

# Suitable activities for and possible factors influencing the outcomes of challenging mathematics in and beyond the classroom<sup>#</sup>

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**Abstract.** This contribution presents several activities whereby challenges may be promoted in and beyond the classroom. Because some challenges might not be suitable for a particular learner or environment, the contribution also examines factors that may influence the outcomes of the use of challenges.

## Introduction

To reduce the complexity of mathematics education making educational things (more) manageable, the design and evaluation of this education should take into account few distinctive types of tasks, learners, instructional designs, and other critical educational issues recognized by research and practice. By assuming this approach, concern “How to make mathematics accessible to more people?” becomes question “Which mathematical objects should be used for such and such kinds of learners (students, teachers or adults) under that (those) instructional design(s)?” By realizing the importance of this question, it is easier to understand why an initiative attracting students with mathematical challenges may not meet an expected degree of success because, for example, those mathematical objects challenged are not be suitable for many learners in that learning environment.

Having explained how mathematical challenges should be viewed, the ICMI Study 16 Discussion Document (IPC, 2004) lists challenging situations, areas and venues of challenge, underlying that providing students with mathematically challenging situations is itself a challenge for mathematics teachers who need an improved pre-service and in-service professional development. Although this document also lists a few tenths of relevant questions regarding challenges, it does not deal with suitable activities for challenging mathematics and possible factors influencing their outcomes. This contribution will thus first presents several suitable activities that may be used in challenging mathematics in and beyond the classroom. It will then present possible factors influencing their outcomes, evidencing why these outcomes should, among other things, be examined in terms of learner type and learning environment type. Note that all tasks presented here have been used in the authors’ work with upper secondary students and reported in some of his published papers.

## Suitable activities for challenging mathematics

### *Solving tasks in several ways*

According to Shimizu (1999), finding alternative solution methods is, contrary to mathematics education in Germany and United States, considerably utilized in Japanese mathematics education, which has probably contributed to its continuously top ranked outcomes in the TIMSS mathematical studies (these outcomes can be found at <http://timss.bc.edu/>, for example).

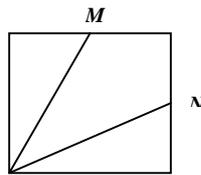
Consider the following task: “A car and a truck set off simultaneously from towns that are 210 km apart. After what time did they meet each other if their speeds were 80 km/h and 60 km/h, respectively?”, which is particularly suitable for eight- or ninth-grade students. It can be solved arithmetically, algebraically and graphically, and pupils should be encouraged

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to come up with different solutions. What makes this problem (or other problem on piecewise uniform motion) a challenge rather than an exercise can be found in the fact that although one solution may be routinely accessible to a pupil, other solution is much less of this nature. And even if all three solutions are routinely accessible, an additional condition like “soon after the departure, the car driver stopped for 15 minutes to take a cup of coffee” is likely to provoke a challenging situation. From all these solutions pupils would learn to work with and coordinate among different solution “microworlds”, which in Papert’s (1987) sense, enables the development of conceptual knowledge out of procedural knowledge fractured in those microworlds.

Some tasks can be solved in ten or more ways, enabling links to be made among various mathematical topics. An example of such task can be found in Barry (1992): “In the square below,  $M$  and  $N$  are midpoints of the corresponding sides. Determine the numerical value of  $\sin\alpha$ .”



This task should be given to students who have learned all relevant issues concerning trigonometry, vectors, analytic geometry and complex numbers (usually eleventh- or twelfth-grade students). To maintain that the task remains to be a challenge than rather an exercise, the teacher should require his/her students to find a solution by using a particular, not previously utilized topic (e.g. vectors). This approach would help students realize that mathematical topics are still connected (as one surprised student exclaimed during the work) although they do not appear so under traditional instructional designs. A simple way to solve this task discovered by students talented for mathematics is given in Kadijević and Krnjaić (2003). Note that some students may start with relation  $\sin\alpha/2 = \sqrt{10}/10$  and use a calculator to do the rest.

#### *Solving tasks on connecting knowledge and its application*

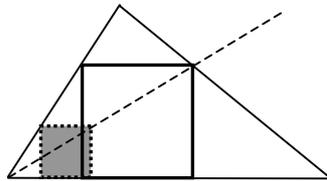
Because real-world modelling tasks such as “Design a rear windscreen wiper” (see Clatworthy & Galbraith, 1991) are, despite their high challenging potential, out of reach of many students (even at the tertiary level of mathematics education), simpler, still challenging, modeling tasks are to be utilized. Two such tasks, taken from Kadijevich (1999), are given below. Their challenges arise from finding the conceptual basis for some given application (the first task) and from finding out an application of some given piece of mathematical knowledge (the second one).

