

CHALLENGING STUDENTS IN INTRODUCTORY PROBABILITY COURSES

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Abstract: *This paper describes one method of challenging students in an introductory probability course at a two-year college. In our course project, students were asked to find interesting and challenging curriculum-related problems and prepare group presentations in class. This combination of challenge and collaborative learning appeared to be useful for better understanding probability concepts and rules by all students, including those of a “remedial and struggling” category. We suggest that carefully selected challenging methods can be included into the introductory probability curriculum and used throughout the entire course.*

Key words: *Challenging; Probability; Word problems; Collaborative learning; Group presentations.*

INTRODUCTION

Our studies focus on challenging students in mathematics courses at two-year colleges in USA, particularly at the Community College of Philadelphia. In previous papers, we discussed challenging in introductory and remedial math courses using elementary logic and ancient problems and puzzles (“Identification of gifted students in introductory logic courses”, Proc. 3rd Intern. Conf. “Creativity in Mathematics Education and Education of Gifted Students”, Rousse, Bulgaria, p. 314–317, 2003; “Teaching capable students in developmental mathematics courses”, Proc. 10th Intern. Congress on Mathematical Education, TSG4, Copenhagen, Denmark, p. 67–70, 2004). The present paper addresses the issue of challenging students in an introductory, non-calculus based, course of probability.

Many students in community colleges need to take remedial and lower level math courses to repeat basics of school mathematics. A population of students in such courses is non-homogenous in background; some students take classes because they either failed or never had a chance to complete school mathematics, whereas others only need to refresh their knowledge. Students that consider their past math education as a failure would bring bad memories and doubts in their abilities to the college classroom. For those students, routine methods of teaching would hardly repair damage made by their previous learning experience. Furthermore, time constraints and attempts to study mathematics “by templates” usually result in additional confusion. Students face a hard task when they attempt to memorize a number of formulas and rules during a short time and to restore rapidly skills that young children achieve smoothly over years. Under these circumstances, challenging students in the classroom is a powerful addition to routine lecturing and problem solving.

At community colleges, the introductory probability course is a part of the curriculum for majors in elementary education. Although about half of students in this course choose a career of elementary school teachers, most of them may be placed in a “remedial and struggling” category. Probability is the last math course for many students in this curriculum, and a pressure of getting a higher grade and time constraints typically result in a “studying-for-exam” style. Curriculum-related challenging of students in this course has a high potential in developing logical thinking and creativity that are critical for future teachers.

CHALLENGING IN INTRODUCTORY PROBABILITY

Course project

In the introductory probability course, students have difficulties in seeing connections between basic probability models and word problems of a varying verbal content, which are based on these models. Furthermore, a typical dilemma for students in this course is to combine, in a proper way, intuitive and strictly

mathematical approaches to problem solving. In order to overcome these difficulties and stimulate students' creativity, we offered them a course project. Collaborative learning was employed to engage all students in this activity; study groups were arranged by random. We asked students to pick challenging, curriculum-related problems and present their solutions in class. Textbook problems were excluded; students were expected to find problems in any other sources such as Internet, books, and magazines. Only problems interesting in their verbal content and challenging at a certain level of difficulty could be selected for presentation, and problem solutions should involve basic probability principles and rules. The project started with class discussion about its purpose and possible outcome. Certain amount of points, equivalent to a total of three quizzes, was credited to participants. Students had four weeks to complete the assignment. After presentations, we asked students to fill up a questionnaire and evaluate the course, choice of problems, quality of presentations, and impact of the challenging method on learning.

Results and discussion

Word problems

Word problems selected by students could be divided into two groups: first, problems based on a "real life situation" and, second, problems dealing with basic probability models, such as selecting marbles from the urn or rolling a fair coin. Students, which selected second group problems, intentionally complicated them to compensate their formal content and stimulate interest of audience (they evaluated conditional probability under various conditions, compared models for selecting objects with and without replacement, etc.). Two study groups chose problems that involved the Bayes' formula, which was optional and was not covered in class. Some word problems, selected by students, are shown below in their original version.

