In the Name of God

# Comparing Two Methods of Teaching Calculus in University (The challenge for teaching mathematics for Non- mathematics students)

**Mohammad Hossein Pourkazemi**

Associate Professor, Shahid Beheshti University, Iran

## Abstract

*Teaching calculus to non-mathematics students, like Economics, Management, Accounting, etc., is different from teaching the students with major in pure mathematics. In some universities, the lecturers teach pure theory and don't pay much attention to applications of the topics. Non-mathematics students should learn mathematics as a tool that they need to use it in their other courses, and they are not interested in the theory.*

*There are two methods of teaching calculus to non-mathematics students. In the first method only the pure theory is taught without giving any real life example of the applications. In the second method the emphasis is on the application rather than theory. The question is: which one of these methods will increase the motivation of students to learn mathematics? In this research we want to answer this question.*

*Although it is an important tool for economics and management analysis, most of the students are not very interested in mathematics,. Using my long experiences in teaching mathematics, I prove that giving applied examples in Economics and Management will create interests in mathematics among students. Also, by the help of an opinion poll, we observed that the students prefer the second method of teaching mathematics. Also we observed that students who have learned mathematics using the second method are more successful.*

## Introduction

There are 9 years of general schooling in Iran that consists of 5 years of elementary school, 3 years of guidance school, and the first year of high school. The students who want to continue their studies have two options; Vocational Studies or Theoretical Studies in the high schools that lead them to higher education at universities.

The theoretical studies have different branches. The students who are strong in mathematics usually continue their studies in mathematics and physics, those who are not interested in the mathematics study human sciences and literature, and the rest study experimental sciences. [1]

After finishing high school, the students who want to enter higher education institutes study a one year pre-university courses in the related branches, i.e., mathematics and physics, experimental sciences, and human sciences and literatures. Then they take part in some

university entrance examinations.
Usually, graduates of the mathematics and physics discipline study engineering, and
Graduates of the experimental sciences study medical sciences, Dentistry or Pharmacology, and graduates of Human Sciences study law or political Sciences. This is the interest of around 80% of high school graduates. [2]

Due to limited capacity of universities in the above fields, most of the students have to
Continue their studies in other branches, which they are not very interested in those subjects.
Same is true for the students of economics and management need to know and use mathematics for their economic analysis and planning.
As a lecturer at university for many years I ran to a challenge which was the challenge of teaching mathematics to students who did not have any backgrounds in mathematics. I usually teach mathematics to students of economics or management sciences. These students are usually graduated from high school by passing courses in humanities and they mostly dislike mathematics and are not interested in learning this subject in university. The other problem is that in their field of studies, they are not using mathematics, because most of their professors look at economics and management sciences as non analytic subjects and teach them as a course in humanity. So while they dislike mathematics and have no feeling for the use of it in their studies, we are sure that they need it in their future research or work, because their field are recently more analytical. I know that the economics and management sciences are getting more analytical than being verbal. The students need to know mathematics and statistics to work with real data and model the situations arising in these fields, so they should be prepared for using mathematics, and without understanding and not hating the subject, no one can use it for his or her research or work activities. Having this in mind, I tried to face this challenge and solve the problem by changing my traditional method of teaching calculus. The main problem is lack of interest in mathematics among these students. The challenge is how we can encourage them to become interested in mathematics, and which teaching method is the best in these situations. This method of teaching may be practiced in
In other countries the case may be different, so I looked on it as a challenge and by presenting the course differently I could solve the problem.

# 1) Economics and management student's lack of interest in Mathematics

At the beginning, some of the students accepted in economics and management branches
of the public universities are not interested in their field of studies. In 2005, around 1500 students were accepted in economics schools, and around 1300 students were accepted in the management sciences schools through 20 public universities in Iran. [3]
Most of the students in economics and management branches, especially those who study experimental and human sciences in high school, are not interested in mathematics. Some of them actually studied experimental and human sciences in high school because they did not like mathematics. Now they are studying economics and must study differential equations, Calculus I, and Calculus II courses compulsorily. So the first basic problem is, the knowledge of students in mathematics is not homogeneous. The second problem is that they mostly do not like mathematics at all. Operation research, Probability, and statistics are very important tools in teaching Macro and Micro economics. The challenge is "how we can encourage them to learn and like mathematics"?