1. How is a rectangular foundation dug? The foundation is marked by a rope, and if the diagonals of the rectangle are (almost) equal, its digging is undertaken. Otherwise, the rope stretching and the equality checking are repeated. Which item of knowledge validates this procedure? (In case of complex ideas, the request for justifying the underlying knowledge may be omitted.)
2. It is known that a transversal intersects two parallel lines so that the alternate-interior angles are equal. Utilize this item of knowledge in order to make an optical instrument. (The area of application may or may not be given.)

Through solving tasks connecting knowledge and its application, the knowledge of mathematics would become alive and more personalized, and thus more accessible to students and their teachers. Solving such tasks would also help learners relate procedural and conceptual mathematical knowledge, which continues to be a major challenge of mathematics education (see Haapasalo and Kadijevich, 2000; Kadijevich and Haapasalo, 2001).

### Solving tasks on extending the applicability of problem solving methods

In his outstanding book “How To Solve It”, Pólya (1990) highlighted a number of heuristics such as analogy, specialization and generalization, which help the solver to understand the examined problem or to make progress in solving it. However, it was Schoenfeld (1985) who underlined that each of Pólya’s heuristics denotes a set of strategies that are context dependent. In applying specialization (focusing on a subset of some set of objects), we may only consider factorable polynomials, regular polygons, special integers (such as 0, 1 and -1), etc. To solve problems successfully, it is therefore very important to be able to apply heuristics in different contexts. To achieve this skill, the learner should gain experience in extending the applicability of a problem solving method or in realizing its utilization in different contexts. An ancient method termed *regula falsi*, or the method of false presumption, can be applied in solving very diverse tasks (Kadijevich, 1990; Marinković & Kadijević, 1990). Suppose that one knows how to solve the problem “What is the value of a heap if it and its third is equal to 12?” by assuming that the heap contains 3 objects and then by correcting this presumption by a multiplication. A task to extend this method to geometry would probably help us inscribe a square into a given triangle by enlarging a smaller square as indicated below.



Further extensions of the method (initially dealing with an increase or a decrease of a number or a figure) may yield quite unexpected applications such as those given below (see Kadijevich, 1993). Note that these hints are just given for the reader. An able learner (a teacher or an upper secondary student) who is coping with this task may come up with other solutions reflecting his/her knowledge.

1. Solve difference equation  $x_{n+1} - \frac{x_n}{2} = \frac{1}{n}$ .

*Hint.* Find its solution in the form  $x_n = c_n y_n$ , where  $y_n$  is the solution of the equation  $y_{n+1} - \frac{y_n}{2} = 0$ . Then

try to solve the differential equation  $y' - y = x$ , by modifying the solution found for the equation  $y' - y = 0$ .

2. By using the compass and ruler, construct a tangent plane to the sphere  $L(L', L'')$ , which passes through the point  $M(M', M'')$  forming an angle of  $60^\circ$  with the horizontal plane of projection.

*Hint.* Use a plane perpendicular to the vertical plane of projection that is tangent to the sphere forming an angle of  $60^\circ$  with the horizontal plane of projection. Rotate this tangent plane, “a false plane”, around the axis that passes through the centre of the sphere being perpendicular to the horizontal plane until it passes through the given point.

3. Write a computer program (in Pascal, for example) that computes the sum of  $n$  given numbers.

*Hint.* Start with a sum that is equal to 0 and then add to it one number at a time. What presumption should be used in determining the product of  $n$  given numbers?

It is true that the ICMI Study 16 Discussion Document (IPC, 2004) underlines the importance of challenging solution methods used in traditional education [that dividing  $4/5$  by  $2/3$  can be done through calculation  $(4 \times (6/2)) / (5 \times (6/3))$  is examined]. However, the need for challenging solution methods of the same task or for challenging the same solution method in different (knowledgeably distant) problem situations is not emphasized.

### Solving tasks on multimedia instructional design

Those who learn more from the instructional materials are their developers, not users (Jonassen, 2000). The learners of mathematics should thus design multimedia lessons and

become knowledge constructors rather than knowledge users. By respecting general principles of multimedia learning (Mayer, 2001), these lessons, realized in, for example, the form of multimedia HTML pages (i.e. simple hypermedia) utilizing Java applets (specially developed and/or downloaded from the Internet), may respect the following requirements: (1) present historical and epistemological issues of the chosen topic; (2) show its underlying mathematical structure; (3) present its contribution to applications and modelling; (4) enable various learning paths within it; and (5) promote links between procedural and conceptual mathematical knowledge. Such a multimedia design, which can be a rewarding learning experience for future mathematics teachers (Kadijevich & Haapasalo, 2004), should, whenever appropriate, involve two other kinds of technology-supported learning: learning through applications and modeling and learning through on-line collaboration (see Kadijevich, 2004).

