Challenging Mathematics in Primary School National Examination in Singapore

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Abstract

This paper is on the culture of challenge that has developed in the Singapore mathematics classroom. The first part provides an analysis of test items from recent primary school national examination. It was found that a significant proportion of the items are challenging. The second part describes a study on a typical class of pupils (aged 11 – 12 years). Based on observations of their problem-solving efforts in pairs and individually over a period of three months, the role of 'big math ideas' such as number sense and the ability to visualize, see patterns and model situations in handling challenging mathematics became apparent. It was also found that the heuristics pupils used and the habits of mind they engaged in played important roles when pupils handle challenging mathematics.

Introduction

Pupils in Singapore schools have consistently done well in international comparative studies in mathematics achievement. While many have attributed this to an array of factors, we have proposed to find a framework to synthesize and explain these factors. The main hypothesis is that a culture of challenge has developed in Singapore mathematics classroom. In such a culture of challenge, every pupil, and not just the high-achievers, is expected to do challenging mathematics. The main study aims to investigate the nature, development and management of this culture of challenge so that all pupils can be engaged at their own pace in meaningful and challenging mathematics. One component of the study is to investigate the kind of mathematics pupils are expected to do. This paper reports some of the initial findings from an investigation into challenging mathematics included in the primary school national examination.

The main purpose of the investigation was to identify the extent and nature of challenging mathematics included in the national examination. The first part of the paper outlines mathematics curriculum reforms in Singapore since 1992, when a problem-solving curriculum was introduced. Recent reforms encourage a shift from teaching heuristics to developing habits of mind to accompany the use of heuristics. Some examples of habits of mind include thinking creatively, thinking critically and metacognition. The second part of the paper describes categories of heuristics used by pupils in the study that have the potential to help many pupils, not just the mathematically-inclined ones, solve challenging mathematics problems.

Problem Solving as a Core Competency

In 1992, the problem-solving curriculum was introduced in Singapore (Ministry of Education Singapore, 1990). Indeed, problem solving is increasingly

acknowledged as a core competency and an important component of school mathematics. Reforms in mathematics curriculum since the early eighties reflect this. In the United States of America, the National Council of Teachers of Mathematics (1989) emphasized that "problem solving should be the central focus of mathematics curriculum" (p.23). The more recent Standards 2000 (National Council of Teachers of Mathematics, 2000) continued to paint a vision for school mathematics where pupils are "flexible and resource problem solvers" (p. 3). In the United Kingdom, the Cockcroft Report (Cockcroft, 1982) stated that "[t]he ability to solve problems is at the heart of mathematics" (paragraph 249). A National Statement on Mathematics for Australian Schools (Australian Education Council, 1990) included as a major goal in school mathematics the capacity "to use mathematics in solving problems individually and collaboratively" (p. 12). This call was echoed in a more recent curriculum document (Australian Association of Mathematics Teachers, 1996). The open-ended approach (Becker & Shimada, 1997), which is a widely practiced instructional method in Japan, centers on problem solving. Most recently, Indonesia adopted a competency-based curriculum where problem solving is a key component (Khafid & Suyati, 2004).

Problem solving, which is the ability to handle unusual and novel situations where routine procedures are not available, is something that education systems around the world aim to help every pupil develop.

Problem-Solving Curriculum in Singapore

Singapore has implemented a problem-solving curriculum since 1992. In 1997, the Ministry of Education made the call for the teaching of thinking skills in key subjects. The initiative Thinking Schools, Learning Nation encouraged the explicit teaching of thinking skills and heuristics. The mathematics curriculum was revised in 2001 to align it better with this initiative (Ministry of Education Singapore, 2000a).

In 2003, the Ministry of Education introduced another initiative to build upon the Thinking Schools, Learning Nation initiative. Schools were asked to help pupils develop good thinking habits or habits of mind under the initiative Innovation & Enterprise. In the National Day speech in 2004, the Prime Minister of Singapore made the call for teachers to teach less to allow pupils to learn more. This call underlines fundamental changes that are required to help pupils acquire a set of competencies that are valuable in knowledge-based economy. Teachers are encouraged to focus of fundamental concepts and use the available time to excite pupils in the learning process and to require them to figure things out.

