Math outside the classroom: recent trends in Russia

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After the collapse of the Soviet Union, many observers of the scientific and educational scene, including this author, predicted that it would be followed by a rapid disintegration of mathematical education and, in particular, of the very diverse and effective extracurricular activities that were the source of its success: the production of world class research mathematicians and theoretical physicists, as well as highly qualified technicians and engineers. The argument was that a more democratic regime would provide interesting opportunities in business and the liberal professions and thus drain most of the talented youngsters away from the scientific specialties which they favored in Soviet times.

However, this did not occur. The main reason is that a new generation of educators, mostly working outside the official educational establishment, not only managed to maintain the existing extracurricular math traditions, but enriched them with new forms, interesting enough to compete for the minds and hearts of the most talented high school students with the seductions of a comfortable life in a free market society. The result is that today Russia is still producing (mostly for export) not only oil and gas and ice-hockey players, but a large number of research mathematicians and physicists, as well as mathematically literate engineers and other technicians.

In this article, I have tried to pinpoint some of the new forms of extracurricular mathematics activities that have surfaced in the post-perestoïka period and deserve to be known (and perhaps imitated) not only in Russia, but also abroad. The text that follows should not be regarded as an objective overview of the present scene: I have only described those activities in which I have been personally involved and use this occasion to digress on the role of challenges and what may be called “challenging environments"in math education.

§1. The Soviet period (1935-1987)

In order to understand the situation in extracurricular math education in present-day Russia, it is useful to have an idea of the heritage left to us by the Soviet regime, so I will begin with a very brief history of the subject.

The most significant development of interest to us before World War II was the creation by B.N.Delaunay of the math olympiads in Saint Petersburg (then called Leningrad) in 1935 and in Moscow in 1936. They served as a model for other cities, but were interrupted by the war and really began to proliferate and increase in size at the end of the 1940ies. That period was also marked by the resurgence of mathematics lectures for the general public and high school students, and the proliferation of math circles.

The first math olympiad of the Russian federation of the USSR took place in 1961 under the impulse of the leaders of the Leningrad and Moscow olympiads, who were all young research mathematicians (A.Kronrod, N.Konstantinov, M.Bashmakov, V.Arnold, A.Kirillov, D.Fuchs, N.Vassiliev, to name only the most famous).

In 1962-63 the first physico-mathematical boarding schools were founded by the leading mathematicians of the elder generation (M.Lavrentiev in Novosibisk and A.Kolmogorov in Moscow). These were followed by specialized math high schools in the leading cities, created by such outstanding mathematicians as I.Gelfand, E.Dinkin, A.Kronrod, and D.Faddeev.

Among other noteworthy and influential events, let us mention the 6th and 10th International Mathematics Olympiads that took place in Moscow and Leningrad in 1964 and 1968, and the appearance of the first summer math institutes in the country.

A measure of the remarkable development of interest in mathematics and theoretical physics, boosted by the launching of the first *sputnik*, was the immediate success of the popular science magazine for high school students*Kvant* (Quantum), which was created in 1970 by the nuclear physicist I.Kikoin and A.Kolmogorov and achieved the staggering circulation figure of 350 000 per month in 1972.

All of these developments were the work of enthusiasts of different generations working outside the official educational establishment, which was often wary of the overly liberal and democratic atmosphere of these activities and structures. Not everything went smoothly, as exemplified by the crackdown of the authorities on some of the specialized schools (e.g. School # 2 in Moscow), the partial purge of the Kolmogorov boarding school from "politically unreliable elements"(the author of these lines was one of the first to go), and the total replacement of the Jury of the national Olympiad (mainly consisting of the people who had created it) when it was taken over by the Ministry of Education in 1973.

The latter event was not without a positive result: deprived of the official olympiad, one of its founders, N.Konstantinov, created his own olympiad in 1978, the now famous (and international) Tournament of Towns (ITT).

In the following years (often called the years of stagnation by political observers), few significant new developments occurred, but the wide network of extracurricular activities, specialized schools, and competitions continued to function very efficiently despite the political and economic difficulties.

