

Problem solving strategy-challenge for gifted students

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Abstract: Using problem solving strategy gifted students may experience the excitement of thinking deeply about mathematical ideas and discovering new concepts. They may learn to explore problems to find that the fun has only just begun when the original problem has been solved, because problem solution is often just the beginning of real mathematics.

The aims of this study are introducing some ideas and principles of problem solving strategy and showing practical tasks using mentioned methods by students individually.

At the end of this study the effects of problem solving will be analyzed.

In my work with students, I see that, while they can readily do exercises, they have difficulty with solving problems. We need to emphasize the development of a type of thinking that promotes solving problems, rather than just give students a lot of tricks as seems to occur in many schools.

A good problem tests the understanding of a student. While a student might be able to solve an equation, if there is no understanding of where the equation comes from or how it can be used, then it is just a memorized skill that can be used only in restricted circumstances. On the other hand, a student with understanding has a repertoire of skills that include appropriate application, transferring, generalizing, representing in new ways, making analogies and metaphors, and so on. Teachers need to reorder their priorities in teaching for understanding in addressing questions such as what should be taught, what is worth understanding, how can students and teachers know when understanding occurs, and how can understanding be deepened. Perkins (1993) has identified some of these priorities: (a) facilitating learning as a long-term, thinking-centered process; (b) providing rich ongoing assessment with supportive feedback and opportunities for reflection; (c) supporting with powerful representations; (d) recognizing flexible conceptions of what students can and cannot learn at certain ages; and (e) requiring the learner to carry facts and principles they acquire in one context into other contexts.

These priorities embody the induction of talented students into mathematics as a system of thought that recognizes understanding. Students should be encouraged to discuss their work with others. Mathematics is a shared activity, and students can motivate and enlighten one another.

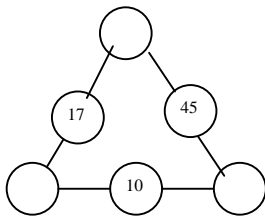
A mathematical exercise becomes a problem when its solution is not routine and not immediately apparent, and one needs to use heuristics and strategies to unravel it. Polya identified four stages of solving a problem: (1) understanding the problem and recognizing its setting; (2) making a plan to relate the underlying elements; (3) executing the plan in a mathematically correct way; (4) reflecting, or looking back, with a view to recognizing applications, generalizations, simplifications.

Among these four, students find making a plan most difficult. Therefore, the teacher's aim should be to build the connection between the concepts required, the strategy used and the heuristic principles. To improve problem solving skills, students need to be exposed to many problems and the usual routine exercises should be modified to require the problem solving process.

Open problems

We distinguish three types of open problems: open-ended, open-beginning and both-ends-open. These will be illustrated by examples.

The following Number Triangle Problem is an example of an open-ended problem. Example 1. The starting situation is given as follows:



(fig.1.)

What numbers should be placed in the blank circles at the vertices of the triangle in order to make the side-sums of the numbers all equal? Since there are many solutions to the problem, we can follow up with other questions:

- Can you find another solution?
- How many different solutions could there be?
- Is it possible to use negative numbers in the circles?
- Can you find a solution where the triangle's side sums (i.e. the sum of the numbers on the same side) is 80?
- Try to pose a similar question. Can you answer it?

The following construction problem represents an open-beginning problem. Here the goal situation is exactly given: Find a box with the biggest volume.

Example 2. Construct an "open box" from an A4 sheet of paper by cutting off square pieces from the corners and fixing the box with tape. Compute the volume of the box constructed and compare your result with your peers' answers. How could you find (and construct) such a box that has the biggest volume?

The spiral of Ulams is an example of both-ends-open problems.

Example 3: Positive integers are written in the form of a spiral (Fig.2). Find out a problem based on this number scheme, and solve it.

In the Example 3 a solver should first invent problem and formulate it (e.g. Can you see any pattern in the diagonal number sequence 1,3,13 ... beginning in the center? What is the n-th term of the sequence? e.c.t.)

Example 4: Figure 3 shows scattering patterns of marbles thrown by three students A, B and C. In this game the student who has the smallest scatter is the winner. It is convenient if we have some numerical measure to indicate the degree of scattering. The key of the open approach method is to develop a variety of solutions-in this case many different ways of generating a numerical measure to indicate the degree of scattering. The next step is to evaluate them.

Solving problems

A problem solving strategy is a previously learned method to solve a problem. LeBlanc (1997) categorizes strategies as general and helpful.

Examples of general strategies are :

1. Trial and error
2. A systematic list of the different possibilities
3. Simplifying the problem
4. Searching for the formula in the problem
5. Experimentation
6. Reasoning
7. Generalization
8. Working backwards

Examples of helpful strategies are : diagrams, tables, drawings, lists and equation.