More than a decade after the implementation of the problem-solving curriculum, schools have been encouraged to develop strategies to help every pupil learn competencies that are important for the 21st century. Alternative strategies are encouraged for pupils who have not done well in schools to also acquire ability to solve problems. A culture of challenge probably has developed in the Singapore mathematics classroom. How can all pupils be embraced in this culture of challenge?

The Present Study

The present study was conducted in two phases. In the first phase, released items from the primary school national examination (called the Primary School

Leaving Examination or PSLE) in the last five years were analyze to identify items that assess competencies beyond procedural knowledge. In the second part, these items were used in an intact class of 38 pupils in a typical primary school in Singapore. While this was a study on just one class, there was a conscious effort to select a class that could be found in any schools in Singapore. The pupils were observed as they solved these problems individually or in pairs over a period of three months.

The PSLE Mathematics is a two-and-a-quarter hour paper-and-pencil test that comprises fifty items of which fifteen are selected-response type items. The remaining thirty-five items were constructed-response type items of which fifteen required pupils to communicate their solution methods. These fifteen items make up 55% of the total score. Released items from 2000 to 2004 were selected for analysis. A total of 196 of the 250 items were released. About 80% of all the items were released each year. The examination is in the English Language which is also the language of instruction, although not necessarily the home language.

The released items were classified as procedural items or challenging items. Procedural items assess knowledge, basic skills, routine procedures and familiar word problem solving. Challenging items require competencies that are beyond routine procedures.

Figures 1 includes some examples of items classified as procedural items. The first item assesses knowledge. The second one assesses basic computation skills. The third one assesses a routine procedure to find area. Although the last item involves several steps in the solution, this type of word problem is familiar to the pupils. Such word problems are typically solved in a linear manner by identifying suitable operations and carrying out those operations.

Item 1
What is the value of the digit 4 in 854 013?
    (1) 4000
    (2) 400
    (3) 40
    (4) 4

                                                        (SEAB, 2005, p.1)
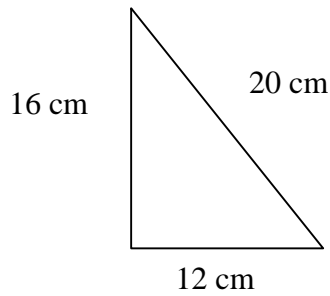
Item 2

Find the value of $\dfrac{3}{4} \div 6$.

                                                       (SEAB, 2005, p.11)

Item 3
A piece of wire is bent to form the right-angled triangle shown below. Find the area of the triangle.

16 cm   20 cm

12 cm

(SEAB, 2005, p.24)

Item 4
Lynn joins Tang Fitness Club. She pays a membership fee of $45. She also pays $5.50 each time she books a badminton court. She books the court 30 times. How much does she pay the club altogether?
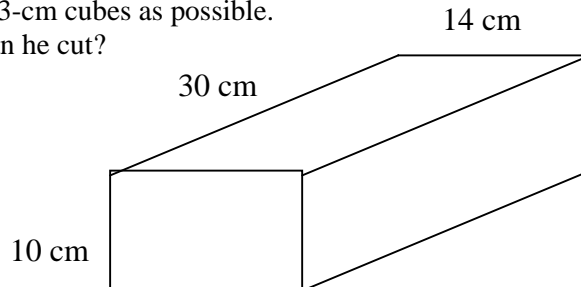
(SEAB, 2005, p.15)

Figure 1   Examples of Procedural Items

Figure 2 includes some examples of items classified as challenging items. The first item require pupils to select the appropriate computation to perform. An inappropriate computation includes the dividing the volume of the block by the volume of a 3-cm cube. In the second item, pupils are unable to succeed even if they possess the necessary computation skills.