Overall, the period under review is characterized by a widespread early specialization in the mathematical sciences, based on efficiently functioning specialized schools and boarding schools, and a nationwide talent search via the olympiads (which had acquired a pyramidal structure: school, city, region, republic, national olympiad, IMO). The main challenges were olympiads and olympiad-like competitions, and the people who implemented this system were mostly young research mathematicians (often former olympiad winners) with a penchant for teaching. The Ministry of Education was usually suspicious of (if not in opposition to) these activities, and the ordinary teacher, as a rule, did little to support them (in this connection, see §6 below).

§2. Math competitions today

In this section a list of the main mathematics competitions now existing in Russia is presented.

- Saint Petersburg Olympiad (since 1935). The first one in Russia, with its special traditions, including an oral final round.

- Moscow Olympiad (since 1936). A classical olympiad with its own traditions, in particular a very high level choice of problems.

- Classical olympiads (since the end of the 1940ies). Form a nationwide hierarchical structure, from the school to town (or city district) to city to county to region to the national level, culminating in the IMO.

- Lomonosov Tournament (since 1977). Multidisciplinary twice yearly competition for middle school pupils in all subjects, including the humanities. Now spreading from Moscow to other cities.

- Tournament of Towns (since 1978). Konstantinov's alternative (now international) to the IMO, taking place twice a year simultaneously in any city willing to participate, a problem solving competition with a tradition of lovely, non-technical problems.

- A.P.Savin Tournament (since 1995). An online tournament for middle school pupils conducted by *Kvant* magazine, culminating in math battles.

- The Urals Tournament (since 1996?). A multi-level invitation math battle tournament for middle school pupils taking place in Ekaterenburg and other cities of the Ural region.

• Math battles. A new type of team competition, described in §7 below.

• Math regattas. Another team competition, see §8.

Overall, the tradition of competitive mathematics have not only been maintained, but enriched with new forms. They remain the principal tool in the nationwide talent search for scientifically talented students going on since Soviet times.

§3. Digression: negative aspects of competitive math

Mathematical competitions (e.g. olympiads) are often regarded as a training ground for future research mathematicians. However, there are obvious contrasts between participating in an olympiad and doing research:

• the key to success in olympiads is *speed*, while significant research is characterized by *depth*;

• olympiads are conducted within a *rigid framework* (both in subject matter and time wise), while the basic ingredient of research is *freedom of search*;

• in olympiads contestants address *solved problems*, while *open problems* are addressed by research mathematicians;

• the results of olympiads are *immediately evaluated*, while the results of research (especially significant research) often require *long term appraisal*.

The result of this state of affairs is that successful olympiad performers usually acquire a warped understanding of mathematical research: they are usually convinced that research is problem solving, the only difference being that the problems to be solved are open and the time alloted is more than a fixed number of hours. And so successful olympiad performers, once at university, often immediately look for popular problems to solve instead of learning the mathematics really necessary to reach the frontiers of existing knowledge, and then move forward, introducing new concepts, developing new theories and setting new significant problems in the process (which is sometimes more important than solving given problems).

A more serious negative aspect of mathematical competitions is that they discourage the strong and deep (but slow) thinkers from doing mathematics: having failed at olympiads, they decide that math is not their calling, and move on to other fields of activity. Imagine a 15-year old Hilbert or Einstein in Hungary or Russia in the 1970ies – having failed in olympiads (both were notoriously slow thinkers) they would never have done mathematics. Einstein would have become a second rate violinist and Hilbert, a dull school teacher, despised by his pupils?

The issue here is not how many Hilberts and Einsteins we have lost because of olympiads (there weren't so many out there anyway), but how to give high school students the right ideas about scientific research, so as to orient the right people in that direction, and discourage the wrong ones without inoculating a fear of mathematics for the rest of their lives.

§4. Summer schools of the ITT

Orienting high school students, giving them a correct idea of what research is like, is one of the main goals of the Summer Schools of the International Tournament of Towns.

It takes place in different locations in Russia and abroad in the first two weeks of August. Some 60 to 100 students participate – they are the best performers at the ITT of the previous academic year (usually about half are from Russia), more precisely those best performers who were able to obtain sufficient financial support to come (not easy to do for students, say, from Argentina or Australia); if there are more than one or two from a given town, they are usually accompanied by a teacher, who is often the person who ran the Tournament there. The school is headed by N.N.Konstantinov and its math program is run by his younger collaborators.

The style of the ITT summer schools is completely different from the Tournament of Towns itself: there are no problem solving competitions in limited time, no first, second, etc. prizes awarded, no olympiad style mathematics. What the school tries to do is to is to give an imitation of mathematical research, to involve the students in a two week long model of how professional mathematicians work.