To fulfill requirements (1)-(5), on the basis of a didactical analysis of the covered topic (see Marjanović, 2003; Marjanović & Kadijevich, 2001), the designer may make use of the three above-mentioned activities: solving tasks in several ways, solving tasks on connecting knowledge and its application, and solving tasks on extending the applicability of problem solving methods. For example, along with solving some tasks in several ways, a multimedia lesson on problems on uniform motion may call for an explanation how the time table for a new express train can be adjusted graphically (hint: slower trains are overtaken during their stops in stations along the route of the express train). It may also call for an explanation how generalization (when the motion of an object is considered as the motion of two objects) and specialization (when the motion of two objects is considered as the motion of one object) can be used in the solution of the following task: “A bus commonly goes between towns A and B moving at the speed of 70 km/h. Due to a motor damage, the bus started from town A with a 15 minute delay, but it arrived at town B on time (according to the timetable) since it moved 10 km/h faster than usual. Find the distance between the towns.” These explanations may be generated through on-line collaborations of the participating learners.

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Although, as mentioned earlier, the ICMI Study 16 Discussion Document (IPC, 2004) underlines that “the process of providing students with challenging situations itself presents challenges for educators” requiring better pre-service and in-service professional development, it primarily views challenging mathematics as a student-directed process. Because the subject of challenging mathematics is not learner (both student and teacher, and even adult; probably more appropriate for this study), concrete challenges teachers should be faced with are not listed in this document. Along with utilizing the above-presented activities, teachers may not only develop some of the examined tasks, but also design and then elaborate a humanistic, technology-supported teaching approach (some guidelines can be found in Kadijevich, 2004). But, no matter which teaching approach is being design and elaborated, its designer/elaborator should also realize (probably more on a practical than on a research level) critical variables influencing its utilization, which would enable him/her to manage them in a way yielding better learning outcomes.

### **Possible factors influencing the outcomes of challenging mathematics**

#### *Factors concerning learning environment type*

- Skilful qualitative reasoning may not necessarily imply competent quantitative reasoning as recognized by Behr *et al.* (1992). Such an unfavorable outcome was obtained in Kadijevich (2002a) who evidences the need for problem solving that require solvers to, not paying particular attention to concrete numerical or textual data, generate contextually different problems having the same underlying structure. This “context-play” activity is

particularly relevant to solving tasks on extending the applicability of problem solving methods. Further research may examine how quantitative and qualitative kinds of teaching involving the “context-play” activity should be combined to promote the acquisition and coordination of both types of reasoning having in mind that procedural and conceptual knowledge seem to develop iteratively (Rittle-Johnson & Koedinger, 2004). Note that the outcome of multimedia instructional design depends on the applied design approach (procedural, conceptual, or its combination; see Kadujevich & Haapasalo, 2004).

- Although link between procedural (P) and conceptual (C) mathematical knowledge can be established through computer-assisted learning activities requiring production rules utilization and multiple representation transformation (Kadujevich & Haapasalo, 2001), most learning environments have not been designed to promote P&C issues. Such an example is CAS (Computer Algebra System) that may exhibit various conceptual and procedural limitations. For example, CAS may allow the user to simplify the equation  $x(x-1) = x$  to  $x-1=1$ , which is not equivalent to the initial one since the solution  $x = 0$  has been lost. Or, if the user wants to simplify the equation  $2x = 5x$  by dividing it by  $x$ , CAS may not protest and return an absurd fact “ $2 = 5$ ” (see Kadujevich, 2002b).

Java applets, small programs used for demonstration, visualization or simple modeling written in the Java programming language, are also learning environments that usually have not been particularly design to promote the P&C issues. Because applet can deal with P (procedural knowledge), C (conceptual knowledge), P&C (both knowledge types) and P-C (link between the two) in nontransparent, semi-transparent or transparent way, we can speak about a *black box C applet* where basic concepts are not defined and related themselves, or a *white box P applet* where all calculations are transparently carried out. Despite the fact that applets usually appear on electronic pages with some explanations, they can, taken by themselves, rarely be considered as white box P&C and P-C applets (Kadujevich, 2004a).

