The Creative Problem Solving Course at Quincy High School

by

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Abstract

More and more juniors (eleventh grade students) in American high schools are taking calculus. When these students are seniors, they usually take more calculus or a statistics course. At Quincy High School, students are given the opportunity to enroll in an innovative Creative Problem Solving Course introducing topics not usually seen by a typical undergraduate mathematics major. This course challenges our brightest mathematics students and provides them unique educational opportunities.

THE CREATIVE PROBLEM SOLVING COURSE

—Introduction and Motivation

Most large high schools in the United States offer calculus to their advanced mathematics students. Some, such as Quincy High School, have a significant number of juniors (eleventh graders) enrolled in calculus.

What is usually done to challenge these students during their senior year (twelfth grade)? Typically such seniors either take an Advanced Placement statistics class, or enroll in a subsequent semester of calculus at a local college or community college.

At Quincy High School, we offer our students a unique challenge. Seniors who have taken calculus as juniors may enroll in the Creative Problem Solving Course, an innovative course designed by a team of faculty, students, and parents under the direction of Dr. Sandra Spalt Fulte in 1995. This course, currently team taught by Todd Klauser of Quincy High School and Dr. Vince Matsko of Quincy University, offers students challenges rarely encountered even at the undergraduate level.

—A Specific Challenge

Before discussing the content of the Creative Problem Solving Course (CPSC), we take a moment to discuss the group project in the CPSC for the year 2005–2006 as an example of how we challenge advanced mathematics students. It might be helpful to refer to

`http://websites.quincy.edu/~matskvi/qball2.html`

for images to accompany the text.

Briefly, students were given the task of coming up with a group project which the class (17 students at the time) could work on together. Searching the Internet, they decided upon a specific geometric figure called a *zonohedron.* At that time, neither Mr. Klauser nor myself had ever built such a figure, nor did we have any clear idea how to do so.

Some months and a lot of mathematics later, the project became a spherical model nine feet in diameter consisting of 2040 compact discs on an aluminum frame. It had been on display in the Quincy High School library for several months, but had to be disassembled since the CDs started to crack under their own weight.

—Course Content

The overriding philosophy of the course design is to expose students to interesting mathematics they might not have a chance to see even as undergraduates. For example, mathematical envelopes are an elegant application of calculus, but are usually not mentioned in an undergraduate mathematics curriculum. Topics in past years have included various aspects of geometry, number theory, problem-solving, and probability, among other topics.

The Academic Year 2005–2006 marked our second year using a draft of *Polyhedra and Geodesic Structures,* a textbook written by Vince Matsko for use in a geometry class for mathematics majors. The idea was to pilot its use with these bright seniors at the high school level and see what they were able to do with it. The first year was a great success, with the highlight being the spherical model described above. Draft versions of *Polyhedra and Geodesic Structures* have been successfully used in Quincy University's Higher

Geometry course since 1995.

What made the year so successful was a strong emphasis on hands-on work. Todd Klauser's expertise with *Geometer's Sketchpad* was indispensable. The many, many construction projects enabled students to see a real application of their work. Mr. Klauser even insisted they design their own nets for the Platonic solids with *Sketchpad.*

The course outline for the CPSC for the Academic Year 2005–2006 is as follows. Chapter numbers refer to chapters in *Polyhedra and Geodesic Structures* (2005 Draft).

1. Basic constructions with compass and straightedge (Appendix A);
2. Trigonometry review (Chapter 0);
3. Constructions of regular polygons (Chapter 1);
4. The Platonic solids, including the building of models (Chapter 2);
5. Introduction to spherical trigonometry (Chapter 3), with special emphasis on the non-Euclidean aspects;
6. Taxicab geometry;
7. Simple geodesic structures (2-frequency and 4-frequency icosahedra) (Chapter 4) with an emphasis on non-Euclidean aspects. The building of rather large models (6-frequency or 8-frequency) is included;
8. The Archimedean solids (Chapter 5) – students must follow a guided exercise set and algebraically enumerate all of the Archimedean solids.