To achieve this, a half dozen *cycles of problems* are presented in the form of half hour lectures (followed by hard copy handouts) on the first day of the school by Konstantinov's young (and not so young) collaborators; let's call these people the *problem directors* (I am proud of occasionally still being one of them). Each cycle begins with some simple (but attractive) problems on a certain topic, continues with more difficult problems, and usually culminates in problems (still on the same topic) whose solution is not known to their authors (the problem directors). The students are free to choose one or more cycles of problems to work on (but encouraged to eventually settle on only one cycle), but there are no constraints as to time spent on this work, in fact a student is free to entirely ignore the problems (although very few actually do). Moreover, students can work individually, in pairs or small groups, whatever they prefer; groups from different countries are not unusual. A key feature is that the problem directors must always be accessible for informal discussions with the students.

After a week, the program directors “publish"the collective results of the first week's “research"in one-hour lectures (usually attended only by those working on the given cycle of problems) and reformulate the remaining problems (new ones may surface and be added). In these lectures, the names of the students who have moved ahead the most or have come up with unexpected solutions are usually mentioned, but no hierarchy among is established.

The students start the second week on an equal footing and continue working (if they like) in the same rythm or in a different one. On the last day they may (but don't have to) hand in their "new research results"in written form to the problem directors (who usually spend the last night of the school assessing the work handed in). On the morning of the day of departure, in a gathering of the whole school, the results are made public. There are no first, second, etc. prizes, the active participants receive diplomas, the best performers get honor diplomas.

Besides the scientific program described above, there are many other activities and traditions, sports (with chess and ping pong tournaments, and, on the last day, a highly disputed football match between the students and the teachers), lectures about history or literature, musical evenings and the traditional picnic in a nearby forest featuring the famous 20 liter Konstantinov *samovar*, the main icon of the Tournament of Towns.

One should not think that the principal goal of these school is to recruit future research mathematicians (not very many are needed anyway). Equally important, to my mind, is the fact that many excellent math olympiad performers learn that research is not really their thing, and begin to correctly determine what the role of mathematics should be in their future careers – perhaps not the main object of study, but simply an important tool, or just another tool (along with the computer).

§5. The Dubna summer school

Another summer institute that helps students interested in math understand what the role of mathematics in their lives should be (research, application to science or technology, or simply a tool used in parallel with the computer in other activities, e.g. law or business), is the Dubna Summer School "Contemporary Mathematics".

It takes place five kilometers from the physics research center Dubna, some 120 kilometers from Moscow, at the place where the Dubna river flows into the Volga, in a nice vacation facility surrounded by pine forests. The school is only five years old, it is held in the last two weeks of July, bringing together high school juniors and seniors, university freshmen and sophomores (approximately 80 students in all) and a sizable group of distinguished research mathematicians and university teachers.

Most of the students are current or former olympiad prizewinners, but a certain number of places is reserved for students without such distinctions, who apply to the school via the Internet. In the application forms, they are asked to describe, in brief essay form, their interest in mathematics, answering such questions as: What was the last math book that you have read and how did you like it? or: What mathematical proofs are your favorites (present two)? or further: What mathematical constructions have most impressed you? The idea is to attract those students who like math but do not perform well at olympiads, in the hope that there are potential Hilberts and Einsteins among them (see §3).

The highlight of the school are lectures by famous mathematicians, such mathematical superstars as D.Anosov, V.Arnold, P.Deligne, Yu.Ilyashenko, S.Novikov, Ya.Sinai, V.Vassiliev. There is also a wide choice of cycles of three-four exercise classes by younger mathematicians, either on the material of the lectures or on other topics, which (unlike the lectures) take place in parallel, so that the instructors compete for the listeners. Overall, the program is perhaps overcharged, but then the students are totally free to visit whatever courses they like and are, in fact, encouraged not to take too many of them. There are no examinations, no mathematical competitions of any kind (but of course the students are given problems to solve both in the lectures and in the other classes).