Blood Type and Marriage

There are four blood types in humans: O, A, B, and AB. Percentage of people with each blood type in the United States is shown in the table.

| Blood type | O | A | B | AB |
|-----------------------|----|----|----|----|
| Percent in Population | 46 | 40 | 10 | 4 |

- 1) What is the probability that two people getting married both have blood type O? What assumptions are you making?
- 2) What is the probability that two people getting married both have the same blood type? What assumptions are you making?
- 3) Do you think that these assumptions are reasonable?

Foot-in-Mouth Disease

One person per hundred people has the infectious foot-in-mouth disease. The probability of a person with this disease testing positive is 0.9, and the probability of a person who does not have this disease testing positive is 0.2. What is the probability that a person who tests positive has the disease?

Note: positive results in the test are not always correct!

Rental Car

The members of a consulting firm rent cars from three rental companies. 60% from the company A, 30% from company B, and 10% from company C. The past statistics shows that 9% of the cars from company A, 20% of the cars from company B, and 6% of the cars from company C need a tune-up. If a rental car delivered to the consulting firm needs a tune-up, what is the probability that it came from company B?

Head or Tail

A fair coin is tossed twice.

- 1) What is the probability to obtain heads in both tosses?
- 2) If at least one of the tosses is head, what is the probability that both are heads?
- 3) If the first toss is head, what is the probability that both are heads?

Presentations

Students demonstrated positive response to challenging and gave excellent presentations. Some groups prepared more than one problem for presentation. All study group members were familiar well with problems and their solutions. Amazingly, students used correct terminology and described ideas and concepts related to each problem. They explained clearly methods and formulas applied and operated properly with such notions and tools as reduced sample space, tree diagram, conditional probability, complete probability, and the Bayes' formula, all of which were difficult to them in routine class studies. Audience met presentations with a great interest; every presentation evoked a number of questions and was followed by a vivid discussion. In questionnaire and class discussions, students evaluated the probability course as "difficult" and "challenging". There was a general belief that a word problem with an appealing content, even if it is difficult, challenges to find a solution much stronger than an easier but boring problem. Students expressed the opinion that "challenging presentations" stimulated their logical thinking and were useful for understanding probability concepts and rules.

Additional problems

The work on class presentations allowed us to recognize capable students that had logical mind and good presentation skills and were interested in the subject. It is worthy to note that some of them never showed their potential in tests and class studies prior to the project. These students became informal group leaders in the process of problem selection and presentation. We offered capable students to choose an extra-problem from the list of well-known challenging problems and

puzzles and prepare in-class presentation. Two problems chosen by students are shown here.

Life or Death

There was a prisoner sentenced to death. The king offered him a chance to save his life by playing a simple game. He gave the prisoner 50 black marbles, 50 white marbles, and two empty jars. He then said: "Place these 100 marbles into the two jars. You can divide them any way you like, but you have to put all marbles in the jars. Then we will blindfold you and mix the jars around. You then can choose one jar and select one marble. If the marble is white, you will live, but if the marble is black, you will die." How should the prisoner divide the marbles so that he had the greatest probability of choosing a white marble? What is this probability?

Three Cards Problem

Suppose you have three cards: a black card that is black on both sides, a white card that is white on both sides, and a mixed card that is black on one side and white on the other. You put all cards in a hat, pull one out at random, and place it on a table. The side facing up is black. What is the probability that the other side is also black?

The marble problem is a simple puzzle; it was understood rapidly and received a positive reaction of the audience. The well-known three cards problem that represents an example of a counterintuitive problem caused a lively interest. It was not easy for many students to accept the solution of the latter problem, and this stimulated discussion about the relationship between intuition and strict rules in probability.

CONCLUSION

Solving problems in probability is challenging itself, and an introductory probability course provides a good opportunity for challenging students within the curriculum. We offered students in this course to select interesting and challenging curriculum-

related problems and prepare group presentations in class. This work combined challenge and creative collaborative learning. Students found this activity stimulating and helpful for their success in the class. Most of all, students liked the possibility of “independent navigating in the world of math problems”. All students in class participated willingly in this assignment. Even students with a weak background, which struggled for passing the course, were engaged in solving and presenting challenging problems. Capable students, not identified in class previously, were recognized as informal group leaders in the process of problem selection and presentation.

Carefully selected challenging methods can be incorporated into the introductory probability curriculum and used dynamically throughout the entire course.

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