## 2) Teaching mathematics in economics and management Branches

There are two methods of teaching mathematics to the students of economics and management sciences branches in Iranian universities. In some universities they study mathematics together with the students of other branches, like Sociology, Management, and Biology, or the lecturers from the Departments of the mathematics come to their department and teach them mathematics. Most of these lecturers are not familiar with economics and management teaches mathematical theories from books like Thomas, *Calculus and Analytic Geometry* [4] ,with less emphasis on roofs.

The second method is teaching mathematics by a mathematics lecturer who is familiar with economics or management ideas. The lecturer knows exactly what part of mathematics is required for the undergraduate and graduate students in economics. [5] Teaching mathematics as a service course for other branches has it own problems that mathematics teachers are always interested in studying these problems. [6] In ICME-8 which was held in Sevilla 14-21 July 1996 [7] this issue was discussed. [8]

## 3) The Past Experience

About 30 years ago, I started my job as a lecturer of mathematics for non-mathematics students in the Faculty of Economics at Shahid Beheshti University. I have studied mathematics and, while studying for my master degree, I enrolled in an elective course in mathematical economics. I enjoyed the application of mathematics in economics.

I became interested in teaching mathematics for economic students. One of the lecturers had a general mathematics book that he called it “General mathematics for non-mathematics students”. He omitted difficult proofs of the book, but he was not familiar with economic concepts and so he was not interested in the application of mathematics in economics. This book was also thought to students of Management, Accounting, and Biology and so on. I used this book in my classes too. The students who studied mathematics in high school did not have any problem for learning the book, but it was difficult for the other students. After a while, I became responsible for teaching all mathematic courses in the Faculty of Economics. Each term around 250 students enrolled in this faculty to study Economics, Accounting, and Management.

After 3 to 4 years of teaching mathematics, I noticed that using applied examples encourages the students to pay more attention to the course. Especially, the first year students paid more attention to mathematics with applied examples. So I decided to fined a book about applied mathematics and I found the book “Mathematical Analysis with Business and Economic Applications by: Jean E Weber” and I translated it into Persian. [9] I could not use this book as the main source of the course, because it was very easy for those with a good background in mathematics, since they have studied about 60 percent of the book in their high school ages, but the applied examples were interesting for them.

In 1988, I wrote a book called “General Mathematics and its Applications”, in which, after each chapter I gave examples of its application in Economics, Management, Physics, Mechanics, and Electronics. The application in Economics was separated from the application in other branches. This book is in two volumes and around 1200 pages divided into 11 chapters.

The contents of the book are: Set Theory, Complex Numbers, Functions, Derivatives,

Application of Derivatives, Integral, Vectors and Matrices, Multi Variable Functions, Series, Differential Equations and Difference Equations. Each section of the book has many applications in economics. The book has three editions and been reprinted 18 times. It is used in all universities in Iran. [10] I do not know of any similar experience in the other economic faculties in Iran.
Nowadays, high level mathematics is being used in economics, so the students should4 know more and should be interested in the subject; otherwise they fail to pursue their studies in economics.

## 4) The Main Challenge

The objective of this report is to show the author's effort to solve the mentioned problems in teaching mathematics to non-mathematics students. The success of the effort, regarding the students who did not like mathematics in high school, and were scared of the subject, is noticeable. The author was successful in creating interests in the students by changing the teaching method, and by this report asks from the other colleagues to use similar method in teaching mathematics to other non-mathematics students.
It is important to mention that, unlike some countries in which top students usually study economics, in Iran top students prefer to study engineering or medical sciences. In Human Sciences students prefer to study Law. So the students, who study economics in Iran, are those who don't like mathematics, and also are weak in other subjects in high school as well. Having influence in these conditions is very important and the big challenge is the slogan of "Mathematics for all". Researchers and the students of economics need to use advanced concepts of mathematics and statistics in their field of study. [5] There are problems teaching subjects such as derivative integral to the non-mathematics students because of their non-homogeneous knowledge in mathematics and their lack of interest in this subject. The challenge is how to teach these students and make them like mathematics.
To solve the problem of student's non-homogeneous knowledge of mathematics, we offered a class for those students who did not study proper mathematics in high school. This class was compulsory for those who studied experimental sciences or human sciences in high school.Those who studied mathematics and physics in high school could also attend this course if they thought they need to improve their mathematics. After finishing this course, the mathematics knowledge of all of the students should be almost similar. This method is very effective in homogenizing the level of student's knowledge in mathematics.
Now, let us have a look at the main problem, i.e. creating interests in the students to learn mathematics. Students want to learn economics or management but, since they are not familiar with the topics we have to make them believe in the usefulness of mathematics as a necessary tool for their field of study. This can be shown to them with proper examples in each section. In my two volumes book, "General Mathematics and its Applications", in each section there are examples to show these applications. This book is in Persian and is based on the knowledge that the students learn in high school.
Now we want to see whether we were able to make the students interested in mathematics or not? What is the opinion of the students about this method? We were successful in our challenge, i.e. encouraging the students to learn mathematics with enthusiasm?
The other problem is; if there are two methods of teaching in Iran, we need to compare these two methods and see which one is more successful?