An important feature of the school, expressed in its title ("Contemporary Mathematics") is that the topics discussed in it are chosen so as to lead up to the frontiers of present-day research, and as yet unsolved problems are often presented at the end of the lectures or courses. The aim, of course, being to demonstrate that mathematics is a living science, still intensively being created, and not a rigid body of knowledge that one must learn and then apply, as many math teachers tend to believe and explain to their pupils. Many of the participants, in their assessment of the school (they are asked to write a little essay about their impressions), stress that only now have they begun to understand what doing mathematics is, that it is not only solving problems previously solved by others in limited time and memorizing theorems and proofs – it is a creative process.

The school has developed stable traditions in non-mathematical activities: informal discussions, swimming, sports (there are always football, volleyball, table tennis tournaments), poetry readings, the traditional boat ride on the Volga, campfire songs. An important feature of all of these is the accessibility to the students in an informal atmosphere of professional mathematicians (including the superstars). Thus the Dubna school not only shows what mathematics is all about, but also what working mathematicians are like as people.

§6. Digression: ordinary teachers against challenges

In Russia (and elsewhere) many teachers are unhappy with university people (in particular young research mathematicians) working with their pupils, and do not support math competitions and other challenges. Why?

First of all, teachers usually dislike mathematical challenges of all kinds, because they feel that challenges put their authority in question. Not being able to solve challenging problems (which some of their students do solve) weakens the teacher's position *vis à vis* of the students; unlike professional mathematicians, who readily admit that they don't know how to solve any given problem immediately and are very happy when their students find a solution faster, ordinary teachers are usually reluctant to admit this inability, regarding it as a professional shortcoming. Further, they are unfamiliar with the material (usually not included in the standard school syllabus) that university people like to teach to their pupils. If the teachers come to such lessons, they are often shocked by the total disregard for pedagogical and didactical principles during them, while observing that these lessons attract the gifted students much more than their own (by the sheer enthusiasm of those giving them and/or by the intrinsic interest of the new material). Another important factor (at least in Russia) is that some (if not most) of the young university instructors or graduate students who work with gifted high school students tend to be rather arrogant in their contacts with ordinary teachers, even good ones.

What should be done in this state of affairs? Obviously, it must be patiently and respectfully explained to the ordinary teacher that the inability of solving challenging problems is by no means a professional shortcoming, it is fact of life that actually makes the teaching process more exciting. Further, it should be stressed that extracurricular work with students is only a useful addition to the school syllabus for which the teacher should also be given credit: it is only possible if the teacher has succeeded in getting some of his/her pupils interested in math.

But there is another way of overcoming a teacher's feeling of insecurity concerning challenges: by getting the teacher involved in the right kind of challenges and challenging environments, and not only in olympiads and other elitist math competitions. Two such competitions, which have become popular in Russia in the last few years, are described in §§7-8 below.

§7. Math battles

The math battle is a relatively recent (10-12 years old) new form of team competition, which has become very popular in recent years. Each battle involves two teams of six high school students and two or three members of the Jury, who are not necessarily university faculty – they usually include (and sometimes consist of) ordinary school teachers. Six problems (not too difficult) are chosen by the Jury and given to both teams, who are assigned different rooms and have three hours to prepare their solutions. During this period, the teams organize their work as they see fit (they may work in pairs, the better students try to solve the less trivial problems, etc.), but by the end of the preparation period one of them, the team captain (who ordinarily plays a key role in the preparation stage) distributes the six problems among the six participants, so that each of them is the "expert"on one given problem (the expert is not necessarily the person who solved the problem, the solution may have been explained to her/him by a teammate). The preparation period is followed by a short lunch break, and then the actual math battle begins.

The battle itself takes place in a classroom with a big blackboard. Each of the teams gathers together facing the board, the jury also sits nearby, leaving room for the spectators (there are usually some) in the back of the class. At the beginning, a coin is flipped and the captain of the team that wins the toss (call it team A) challenges the opposite team (B) to present the solution of a chosen problem (one of the six). The captain of team B can either accept the challenge (then he/she sends the expert on that problem from team B to the board) or refuse it; in the latter case (which occurs if the team B captain suspects that team A has no solution of the problem, and is akin to calling the opponent's bluff in a poker game) the expert from team A is sent to the board.

