Computer Tools for Internet-Supporting of Research Activity in Mathematics

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Abstract

Three projects will be discussed. The first project deals with different kinds of automatically expertise of mathematical problems solutions, the second is devoted to experiments with mathematical objects for acquaintance of students in new mathematical area, the third project is about how to support activity of teachers and students in mathematical internet Olympiads. All of them show an approach to dissemination of mathematical knowledge via internet in interactive form and to involving students in mathematics.

Introduction

We will consider three projects for supporting of problems solving via Internet.

There are two different strategies to support such an activity:

- provide distance connection between students and teachers which will manage them in solving problems;
- give students an access to special expert systems which will produce analysis of their solutions.

We will consider the second way.

When creating such an expert system we must to answer next questions:

- does the system give us new possibilities in teaching mathematics?
- does the system can motivate students to work with it?
- doesn't the system kill the human factor of such activity?

If we can give positive answers for all the questions we can say that such a system can be used in teaching of mathematics. We give positive answers for all three projects under discussion:

1) One of the main advantages of computer system is the possibility to produce large amounts of computing. So to supply students with a tool which use large amount of specially constructed tests for checking partial solutions of particular problem is the same as to provide students with new type of "critical adviser" which will lead student to right answer with his own way. Let's remark that it is impossible for the teacher to construct quickly counterexamples for every student's conjecture. So this system gives teachers new possibilities in teaching of solution of complicated problems.

2) Does such expert systems will motivate students for continuation of their work? Yes but if and only if the system will be used as a directory and not for estimation of student's knowledge's. The role of such systems is similar to role of our friends when we ask them to check the solution proposed by ourselves before its publication.

3) Why such the systems give not an impulse to dehumanization of teaching process? The answer will be similar to previous one: such systems not pretend to be "nonunderstandable intellectual". We clearly understand how they work, what they can do and what can't do. More over they give us new possibilities to expand classes of mathematical problems to be discussed on the school level.

1. First project is "Verifier" [I].

Let's begin with considering of problem solution process. Every solution can be considered as a sequence of statements for objects which "participated" in this problem. Then the problem of verification is to try with the special constructed set if the new statement not contradicts to previous statements. Of course this problem has no common solution. But we will consider the system only as adviser which knows one thing and can have failure in other things. The system “Verifier” will give counterexamples only if it knows them.

How does the system work?

Let’s consider two examples with two different types of problems:

a. problems with parameters (solution is a formula with parameters);
b. constructive problems (formulation of problem include such phrases as “give an example of…”, “find the dependence between set of parameters…”)

Examples

Here we can see examples of dialog between students and the system. The “Verifier” gives the reaction in three forms:

- signal of contradiction;
- counterexample in text and/or formula and/or graphic form;
- probable sources of mistake.

1. Find the conditions for parabola $y=ax^2+bx+c$ to have no points in 3-th quadrant.

a. First conjecture: $a > 0$. Counterexample: “$y = x^2 - 1$ has point in 3-d quadrant so this condition is not sufficient” (the picture accompanies this message).

b. Second conjecture: $a > 0$ & $b^2 - 4ac < 0$. Counterexample: “$y = x^2 - 4x+3$ has not points in 3-d quadrant but not satisfy to this condition” (the picture accompanies this message).

c. Third conjecture: $a > 0$ & $c>0$. Counterexample: “$y = x^2+4x+3$ has points in 3-d quadrant but satisfy to this condition” (the picture accompanies this message).

d. So after combining both ideas the student might find the condition which give the way to combine both conjectures: $a >0$ & ($b \geq 0$ & $b^2-4ac<0$ OR $b<0$ & $c>0$).

2. Give an example of periodic function with period equal to π and with a symmetric graph where $x = \pi$ is a mirror.

a. First attempt: $y=(x-\pi)^2$. System advice: “your function has not pi as a period”.

b. Second attempt: $y=sin\ 2x$. System advice: “graph of your function has not $x = \pi$ as a mirror”.

c. Third attempt: $y=sin^2\ 2x$.

We see here other possibilities of verification. Here we need not in arbitrary samples. We check only given conditions in the form of functional equations. It is important that every student can give his own sample. In our example equally possible to receive other answers: $y=sin^2 x$, $y=|cos(x)|$, $y=arccos(cos(x))$ and so on. (By using the mechanism of changing of initial conditions we can add restriction “graph combined of line segments” and to make other problem).