Last semester I had two classes. One class was in Tehran University and the other class was in Shahid Beheshti University. I decided to use each method of teaching in one of these classes, but the students did not let me to continue teaching pure Mathematics, which was an interesting issue. So I used the same method in both classes and I will compare two Methods using another method that I will explain in the next sections.

## 5) The Questionnaire

At the end of the term students of the both classes were asked to fill a questionnaire. In the questionnaire we asked them their type of high school diploma, and then we asked questions about their view points about the teaching methods and the applied examples. Also we asked them if they liked to learn Mathematics taught to them by the lecturers from the Department of Mathematics who are not familiar with Economics.

There were 500 students in the department and the number of sample was appropriate regarding the population. The questions and their answers are as follows:

Question (1): What is your high school diploma type?

There were 62 students in both classes from which 10 students studied Math and Physics, 36 students studied Experimental Sciences, and 10 of them studied Human Sciences in high school.

| Diploma | Mathematics | Experimental | Human Sciences | Total |
|---|---|---|---|---|
| Frequency | 16 | 36 | 10 | 62 |
| Per Cent | 25.8 | 58.1 | 16.1 | 100 |

Question (2): Do you like to study Mathematics with the students of the Mathematical branch of the Department of Science?

The answers of the students to this question fall in 5 groups as follows:

| Answer | Very Interested | Interested | No Differences | Not Interested | Not Interested At All |
|---|---|---|---|---|---|
| Frequency | 0 | 0 | 2 | 18 | 42 |
| Per Cent | 0 | 0 | 3 | 29 | 68 |

Notice that close to 97% of the students are not interested to study Mathematics in the Department of Mathematics. This is why they did not let me to continue teaching them pure Mathematics without applied examples.

Question (3): Do you like to study Mathematics with the students of the other nonmathematical branch like Management, Chemistry, and Biology…?

The answers of the students to this question fall in 5 groups as follows:

| Answer | Very Interested | Interested | No Differences | Not Interested | Not Interested At All |
|---|---|---|---|---|---|
| Frequency | 0 | 4 | 10 | 14 | 34 |
| Per Cent | 0 | 6.5 | 16.1 | 22.6 | 54.8 |

Notice that most of the students are not interested to study Mathematics with the students from the other branches in the university.

Question (4): Were the applied examples explained for the Math I adequate?

The answers of the students to this question fall in 5 groups as follows:

| Answer | Much More Required | More Required | It was Adequate | Less Required | Not Required At All |
|---|---|---|---|---|---|
| Frequency | 12 | 14 | 22 | 10 | 4 |
| Per Cent | 19.4 | 22.6 | 35.5 | 16.1 | 6.4 |

Let us look at this result using inference statistics. 48 out of 62 students said that the applied examples were required and only 14 students said it was not necessary.

p = 0.774

q = 0.226

It means, from this sample 0.774 said applied examples are necessary and 0.226 said it is not necessary. Now, with 95% probability the idea of the student's population about the necessity of the applied examples for the Mathematics I is as follows:

$$P - Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}} < \pi < P + Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}}$$

$$0.675 < \pi < 0.878$$

In which, $\pi$ is the population's ratio. It means that with probability of 95% the positive opinion of the population is between 67.5% to 87.8%, which shows the necessity of the discussing applied examples in the class.

Question (5): How much necessary is to give applied examples for the Math II?

The answers of the students to this question fall in 5 groups as follows:

| Answer | Much More Required | More Required | It was Adequate | Less Required | Not Required At All |
|---|---|---|---|---|---|
| Frequency | 22 | 18 | 16 | 6 | 0 |
| Per Cent | 35.5 | 29 | 25.8 | 9.7 | 0 |

56 out of 62 students said that the applied examples were adequate or they were needed more. It means 90.3% agree that applied examples are required and only 9.7% said that it was not necessary. If we study the opinion of the population from the above relation the result would be:

$$0.830 < \pi < 0.976$$

It means 83% to 97.6% of the population with the probability of 95% agrees that applied examples are necessary.