Item 1
A toy-maker has a rectangular block of wood 30 cm by 14 cm by 10 cm.
He wants to cut as many 3-cm cubes as possible.
How many such cubes can he cut?

14 cm
30 cm
10 cm

(SEAB, 2005, p.25)

Item 2
Peter, James and Ruth had some stamps. James and Ruth together had 3 times as many stamps as Peter. The ratio of the number of stamps James had to the number of stamps Ruth had was 3 : 7. Peter and Ruth had 310 stamps altogether. How many stamps did Peter have?

Figure 2   Examples of Challenging Items

Among the 196 released items, about a quarter of them were classified as challenging. With such a significant proportion of the items being challenging, it is not difficult to understand why the culture of challenge develops in a typical mathematics classroom.

The challenging items were used in an intact class of 38 pupils (aged 11 to 12 years old) in their sixth and final year of primary school. They were observed over a period of three months solving problems individually or in pairs. The final part of the paper described the initial observations and how the data help us how challenging mathematics can become accessible to all pupils.

The Role of Big Math Ideas in Problem Solving

It was observed that the ability to perform basic computations and follow procedures were not sufficient for pupils to be successful in problem solving. 'Big math ideas' were used together with basic computations and procedures in cases where pupils solved the problems successfully. The four 'big math ideas' used by pupils in the study to solve challenging problems are classified as number sense, visualization, patterning and modeling. Four cases are used to show the role of 'big math ideas'.

Alan and Brian were solving the problem shown in Figure 3. Alan did not a sense that 23 is not used in getting one of the three sums. He simply chose 86 because the question asked for the largest number. Alan performed 86 – 23 to obtain an incorrect answer. Brian knew that 23 is not used in getting one of the sums. He performed 71 – 23 and 61 – 23 and selected correctly the bigger number as the answer. Brian was using the idea that the biggest sum is obtained by using the two biggest numbers. The problem in Figure 3 requires pupils to use number sense in the solution process. Pupils who possess the ability to select the correct operation and perform the operation may not necessarily obtain the correct answer.

The smallest number is 23.
When these numbers are added two at a time, the sums are 61, 71 and 86.
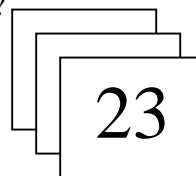What is the largest number on the cards?

23

Figure 3   Three Cards Problem (SEAB, 2005, p.6)

Chris and Delia were solving the problem in Figure 4. Chris knew all the formulae to calculate the areas of squares, rectangles and semicircles but she could not solve the problem because the dimensions of each shape were not given explicitly. Delia could see that the width of the rectangle is the same as the radius of the semicircle, that the length of the square is the same as the length of the rectangle and that the length of the square is also the same as the diameter of the semicircle. These

observations allowed her to use the 70 cm to find the various dimensions of the shapes and to calculate the area of the figure. The problem in Figure 4 requires pupils to visualize. The knowledge of a method to compute area of figures is necessary but not sufficient.

The figure below is made up of 2 identical squares, 4 identical rectangles and 3 identical semicircles. What is the area of the figure? (Take $\pi = 3.14$)
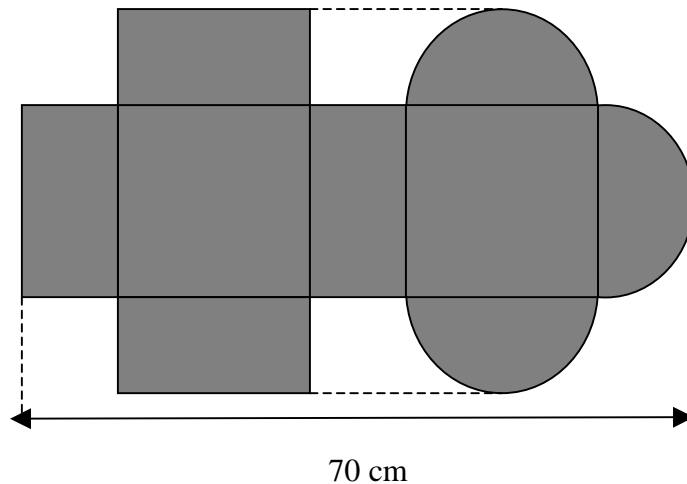