Building of models is included, both with paper and construction kits such as Polydrons;

9. Angles required for geodesic structures based on the Archimedean solids;
10. Inversion (geometrical and algebraic in the real plane);
11. Spherical right triangles and applications to geodesic structures;
12. Antiprisms and snub polyhedra (spherical trapezoids);
13. Duality of polyhedra;
14. Cartesian coordinates of polyhedra in three dimensions;
15. Mathematical envelopes (an application of basic calculus);
16. Matrices and symmetry groups of polyhedra;
17. Graph theory (applied to graphs of adjacency of vertices of polyhedra);
18. More geodesic structures (Class II structures);
19. Deltahedra (algebraic enumeration and metrical properties). Building of models is included;
20. Reading and discussing *Flatland* by Edwin A. Abbott. Included is a discussion of the fourth dimension;
21. Individual projects. Students undertake a small research project on a topic of their choice (which need not relate specifically to course content), write a ten-page paper, and give a presentation on their topic.

Although ambitious, not every topic is always covered in great depth. However, the common theme of spherical geometry and trigonometry enables students to have a rather firm grasp of these concepts by the end of the year. Further, many results are not merely presented, but their derivations from more basic results are given.

—"State of the Art"

The Creative Problem Solving Course offers to talented high school students an opportunity encountered in very few institutions. This innovative collaboration between Quincy University and Quincy High School introduces interesting topics in an accessible way. Certainly part of the success is due to the choice of subject matter. The construction of geometrical models is always exciting for students, but more than that, students get to see how abstract mathematical ideas take real form. Geometry is an excellent vehicle for exploration, since with the introduction of software such as *Geometer's Sketchpad* and courses such as the CPSC, students can really experience two- and three-dimensional geometry in a meaningful way.

The course is challenging to teach, but the rewards are significant. Mr. Klauser remarked that much of the course content was unfamiliar to him at first, but he eagerly learned as he went. Over the past decade he has become extremely familiar with the course material. As such, it is easier to engage students in active discussion of abstract concepts, such as imagining what a four-dimensional being would be like when reading *Flatland.*

Moreover, Mr. Klauser mentioned that his interest in stellations of polyhedra was energized when one of his early students undertook an individual project on the stellations of an irregular dodecahedron. The scope of student projects over the last several years has been truly remarkable.

—Directions for the Future

There are many steps that can be taken to further the projects described above, in addition to the obvious need to secure a publisher for the textbook *Polyhedra and Geodesic Structures.* Public speaking at regional, national, and international conferences is certainly a part of this endeavor.

Moreover, it would be good to see courses like Quincy University's Higher Geometry course taught at other institutions, especially for prospective high school teachers. Many students who have enrolled in this course build polyhedra in their classrooms in their respective schools. It would be very exciting to see this course become the standard college geometry course for prospective high school teachers. In this way, the excitement and challenge of designing and building geometric models may be made available to all high school students, not just the most advanced.

Educators interested in working with such a course at the high school or college level should contact the authors. We are committed to making whatever resources we have available to ensure course delivery is successful and rewarding.

–Conclusion

Geometry, especially three-dimensional geometry, is often underemphasized in high school curricula. Outside the usual high school course in geometry in the United States, connections to geometric ideas are the exception rather than the rule. Introducing three-dimensional geometry into the classroom, especially for bright high school students, allows for an exposure to a broad array of topics and an appreciation for the interplay of algebra, trigonometry, and geometry, as well as more advanced ideas.

A collaboration between Quincy High School and Quincy University successfully offers a senior capstone course based on these observations. It is hoped that this course may be made more widely available so that other students can take advantage of its innovation.

–References

Hart, George, *The Pavilion of Polyhedreality*; available at

`http://www.georgehart.com/pavilion.html`.

Matsko, Vincent J., *The Amazing Q-Ball!*; available at

`http://websites.quincy.edu/~matskvi/qball2.html`.

Matsko, Vincent J., *Polyhedra and Geodesic Structures*, Draft version, 2005.

Wenninger, Magnus J., *Polyhedron Models,* Cambridge University Press, Cambridge, 1971.

Wenninger, Magnus J., *Spherical Models,* Dover Publications, Inc., Mineola, New York, 1999.