strongly felt that the facing and surmounting of challenges was an essential component of mathematical learning, so that challenges should be present in all sorts of settings that have nothing to do with competition. In particular, if the classroom is to be the locus of challenge, then this has many implications for the design of the syllabus, formation of teachers and assessment.

In the end, about a third of the participants were "competition types," while the rest represented a host of activities including professional development of teachers, educational research, operation of mathematical exhibitions, design of internet material, work with groups of students in research, and investigation of mathematical problems, and assessment. At the Study Conference, three working groups were constituted to prepare chapters for the Study Volume. One dealt with challenges beyond the classroom (including competitions) while the others looked at challenges within the classroom from the viewpoint of the student and of the teacher. They were inspired by two keynote speakers, Jean-Pierre Kahane, from France, and Alexei Sossinsky, from Russia, both of whom remained for the entire time at the conference and participated actively.

The "Beyond the Classroom" group worked under the direction of Petar Kenderov from Bulgaria. Their deliberations led to the first three

chapters of the Study Volume. Chapter 1 provides many examples with brief commentaries of challenges that arise from contests of all levels, as well as from traditional puzzles and adaptations of textbook material. Alexander Karp ties the chapter together with a discussion of how such examples can be constructed and used. It is abundantly clear that the creative impulses in twentieth century mathematics are not restricted to frontier research; the insatiable appetite for material of contests, projects and investigations has led to the recent creation of problems that are novel, ingenious and compelling. Consider for example this recent IMO problem: Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P. Show that the sum of the areas assigned to the sides of P is at least twice the area of P. There are easy cases that serve as entry points, but the full solution is difficult.

The second chapter surveys all the settings where challenges can be found. Along with competitions, students, teachers and the public have recourse to journals and books, conferences and camps, clubs and circles, exhibitions and houses, public lectures, open houses at universities, correspondence programmes, workshops and fairs, and finally websites. People can work individually or in teams, in a competitive or cooperative environment. The challenge can take the form of a problem with a single answer or a written-out solution or it can be an extended investigation. The sheer variety of contests is illustrated by the examples discussed, the *Australian Mathematics Competition*, *Euromath*, *Kapp Abel*, *Ontario Math Olympics*, *Tournament of the Towns* and the *A-Lympics*. Mathematical publications directed to the young have burgeoned over the last century from humble beginnings in Eastern Europe to books and magazines in every region of the world. There are now many opportunities for students to make their own investigations in countries such as Germany (*Jurgend Forscht*), the USA (*Research Science Institute*) and Bulgaria (*High School Students' Institute for Mathematics and Informatics*). For the general public, many countries boast mathematics exhibitions, such as *Mathematikum* in Giessen, Germany. The numerous activities described in the chapter that illustrate the different settings for challenge show that the time is ripe to take stock, analyze their structure and account for their successes. An appendix of the chapter discusses in more detail the Mathematical

Houses in Iran and the *Archimedes* mathematics organization in Serbia; the Iran case is interesting in its involvement of the local and national governments.

Of course, any discussion of challenges would be incomplete without a discussion of technology, which has become incredibly powerful and pervasive during the last two decades. Computers have not only allowed the dissemination of material and communication over long distances, but have affected how we perceive the whole learning environment, the classroom in particular. We can provide guidance that is tailor-made and designed to bring challenges to within Vygotski's Zone of Proximal Development, that is, to the point where the past experience of the solver can be brought to bear. The third chapter takes up the sort of challenge best suited to the technology environment, the tools that can be employed, the support that technology can provide in more traditional settings, psychological issues and cost. A section treats in detail the characteristics of the Internet-based *Numerical Working Spaces* that can bring together in one place resources for the users. There are numerous cases detailed in the chapter that illustrate design of material and types of context.