These are indirect steps for implementing general strategies and are suitable for all general problem-solving strategies. Helpful strategies clarify and help to perceive problems that are posed.

Problem solving skills

Categorization of the following skills needed in problem solving used by Moses (1982, 11.) is:

Level1.Familiarization with the problem

*Motivation

Level2.Basic skills

*Mathematical skills and reading skills

Level 3.General cognitive skills

*Forming analogies

*Flexible thinking

*Visualization

A solving map

Usually the fastest and most talented pupils try to solve problems as an extra workload, without teacher's help. Most of them haven't the tools to solve verbal problems. The other pupils do routine tasks and they are not at all involved in problem solving. I think it is very important to teach some kind of tool to all pupils, especially for talented. The solving map is a method that helps pupils to understand and verbalize word problems, for example.

Instruction how to make a solving map

1. Familiarize yourself with the task and read it through as many times as you need to understand it!
2. Pick out information about task and write it down on a separate piece of paper at the beginning of the solving map.
3. Choose a solving strategy

4. Write down your own ideas on the solving map. Always make a drawing or diagram of the task if it is possible.
5. Evaluate and check your solution.
6. Improve your solution. If you find a lot of errors, make a new solution after the wrong solution.

Note! Don't delete the wrong solution! It is a part of the solving process too! Discuss an example of the problem.

Example of the solving map method

Word problem 1.

Some boys have a flashlight, which will last for only 12 minutes. They must get safely through a tunnel before the tunnel collapses. The boys have no escape route other than to go through the tunnel. Only two boys can go into the tunnel at the same time. It is too dangerous to go into the tunnel without a flashlight. It will take one minute for Oliver to go through the tunnel, for Tim it will take two, for Pete four and for Mark five minutes. Each couple must walk in the tunnel at the speed of the slower one. In which order must the boys walk through the tunnel so that they can go safely while the flashlight is on?

There are two parts to the solving map. The pupil makes his/her own map on a separate sheet of paper. First he/she tries to find information about the problem.

A) Problem information

-A flashlight will last for 12 minutes and it must be in the tunnel with boys

-Only two boys can go into the tunnel at the same time.

The boys' speeds in the tunnel are determined by the following times:

Oliver 1 min

Tim 2 min

Pete 4 min

Mark 5 min

-Each couple must walk in the tunnel at the speed of the slower one.

After that the student starts to solve the problem. Sentences and drawings are more than desirable!

B) Problem solution attempt:

1. Oliver takes Tim 2 min and comes back 1 min (sum.3 min)

2. Oliver takes Pete 4 min and comes back 1 min (sum.5 min)

3. Finally Oliver takes Mark 5 min (sum. 5 min)

Altogether $3\text{min}+5\text{min}+5\text{min}=13\text{min}$

Wrong answer because the flashlight lasts only 12 minutes.

Now it is important that the pupils don't delete the wrong answer. The teacher must encourage him/her to continue and improve his/her answer.

New attempt:

Time must be saved: at least 1 min.

-Because each couple must walk in the tunnel at the speed of the slower one, it would be good to put the two slowest boys into the tunnel together!

-It would be useful to bring back the flashlight as fast as possible.

1. Oliver takes the flashlight and takes Tim 2min and Oliver comes back with the flashlight 1 min sum 3min.
 2. Oliver gives flashlight to Pete who goes through the tunnel with Mark 5 min. Tim comes back with the flashlight 2 min.sum.7min
 3. Lastly Oliver and Tim come through the tunnel together sum 2 min
- Altogether $3\text{min}+7\text{min}+2\text{min}=12\text{min}!$

What kind of problems would be good?

Changes in the word problem 1

First, we remove time limit of 12 min. Now everyone will certainly find some kind of answer.

Then we gradually increase the difficulty of the problem's next points so that the last point of the problem is closed.

a) In which order do you send the boys into the tunnel so that they can go quickly and safely?

b) Can you get the boys through the tunnel in 13 min?

c) Can you get the boys through the tunnel in 12 min? (It is possible!!)

The task should progress from an open to a closed problem so that no one hints a dead end right away. In this way we tempt pupils to familiarize themselves with the problem. When the first part of the problem has been solved, the threshold to try more demanding tasks is lowered because:

Success encourages the student to go on.

The problem has become familiar.

The students learn to work with the same problem longer than they are used to doing.

Problem solving makes challenging situation and open approach for gifted students: they can always go further, go beyond situations, ask new questions, initiate their own investigations, and be more creative in their mathematical work.

At the same time this gives us as teachers a chance to understand better what it is that makes mathematical talent appear and grow and thus leads to the creation of more efficient didactical tools that would help to keep their interest in learning mathematics.

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