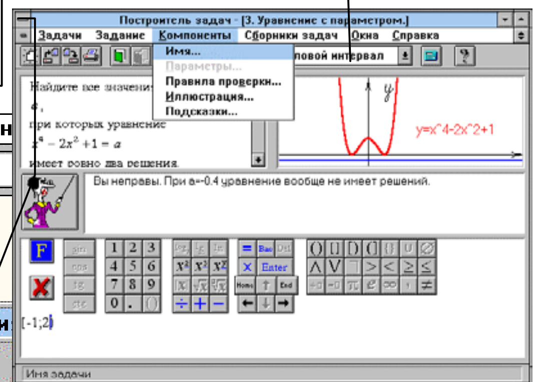
So this project shows possibilities for design of internet sites with sets of mathematical problems and expertise system for checking of partial solutions of the problems and giving counterexamples to correct students in solving problems.

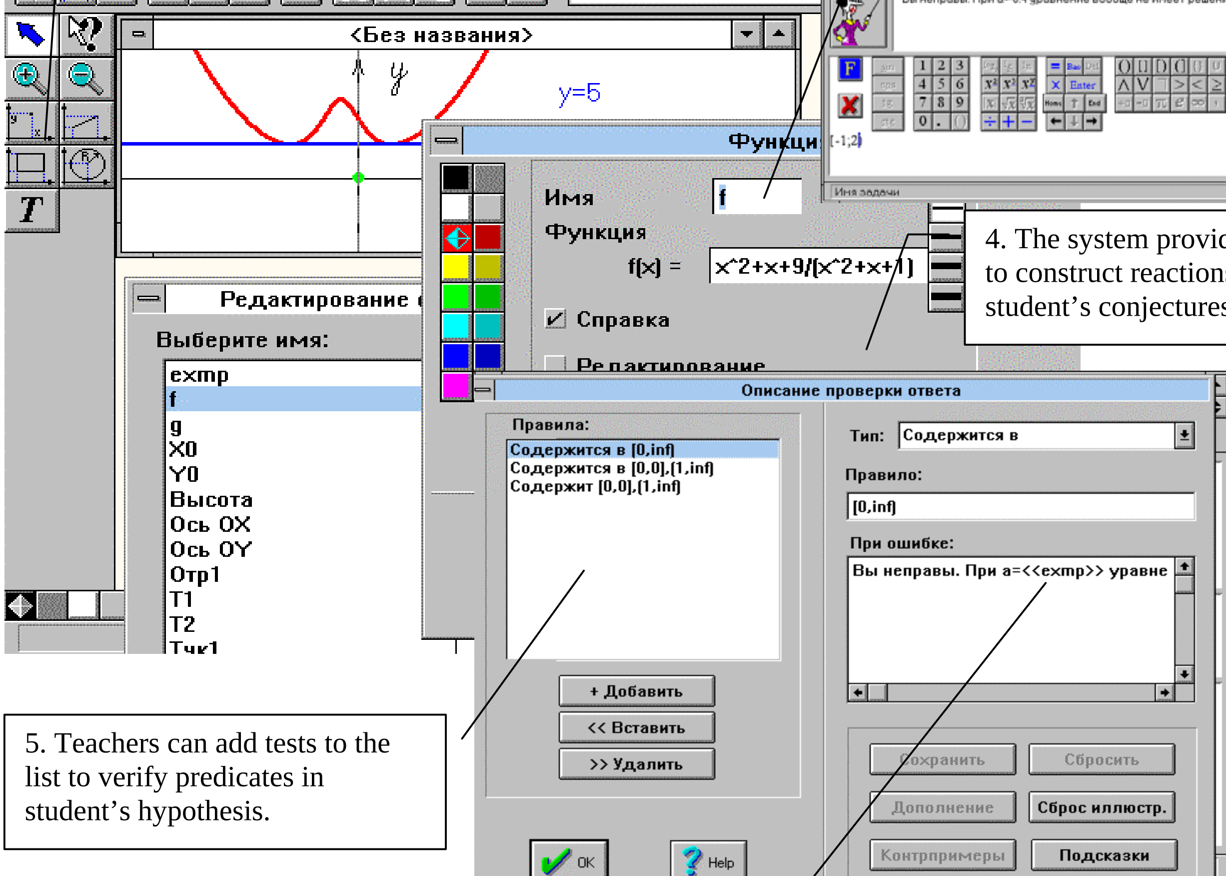
Verifier system architecture

1. Both teachers and students operate with the same (common) objects. Teachers have possibilities to do experiments in parallel with the design of new problems.

2. Graphical subsystem provides visualization for all kinds of examples, which are generated by the “Verifier” or are planned by the teacher.