Now we move to the classroom with Chapters 4 and 5. In the first of these, the focus is on challenging problems and how they can be organized into strands. It is through the orchestration of challenges that students become aware of underlying structure and gain the ability to articulate their reasoning. Many examples in different settings are described. The fifth chapter opens with a discussion of the social learning environments, time constraints, instruments and objects, and pedagogical method. More examples are discussed, in this case, of activities for school students apart from the regular curriculum, such as visits to museums and art galleries, rallies, additional courses and original research. All of these illustrate how highly students can aim with the proper leadership.

We have to rely on teachers themselves to provide much of the leadership; Chapter 6 addresses the issue of professional development. Teachers must consciously consider the nature of mathematics and the importance of mathematical challenges in the classroom, understand what makes a suitable challenge, and be aware of factors, both psychological and

mathematical, that might undermine success. It concludes with what research tells us about the affect of teacher knowledge and beliefs, psychological development and appropriate pedagogy. Chapter 7 offers some examples to show how classroom tasks can be designed in order that students are willing to accept and appreciate challenges. The final chapter of the Study Volume treats assessment issues. Singapore, in particular, has mandated challenge as part of the curriculum and designed its tests to incorporate challenge items along with procedural ones. Norway too has recently revamped its syllabus with attendant modifications to its examination system. Many questions in this area are posed that require research.

The Study occurred at a critical time, when considerable experience is available to build upon, but when systematic research into the role and use of challenge is only beginning. We expect that the Study Volume will become a standard reference that will help to focus this research.

1 Appendix

The Study Volume is really a related collection of papers, each written by a lead author with the collaboration of a team of participants. Here is a list of the chapter titles along with the authors. The lead author is given in italics.

1. Challenging Problems: Mathematical Contents and Sources (41 pages) Vladimir Protassov (Russia), Mark Applebaum (Israel), Alexander Karp (USA), Romualdas Kašuba (Lithuania), Alexey Sossinsky (Russia), Ed Barbeau (Canada), Peter Taylor (Australia)
2. Challenges Beyond the Classroom—Sources and Organization Issues (44 pages) *Petar Kenderov* (Bulgaria), Ali Rejali (Iran), Maria G. Bartolini Bussi (Italy), Valeria Pandelieva (Canada), Karin Richter (Germany), Michela Maschietto (Italy), Djordje Kadijevich (Serbia), Peter Taylor (Australia)
3. Technological Environments beyond the Classroom (29 pages) *Viktor Freiman* (Canada), Djordje Kadijevich (Serbia), Gerard Kuntz (France), Sergey Pozdnyakov (Russia), Ingvill Stedoy (Norway)

4. Challenging Tasks and Mathematical Learning (38 pages) *Arthur B. Powell* (USA), Inger Christin Borge (Norway), Gemna Inez Fioriti (Argentina), Margo Kondratieva (Canada), Elena Koublanova (USA), Neela Sukthankar (Canada/PNG)
5. Mathematics in Context: Focussing on Students (33 pages) *Maria G. Bartolini Bussi* (Italy), Sharade Gade (India), Martine Janvier (France), Jean-Pierre Kahane (France), Vince Matsko (USA), Michela Maschietto (Italy), Cecile Ouvrier-Buffet (France), Mark Saul (USA)
6. Teacher Development and Mathematical Challenge (38 pages) *Derek Holton* (New Zealand), Kwok-cheung Cheung (Macau), Sesutho Kesianye (Botswana), Maria Falk de Losada (Colombia), Roza Leikin (Israel), Gregory Makrides (Cyprus), Hartwig Meissner (Germany), Linda Sheffield (USA), Bharath Sriraman (USA), Ban Har Yeap (Singapore)
7. Challenging Mathematics: Classroom Practices (40 pages) *Gloria Stillman* (Australia), Kwok-Cheung Cheung (Macau), Ralph Mason (Canada), Linda Sheffield (USA), Kenji Ueno (Japan)
8. Curriculum and Assessment that Provide Challenge in Mathematics (31 pages) *Maria Folk de Losada* (Colombia), Ban-Har Yeap (Singapore), Gunnar Gjone (Norway), Mohammad Hossein Pourkazemi (Iran)

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