The pupil at the board presents the solution of the problem, while his/her counterpart from the other team is allowed to ask questions (whose aim is to pinpoint possible errors or insufficiencies in the presentation). After the presentation is over (it may take from five minutes to half an hour), the Jury divides the "value"of the problem (usually all the problems are worth the same: 10 points) between the two teams: all ten of the points go to the team presenting the solution if the latter is correct (and no significant defects have been pointed out by the opponent), some of the ten points may go to the opposing team if its representative has convinced the Jury that there were faults in the solution or that that there is a much better one, and finally most (or even all) of the points go to the defending team if it has shown that the presented solution is wrong and that they have a correct one. Sometimes, a team can fool their opponents by getting away with an erroneous solution; in that case, the Jury explains the mistakes and ... gives all the ten points to itself, becoming the third contestant in the battle.

After the first problem is disposed of, the captain of team B challenges team A to solve a problem chosen among the remaining ones, and the proceedings continue until all the six problems have been addressed (this may take from two to three hours). The Jury then adds ups the points and congratulates the winning team.

If the level of the problems is appropriately chosen (this is the key point in the success of math battles) the battle can be highly emotional, exciting and enjoyable (especially for the eventual winners). It should be understood that the format is not intended for the gifted elite, the problems can be quite simple, any class in an ordinary school can choose six pupils who don't do too badly in math, including one (the captain) with leadership qualities (who does well enough in math). Among other things, this competition teaches high school students the strategy of teamwork and performance under stress, something they will may well need in their further career even if mathematics will not be significantly involved in it. To many pupils not particularly interested in mathematics, these battles show that math can be fun, that the subject is not only computational drills and abstruse reasoning.

Teachers also like this format, because they are involved in the proceedings (if not in the Jury, they enjoy following the battle), they see how their pupils perform and get significant feedback about the efficiency of their everyday lessons. Also, they realize that math battles not only give an incentive for solving problems, but also teach students to present them convincingly.

An important drawback of these competitions is that they are rather long, they take almost a whole day to carry out, and therefore it is difficult to use this format to select one winner from a large number of teams. But that can be achieved by other team competitions, e.g. in the framework of "math regattas"described in the next section.

§8. Math regattas

This type of team competition is somewhat "younger"than math battles, but is now becoming increasingly popular in Russia. It has the advantage of being much faster (two-three hours in all) and larger (not two, but several dozen teams can compete simultaneously) than a math battle.

A math regatta is conducted in a large flat auditorium with a big cinema-size screen (the organizers need a projector that sends their computer screen on this large screen). From 20 to 60 teams of four people participate, seated around small tables in the auditorium. A big Jury (10-15 people) is required, as well as a fast and efficient computer operator.

At the beginning, a simple problem is proposed (projected on the screen) and five minutes are given for its solution. Working as a team, each foursome produces a solution in written form and hands it in to the Jury. The first problem, as well as the subsequent ones, are chosen so that the solution can be verified in a few seconds. While the Jury corrects and grades the solutions, one of the Jury members explains the correct answer, projecting it on the screen. As soon as this is done, the computer operator shows the grades of all the teams for the first problem.

Then the next problem (still fairly easy, with ten minutes given for its solution) is flashed on the screen, the answers collected and graded while the solution is explained, the grades summed with the previous ones, and shown on the screen. Then comes the next problem, less easy, with 15 minutes to solve it, and the regatta continues in the same vein for a total of four or five rounds, the last problem (the "half-hour problem") being difficult enough to be solved by only a few teams but simple enough for all the teams to attempt (here, as in math battles, the choice of problems appropriate for the general level of the teams is crucial to the success of the competition). Then the grades are summed up, a few minutes are given to teams that may disagree with the way some of their problems were graded to talk to the Jury, the final total grades appear on the screen and the winning teams receive their prizes.

The whole competition is fast, very dynamic and exciting, interesting for the average student fairly good in math and not only to the "professional problem solvers"coming from high level olympiads. The latter also like math regattas, because this kind of competition is very visible, allows them to show how good they are even as team players. Teachers, for the most part, also appreciate this type of competition.

§9. Don't forget math circles

Although math circles are not part of the subject matter of this article, it should be stressed that this traditional form of challenging environment remains as important as math competitions (old and new) in attracting high school students to mathematics and science in general.

§10. Conclusion

Such is, from this author's subjective point of view, the present Russian scene in mathematical challenges and challenging environments, with its successes and problems. I would like to end this overview on an optimistic note by pointing out that the present generation of young people involved in these activities is at least as good and as successful as the one, soon to leave the scene, that I have had the pleasure and honor to belong to.