The results of the questionnaire and the extracted per cents show that the students accept the above teaching method and like it.

## 6) Comparison of the Teaching Methods

Teaching methods in the universities other than Tehran and Shahid Beheshti Universities are different from my teaching method. Also in those universities the lectures are not familiar with Economics. All the graduates of the universities must take the entrance examination to continue graduate studies in the Master level. The courses of the entrance examination are Micro and Macro Economics, Statistics, Mathematics, and English. Table below shows the number of students from each university who took this exam and their average in Mathematics in 2005. [9]

| No | University | Participants | Average | Standard Deviation |
|---|---|---|---|---|
| 1 | Shahid Beheshti | 202 | 37.98 | 24.78 |
| 2 | Tehran | 247 | 39.42 | 23.98 |
| 3 | Shiraz | 119 | 37.16 | 19.66 |
| 4 | Isfahan | 185 | 35.79 | 35.79 |
| 5 | Allameh | 399 | 31.67 | 31.67 |
| 6 | Ferdousi Mashhad | 163 | 29.53 | 29.53 |

The above table shows that the students of the Tehran and Shahid Beheshti universities are more successful in the Mathematical examinations than the students of the other universities. We divide these universities into two groups and compare their average. In the first group we put Tehran University and Shahid Beheshti University, and the other universities form the second group. We use the test of hypothesis and show that there is a meaningful difference between the averages of these two groups of universities.

| Universities | Average | N - number of students | S -Standard Deviation |
|---|---|---|---|
| Tehran and Shahid Beheshti | 38.77 | 449 | 24 |
| Shiraz, Isfahan, Allameh, and Ferdousi Mashhad | 33.9 | 866 | 25 |

Using the test of hypothesis, if $\mu_1$ is the average of Tehran and Shahid Beheshti universities students, and $\mu_2$ is the average of the students of the other universities, then,

$$H_\circ : \mu_1 \geq \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{38/77 - 33/90}{\sqrt{\frac{576}{449} + \frac{625}{866}}} = 2/05$$

Using test 1, $Z > 1/96$ is the range of $\alpha = \circ/\circ 5$, so with probability of 95% $\mu_1 > \mu_2$. This concludes that, this teaching method is more successful than the other methods for the Mathematical examination.

## 5) Conclusion

As we show, the non mathematical students are more interested in the courses other than Mathematics. So we have to clarify the importance of the Mathematics for them. Therefore, the courses of Mathematics have to be planed regarding the requirements of these branches. It would be more interested for the students if the presentation of each section includes the applied examples of the Mathematics in those areas. If the lecturer cannot give applied examples, then, s/he should encourage the students to find applied examples of the Mathematics in their other courses and represent it in the classes. This will increase the interests of the students to the Mathematics.

## References


[1] Pourkazemi M. H., "*The Effect of Teaching Mathematics on Achievement of the Students in the National Entrance Examination for the Iranian Universities*", ICME 7 (17-23) Canada 1992.

[2] "*Studying the University Selections of the National University Entrance Examination Applicants*", NOET, 2005.

[3] "*Statistical Reports of the National University Entrance Examination*", NOET, 2005.

[4] Thomas G. B., Finney R. L., "*Calculus and Analytic Geometry*" 9th edition, ISBN: 0201531745, Addison Wesley.

[5] Takayama Akira, "*Mathematical Economics*" Second edition, Cambridge University Press 1987.

[6] Clements R.R. et.al. (eds), "*Selected papers on the Teaching of Mathematics as a Service Subjected*", Springer-verlag, (1988).

[7] Proceedings of the 8th ICME Working Group 17 as service subject at the tertiary level, led by Alverza (177-181), (Sevilla 14-21 July, 1998).

[8] Pourkazemi M.H., "*Control Problem and its Application in Management and Economics*", Iranian Economic Review, vol. 9, No 10, spring 2004.

[9] Weber J.E., "*Mathematical Analysis and Business Application*", Translated by M.H. Pourkazemi 8th edition, Printed shahid Behshti University 2003.

[10] Pourkazemi M. H., "*Calculus and Their Applications*" 18th edition, Press in Iran Nashre Nei.