70 cm

Figure 4　　　Composite Figure Problem (SEAB, 2005, p.32)

Eddy and Fan were solving the problem in Figure 5. Eddy was thinking of adding up all the numbers from 1 to 97. This would have been too time-consuming. Fan did think of doing the same but knew she would not do that. She tried to look for a pattern and found that 97 can be paired with 3, 96 can be paired with 4, 95 can be paired with 5 and so on. As each pair form a sum of 100, the digit in the ones place for $3 + 4 + 5 + \ldots + 95 + 96 + 97$ is 0 as the number that does not pair up to form 100 is 50. Fan knew that the ones digit comes from the first two whole numbers. The problem in Figure 5 requires pupils to observe a pattern and use it to solve the problem in an elegant manner. The ability to add numbers does not guarantee a correct solution.

$1 + 2 + 3 + 4 + \ldots + 94 + 95 + 96 + 97$
When the first 97 whole numbers are added up, what is the digit in the ones place of this total?
(1) 1
(2) 2
(3) 3
(4) 8

Figure 5　　　Ones Digit Problem (SEAB, 2005, p.4)

Gary and Huiling were solving the problem in Figure 6. Gary did $12 \div 4 = 3$ and $3 \times 9 = 27$ to obtain an incorrect answer. He did not model the situation but instead used a computational approach to solve the problem. Pupils who use a computational approach perform operations on the numbers in a superficial way. Huiling did $12 \div 3 = 4$ and $4 \times 8 = 32$. She was able to model the situation by sketching a simple diagram of the situation. Pupils who use a modeling approach consider the situation before deciding on the computations to be done. The problem in Figure 6 requires pupils to model the situation by drawing a diagram or by visualizing

the situation. The ability to select the correct operations and the ability to perform computations do not always lead to the correct solution.

> Mr Lau planted 9 seedlings in a row.
> The seedlings were planted at the same distance apart.
> The distance between the first and fourth seedlings was 12 cm.
> What was the distance between the first and ninth seedlings?
> (1) 24 cm
> (2) 27 cm
> (3) 32 cm
> (4) 36 cm

<p style="text-align:center">Figure 6   Seedlings Problem (SEAB, 2005, p.7)</p>

The Role of Heuristics in Problem Solving

Mathematics textbooks used in primary schools in Singapore were analyzed to show common heuristics used (Yeap, 2005). Based on the two most common textbook series used, it was found that pictorial heuristics are common.

> Tom and Gary ran in a race.
> When Gary had completed the run in 20 minutes, Tom had only run $\frac{5}{8}$ of the distance.
> Tom's average speed for the race was 75 m/min less than Gary's.
> (a) Find the distance of the race.
> (b) What was Tom's speed in m/min?

<p style="text-align:center">Figure 7   Race Problem (SEAB, 2005, p.40)</p>

Indra solved the problem in Figure 7 by drawing a simple sketch shown in Figure 8. Using the basic idea of speed, Indra knew that every minute, the gap between Gary and Tom increases by 75m. Thus, after 20 minutes, the gap between Gary and Tom is 1500m, which is 3 eighths of the distance of the race. The subsequent computations that Indra did to answer the first question were $1500 \div 3 = 500$ and $500 \times 8 = 4000$. The computations were simple because a diagram was used. A similarly simple computation was done to answer the second question.
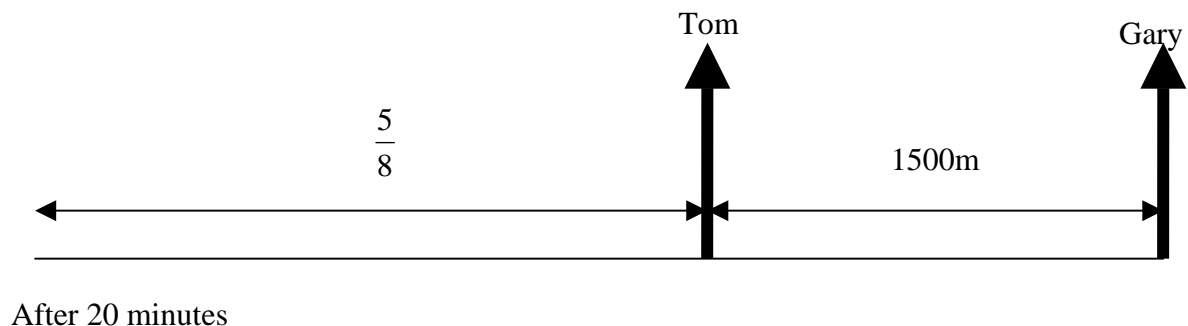