An exception to this inappropriate approach concerning the P&C issues can be found in ClassPad 300 (see [www.classpad.org/Classpad/Casio\\_Classpad\\_300.htm](http://www.classpad.org/Classpad/Casio_Classpad_300.htm)). Its drag&drop utility is a versatile tool because it helps learners not only to realize how a change in the symbolic representation of an object (function, vector, etc.) affects its graphical representation, and vice versa (when *Geometry Link* is inserted just one drag&drop is needed), but also to relate abstract concepts when their symbolic and geometric representations are examined together (e.g., a function is maximized where the graph of the first derivative of that function crosses the  $x$ -axis). Examples of such a sophisticated use of this tool can be found in Haapasalo and Kadujević (2003).

#### *Factors concerning learner type*

- May, because of their less flexible (say more field dependent) cognitive style, some students demonstrate unbalanced gains in procedural and conceptual mathematical knowledge resulting in missing or poor links between them as claimed in Kadujevich, Maksich and Kordonis (2003)? Such an impact of cognitive style (where “field-dependence-independence” had a very specific perceptual connotation) on link between procedural and conceptual mathematical knowledge was found in Kadujević and Krnjaić, (2003). Does the same relation apply for ordinary (not mathematically talented students)? This important question, the answer to which should help us to approach the attainment of the P-C link in a realistic way (when we wish to achieve an educational goal, we should know what learners may do so!), has not, to the author’s knowledge, been studied so far.
- Mathematical problem solving performance results from a complex interplay among solver’s cognitive, metacognitive and affective domains, the last of which determines the global context where cognition takes place monitored and controlled by metacognition

(Schoenfeld, 1992). Can thus paired problem solving performance be predicted by paired students' features concerning mathematical self-concept and cognitive empathy, which may be taken as good representatives of affective and metacognitive domains, respectively? According to Kadijević (2004b), collaborative problem solving performance was positively influenced by average mathematical self-concept for paired talented students. Furthermore, the talented (average) pairs' bootstrapped data evidenced (indicated) that this performance could be explained by a multiple linear regression model, where average mathematical self-concept for paired students and average cognitive empathy for paired students had zero or positive influence, whereas absolute mathematical self-concept distance for paired students and absolute cognitive empathy distance for paired students had zero or negative effect. Further research may test and elaborate such a regression model by using a measure of cognitive empathy or perspective taking that is more related to collaborative problem solving. It may also examine variables concerning cognitive style bearing in mind that mixing students with different thinking styles can empower group learning (Lee & Tsai, 2004).

- Without limiting learners to particular tools or technology, technology-supported learning of mathematics should be based upon the following principle: "When using mathematics, don't forget available tool(s); when utilizing tool, don't forget the underlying mathematics." (Kadijevich, Haapasalo & Hvorecky, 2005a). However, as Galbraith (2002) underlines, students may view the utilized technology (graphical calculators to be precise) in several ways: technology as master, technology as servant, technology as partner, and technology as an extension of self, each of which promotes a different kind of technology-assisted learning. This may mainly be result of diverse attitudes toward technology since, to paraphrase Woodrow (1991), attitudes toward technology influence not only its acceptance, but also its use as professional tools or teaching/learning assistants. As these attitudes are shaped by the received institutional support concerning technology-supported learning, which of the two (attitude or support) is a better predictor of the use of technology? According to Kadijevich, Haapasalo and Hvorecky (2005b), subjects' interest to achieve educational technology standards was primarily influenced by his/her computer attitude not the support concerning the standards. Note that, compared with mathematics attitude, student's computer attitude was a better predictor of his/her active involvement in computer-based activities in learning mathematics (Galbraith, 2002), which may particularly be important to learning through multimedia design.

## Conclusions

Challenging mathematics may be quite demanding for many teachers who lack the skill and confidence to deal with new material in a more open pedagogical way not covered in their professional development (IPC, 2004). A catalog of suitable activities for challenging mathematics and possible factors influencing its outcomes may improve such a state. A set of standards of challenging mathematics may also improve the matters. However, an enthusiastic advocate of challenging mathematics should not forget that, despite some twenty years of dedicated international activities, mathematical modelling (a valuable challenging activity) has had so far mostly a marginal role in everyday mathematics education at all educational levels (Blum *et al.*, 2002). As Artigue (1999) underlined, substantial improvements of traditional teaching cannot be achieved by easy and inexpensive means without a strong institutional support and a substantial positive change in teachers' knowledge, engagement, and day-to-day practice.

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Each of the hyperlinks given in this paper was tested and found active as of April, 7, 2006.