Mark Saul July, 2006

MATHEMATICS: KEEP THE CHALLENGE

I really first learned about challenges from my son Michael.

Michael was born with multiple congenital anomalies. He learned to sit up at age three, to walk at age six, to use two fingers on a computer keyboard at age 10. Now in his mid-20's, he cannot write with a pencil. Climbing a flight of stairs is still not a simple matter for him.

Michael faces challenges every day, although the casual observer may not see them. He is challenged as he dresses himself in the morning, feeds himself at lunch, and bathes himself in the evening. He has developed ways to perform many of the routine movements that we take for granted, but must devise new 'tricks' for the unexpected. And he encounters the unexpected in places we would not think it lurked.

The word 'challenge' is too often reserved for a feat of epic proportions: the conquest of a mountain peak, the performance of a concerto, the proof of a mathematical conjecture. But climbing a staircase can be as much a challenge for Michael as climbing the Matterhorn would be for most of us. For the word 'challenge' really refers not to a specific situation, but to the relation between a situation and a person. The person sees a goal, commands certain means that will allow him to reach the goal, but does not know, *a priori*, how to go about achieving that goal. For instance, the computation of $\int_1^5 x^2 dx$ is a challenge for a beginning student of the calculus, as it was for Archimedes. But it is not a challenge for most of us. It is also not a challenge for the usual ninth grade student, who does not understand the goal and does not possess the means to achieve it.

The professional mathematician may see the exhortation of my title as an odd one. The challenges of mathematics provide him with a continuing, if not bounteous, livelihood. How can mathematics ever be anything but challenging? For many others, however, the experience of mathematics is one of routine, of the tedium of computation, or, in a less common but more painful case, of embarrassment at not following the logic of an argument.

That this is the whole of mathematics is a misconception acquired in childhood. And it is easily acquired, given that so many citizens and so many teachers have just this misconception, and pass it on without examining an alternative.

I want my students to know an alternative.

For me, the true challenge of teaching mathematics is to get students to see through the algorithm or technique, to the motivation behind the technique, or the meaning of the algorithm. I want to find ways to get them to practice technique in settings where they need to think, not merely imitate. Sure, sometimes they have to do things automatically:

at age 16, they shouldn't have to think about muffin tins or successive addition to know that 8x7 = 56. But I would like them to come to know this fact, and algorithms that harness this fact, through experiences that teach them what it means and how to use it. And when they do see a muffin tin, I don't want them to have to count the holes individually.

This is why we must devise ways of teaching through challenges. Students can come to know the basic algorithms and skills by facing situations that challenge them, and learning the algorithm or skill while meeting the challenge. That is, challenging mathematics should be located right in the classroom, and right at the center of the work of the teacher.

For example, I often found myself teaching about logarithms to students in algebra II who had trouble in algebra I, students whose success in the course was not at all certain. Rather than giving them the rules for computation with logarithms, I would ask them to solve for x in problems like these:

$$2^{12} = 8^x$$

$$7^x = 49^{10}$$

$$81^{12} = 27^{2x}$$

$$25^{2/3} = 5^{x/4}$$

They had already learned about exponents, and thinking 'backwards' like this about something they were already familiar with slowly opened to them the concept of a logarithm. Eventually, through problems like these, we got to a discussion of logarithms that were not rational. But by that time, the concept was less an abstraction, and more just a new way of talking about something they already knew.

It would have been faster to write down a few algebraic rules for computation with logarithms. Then they could have done problems applying these rules, and probably passed whatever tests they needed to graduate. But I wanted to give them a greater experience. I wanted them to write down these rules themselves, after they knew them simply as new ways of expressing the rules for exponents. Similarly, I want them to see the rules for general exponents as natural extensions of the rules for integer exponents. I want the rules for integer exponents to grow out of their knowledge of multiplication, and so on.

Pedagogically, it is not enough simply to know about these relationships, or to recite them to students as theorems, or even to show them proofs. That sort of teaching ends up a form of ritual. The majority of students let the teacher lull them to sleep with talk about the theorems, then become alert when an example is given which is like one they will have to work.

This is how I was educated. Luckily, I was already interested in mathematics, and was eager to follow the proofs. But most of my students are not ready for this step, and getting them to take it is itself a goal of education. The classroom task is to invite students into the mathematics, to have these logical relationships dawn on them as they work with mathematical objects.

Why is it important that students have these greater experiences? I can think of at least two reasons. One reason involves their survival, and mine, in the classroom. Into each life a little tedium must fall. Students need to practice skills and drill in algorithms. But if I approach the skills as auxiliary to the main purpose, which is to think about how to use them, the basics will be learned in the context of a more intellectual task, and therefore are likely to take on meanings that would otherwise elude the students. And of course they, as well as I, are likely to be more attentive in drill if there's actually something interesting to attend to.

This is a pedagogical reason to think of challenges as central to the classroom. There is also the matter of personal development. Again, it was my son Michael who taught me this. It would have been fast and easy for me to dress him. But it was through meeting the challenge of putting his own clothes on that Michael grew. In just this way, students of mathematics need to be challenged, not just on special occasions such as contests or projects, but daily, as part of the classroom routine.

And there is yet another reason, a societal reason, why students must be challenged in the classroom. Some of our students will eventually go on to careers in technical areas, and will need to find their way in a challenging situation on a daily basis. They will need to know how mathematical skills can be used, and not just what they consist in.

There's a bit more to this observation. It is true that only some of my students will harness their mathematical skills to technical careers. But virtually all of my students will have occasion, in their daily lives, to think through a situation, to find their own pathway through a difficult landscape, to figure something out. It is this challenging quality of life that I would like to prepare them for. It is this challenging quality of life that my son Michael taught me to observe.

A colleague, examining a proposed elementary curriculum, once asked me when students following this curriculum will learn to multiply 527 by 14 accurately and fluently. I answered the question with a question: When will they have to do this? When was the last time you, gentle reader, had to do even a relatively simple computation like this, accurately and fluently? We have machines these days that do them for us, more accurately and more fluently than we could hope to.

The point is that many of the computational skills we used to stress in school mathematics are now rarely required of us. Like the gentling of a horse or the kindling of a fire, these skills were once vital, but are now of marginal use. But the process of learning these skills can be combined with the process of developing their thinking. And this combination brings arithmetic into the mainstream of mathematics. The central experience of elementary mathematics should not be a mastery of arithmetic algorithms, but of the concepts that generate these algorithms.

For mathematics, at its heart, is not about arithmetic figures, or about geometric figures. It is about figuring things out. And the process of figuring things out remains challenging on any level of achievement. It is a challenge for the teacher to keep this challenge in school mathematics.