After 20 minutes

<p style="text-align:center">Figure 8   Indra's Solution</p>

At first Sara had $\dfrac{4}{7}$ of the number of marbles Jack had.

When Sara received 36 marbles from Jack, both had the same number of marbles.
(a) How many more marbles did Jack have than Sara at first?
(b) How many marbles were there altogether?

Figure 9   Marbles Problem (SEAB, 2005, p.17)

Janice solved the problem in Figure 9 by using a method that is known as the 'model method' in the Singapore textbooks. She used rectangles to represent unknown amounts. Her initial diagram is shown in Figure 10a.

Sara

Jack

Figure 10a   Janice's Solution

Subsequently, Janice modified her diagram as shown in Figure 10b and did the computation $36 \div 3 = 12$ and $6 \times 12 = 72$ to answer the first question and $22 \times 12 = 264$ to answer the second question. She did not have to do cumbersome computations involving fractions because she used the diagram.
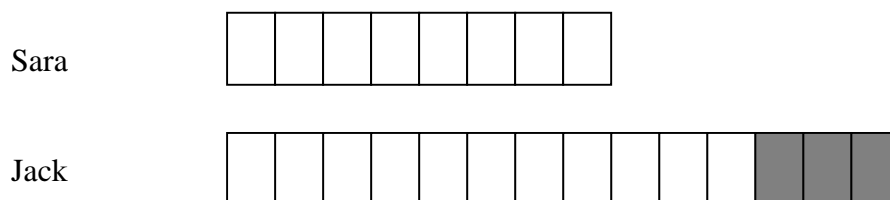
Sara

Jack

Figure 10b   Janice's Solution

The Role of Habits of Mind in Problem Solving

Polya (1957) outlined the main staes of problem solving as understanding, planning, doing and looking back. In each of these stages, pupils who demonstrate good habits of mind have a greater chance of proceeding to the next stage.

Jane had a total of 156 red and yellow beads in the ratio 7 : 6.
After she gave a way an equal number of each type of bead, the number of red and yellow beads left was in the ratio 7 : 3.
(a) Did the fraction of red beads that Jane had increase, decrease or remain the same?
(b) How many beads did she give a way altogether?

Figure 11   Beads Problem (SEAB, 2005, p.46)

Ming and Nick were solving the problem in Figure 11. Ming was initially confused by what he read as he did not read for comprehension. Nick read the first sentence and drew a diagram and did some computation before he proceeded to the second sentence. Nick demonstrated a good habit of mind during the reading phase. Ming was able to answer the first question correctly but did not make use of it to understand that the 7 parts before and the 7 parts after Jane gave away the beads does not represent the same amount.