3. Dialogue among teachers and students includes such objects as numbers, sets, vectors, functions and predicates.



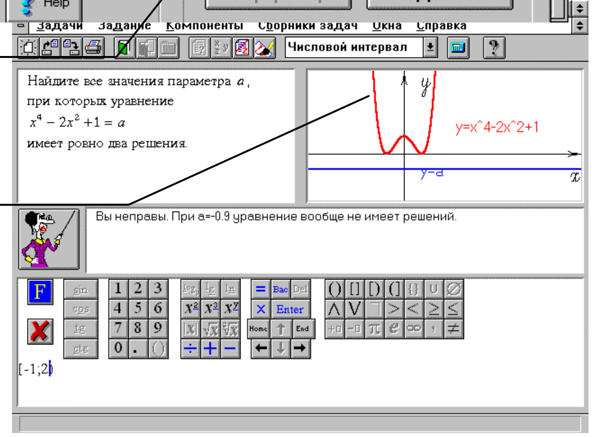


4. The system provides tools to construct reactions to student’s conjectures.

5. Teachers can add tests to the list to verify predicates in student’s hypothesis.

6. Subsystem which generates the set of examples to verify student’s answer in the “Verifier”.

7. An example which contradicts to student’s answer transfers to graphical subsystem.



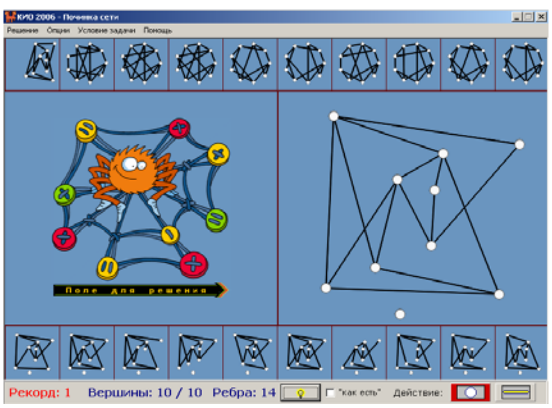
Pic. 1

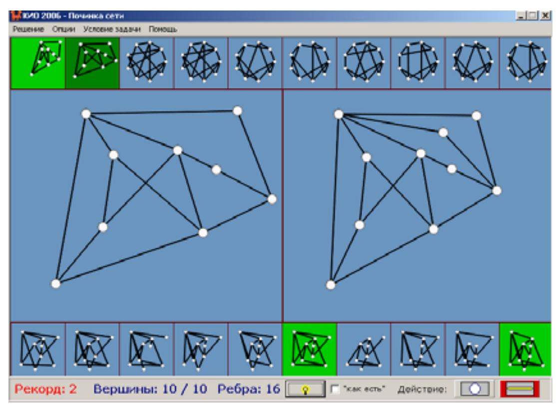
2. “Construct-Test-Explore contest” project [C].

The predecessor of the CTE contest is the “mathematical auction” – the popular form of contests in winter and summer Russian mathematical camps. The essence of “mathematical auction” is the process of solving problems formulated in such a way that all solutions can be compared by using of one or more parameters (for example “find a solution with minima *n*”). After one team or participant will find one of the solutions the price for this problem will increase and if other team or participant know better solutions they can show it and receive more points. This process will be finished if there are no new solutions and then next problem must be “exposit for tender”.

In CTE-contest the computer models for supporting an experimental activity with mathematical problems are used. Every model has tools for design and investigation of mathematical object and for calculation of parameters which are used for comparing different designs. The purpose of contest is to find design with best parameters.

For example, let’s consider the problem “Graphs (How to repair a net)” from CTE-2006. This problem is closely connected with an unsolved mathematical problem if it possible (and how) to reconstruct the graph based on special set of subgraphs (in model under consideration there were subgraphs which were produced one by one eliminating of graph vertexes with all the incident edges). The computer program provided tools for constructing graphs, automatically generation of needed set of subgraphs, finding an isomorphism with given subgraphs and changing the representation of graph to become the isomorphism evident. The parameter for comparing results was the number of isomorphic pairs of subgraphs (one from given set of subgraphs and other from set of subgraphs generated by the constructed graph).





Pic.2

So the role of CTE contest is
1) to give ideas of new mathematical concepts
2) to propose tools for designing of mathematical objects with particular properties
3) to compare own ideas with ideas of other participants

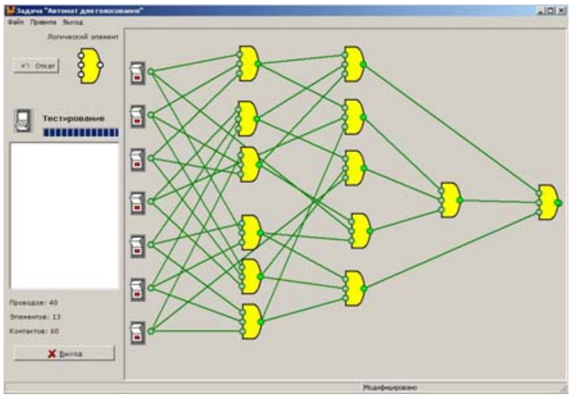
Important remarks

1. The number of comparative parameters can be more than one. It gives possibility for more accurate differentiation of results and open the way to find out small improves of solutions.

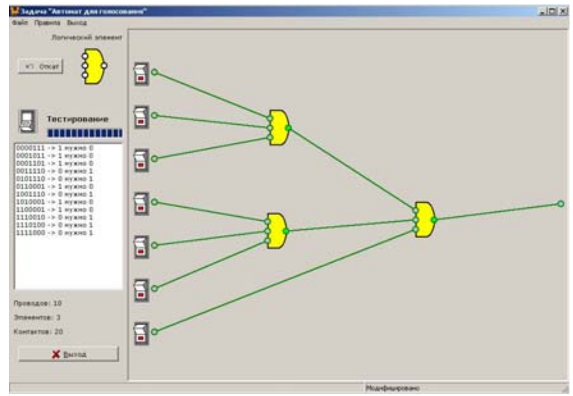
Let’s consider the example from CTE-2005.

The problem was how to construct logic scheme for Boolean domination function with 7 inputs using logical elements which are Boolean domination functions with 3 inputs.

The main purpose is to construct a scheme with given properties and minimize number of elements. But for those who can not construct such a scheme the next level parameter was used: for how many inputs (from all 128) the scheme gives true results.



Pic.3a. All the inputs give right results



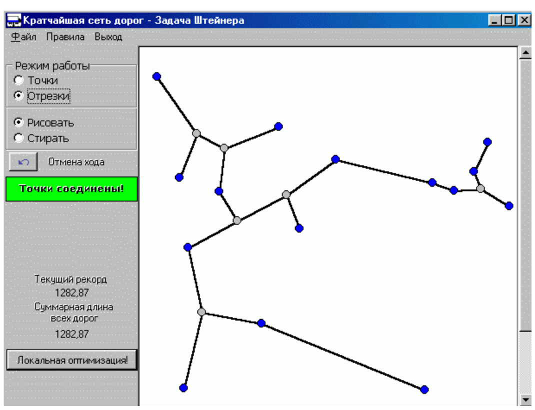
Pic.3b. Only part of inputs gives right result.

2. The CTE contest can be considered as the first step to acquaintance with unsolved or difficult problems in mathematics. The Computer Tools in Education Journal which organize this contest publish articles devoted to those problems after contest is finished.

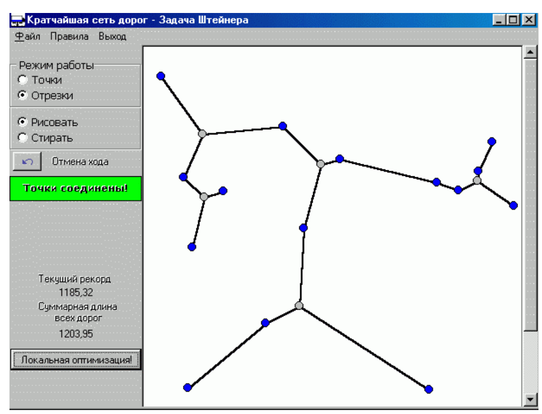
Example

One of problems of CTE-2004 was Steiner problem about how to connect n points on a plain by the net of straight segments with minimal sum of lengths. After the contest the CTE Journal published the results of the Contest with some articles. One of these articles (written by J. Romanovsky) was "The Steiner problem for graphs and dynamic programming".

So the CTE contest is a new form of dissemination of mathematical knowledge and motivation for engaging in mathematics.



Pic. 4a. Local minimum.



Pic. 4b. Global minipum.

The CTE contest is supported by CTE-site. The process of communication with the site during the contest (3 days) contains next stages:

1. Participant loads from the site special software which contains contest problems with all the needed tools for supporting experiments, automatically storing protocols with solutions.
2. Participant loads (during 3 days) file with protocols of solution on site.
3. All protocols are automatically processing and storing in data base.
4. Then on the base of those protocols all the solutions are automatically generating and publishing on the site.