Red

Blue

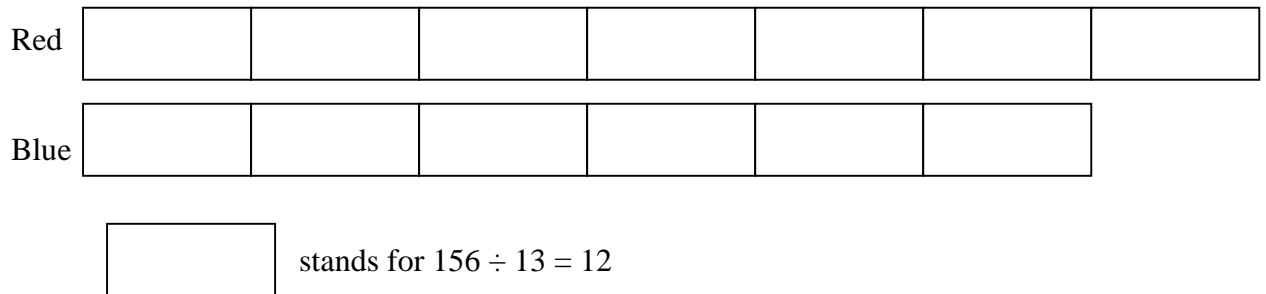stands for $156 \div 13 = 12$

Figure 12   Nick's Solution

Both pupils planned to use the 'model method' and proceeded in a similar way. Both did not achieve much success. Ming tried to draw the 'model' for the situation after the beads were given away but the model was incorrect. Ming gave up after about fifteen minutes. He did not have the habit of mind to change his plan. Perhaps he could have used systematic listing to work with known numbers. Nick, after a while, began to look at the information from different perspectives. He finally stumbled upon the idea that the difference between the red and blue units after Jane gave away the beads is 4 units. This was a major breakthrough for Nick.

Red

Blue
4 units

$12 \div 4 = 3$
She gave away 21 units red and 21 units blue.
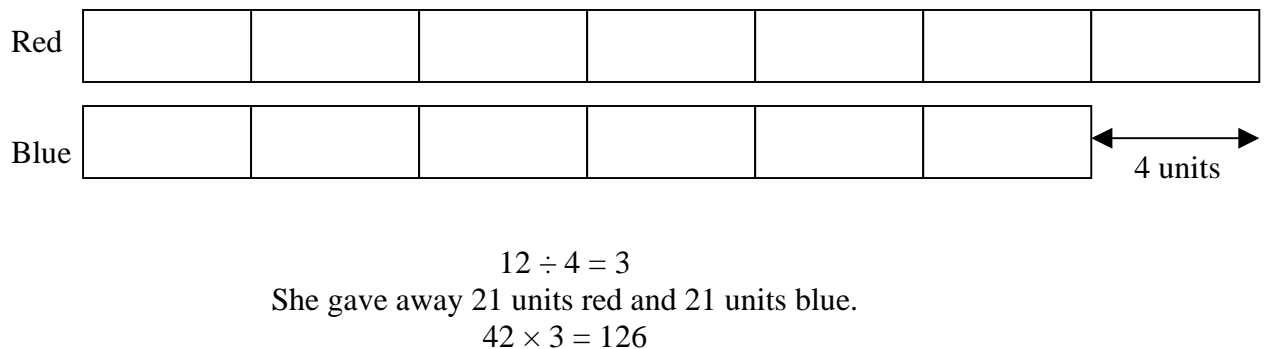$42 \times 3 = 126$

Figure 13   Nick's Breakthrough

Nick's breakthrough during the doing phase was due his ability to look at the information from a new perspective.

In the above case, the role of habits of mind in different phases of the problem-solving process can be seen. Ming did not monitor his reading. He also did not seek other plans when the 'model method' did not work. Nick had a monitoring strategy when he read the problem. He was able to look at the same information in a different perspective perhaps because it had become his habit to seek out new ways when he was stuck.

Conclusion

The findings in the investigation reported in this paper suggest that items in the primary school national examination in Singapore that were classified as challenging were challenging for several reasons. Some were challenging because pupils need to go beyond computational proficiency to succeed. Pupils who succeeded had 'big math ideas' such number sense, visualization, patterning and modeling. Others were challenging because pupils required suitable heuristics to simplify difficult ideas. It was found that diagram-type heuristics were of particular help for many pupils. And yet others were challenging because pupils needed good habits of mind to make the leap to a correct solution.

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