5. At the same time gallery of results is generating and placing on the site.

6. Digital certificates with contest results are automatically preparing and sending to participants.

So the most important "human" part of the contest is to choose interesting problems and to construct instrumental models for supporting of problem investigation. Other steps are being done automatically.

3. Internet support of mathematical Olympiads.

The third project is about how to support the process of constructing and solving Olympiad problems via Internet.

All the Olympiad problems can be divided in groups. For example there are “combinatorial problems”, “pigeon holes principle problems”, “geometrical problems” and so on.

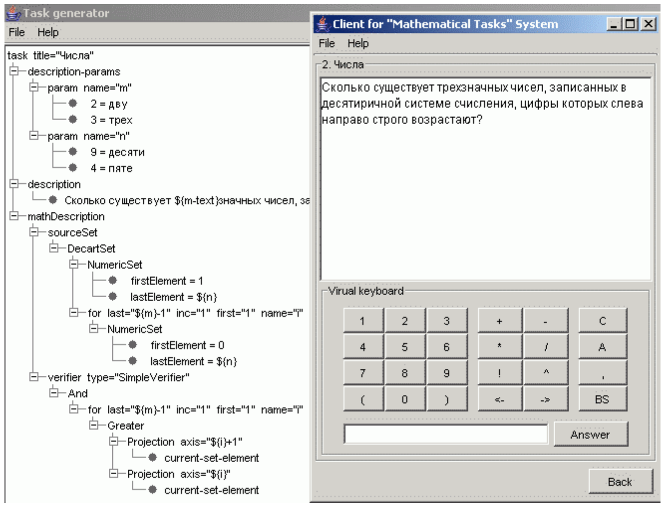
Each type of problems has its own “language” – set of concepts, ideas, methods. Then if we plan to create computer support for math Olympiad activity we need in different models for different types of problems. Such models should give us tools not only for solution of the problems but rather for description of problems [M].

Let’s consider example. Combinatorics.

Problems in combinatorics often have simple description, but at the same time not evident solution. We will construct language for description of such problems. The language will have tools for defining of sets in direct (constructive) way: recounting elements of the set, defining a set in a form of operations with other sets and so on. Also this language will have tools for indirect set definition on the base of predicates which give possibility to check if particular element belongs to the set or not.

After that we need to construct a processor which will generate the algorithm for enumeration of all described combinations. Of course this algorithm will have structure of backtracking algorithm which tries all possible combinations given by language designs and check all restriction given by language predicates.

So the most important "human" part of the contest is to choose interesting problems and to construct instrumental models for supporting of problem investigation. Other steps are being done automatically. Students use special interface for input of solutions which can have different tools for constructing an answer in proper or recurrence form: numbers, variables, functions (such as factorial or binomial coefficients), arithmetic operations and so on. So we have two (or more for different interfaces) ways to describe the same number of combinations. Then program compute both results and compare them. If they are equal then problem is solved.



Pic. 5. Description of problem (from the left). Student's interface (from the right).

Remarks

1. This system can be used for open publishing of new and unsolved mathematical problems with possibility to check of various conjectures.

2. System supposes two types of answers:

- first type means the numeric answer,

- second type means answer with variables; in such a case the system checks the answer with arbitrary values of this variable.

Example

1) The problem of first type.

A tram ticket has 6 decimal digits. We name it "happy in Saint-Petersburg" if the sum of first three digits is equal to sum of next three digits and we name it "happy in Moscow" if the sum of digits on odd places is equal to sum of digits on even places. How many tickets are "happy" both in Saint-Petersburg and in Moscow?

2) The problem of second type.

In how many ways we can color edges of regular *n*-gon with *k* colors, if adjacent edges have different colors.

Conclusions

1. When constructing computer (internet) tools for supporting activity in mathematics we must to base on Vygotsky principle (which was formulated long ago before computers appeared) that "when using the external tools the children study how to manage their mental processes". So the computer tools must not substitute mental activity but give new situations to apply mental activity [T].The special expertise system can give chance to prolong student activity in searching of mathematical problem solution and support student's own way to solution.

2. The special expertise system can give chance to prolong student activity in searching of mathematical problem solution and support student's own way to solution. Using of models which give means for manipulating with mathematical objects open the way for constructive acquaintance with new mathematical concept, give the field for experimental activity in mathematics.

3. One of perspective ways to support internet activity in mathematical Olympiads is to use tools for accurate describing of problems without describing the answer. Then people get possibilities to announce problems before they will know answer or use problems with open description without danger that the program will be "cracked out" and the solution become known.

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