SOME ADDITIONAL MATHEMATICAL ACTIVITIES IN MATHEMATICAL EDUCATION IN LITHUANIA

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Abstract. *The main aim of additional mathematical activities is to ensure for every gifted or interested person his right to have a suitable mathematical education according to his skills and abilities. From another side this deepens and influences the mathematical education in general. In the paper the authors try to show what is being undertaken in Lithuania in order to realize it.*

There is no doubt about it that the solving of mathematical problems is engaging, useful and creative but not very easy and sometimes even rather difficult task. In solving of non-trivial problem the whole human being with all its psychology and mind powers is involved. The main question and most important task in that direction is how to deal with universal problem of mathematical education: what ought and could be done to make the maximal number of people to feel and strongly believe that this is worth doing, worth undertaking efforts of every kind and is extremely useful for increasing of deepness of mind?

There is no doubt that the deepness of mind or ability possibly quick to catch the main features of the situation is one of greatest human treasures.

The attempts in that direction are taken since many years in many when not all countries among them also in Lithuania. After all mathematics as a subject suits perhaps most perfectly to make every engaged person to think deeper and to be able understand more than before.

It is clear that such a talk ought to begin from matters around the school and concerning the teaching of math or, generally speaking, from the education system. Afterwards we are forced to speak what could be done in order to improve the level of students who have to deal with the average school and its standards. Matters of that kind were not very much of importance in former days when only a bounded number of persons were able to get an education as it was e.g., hundred or two hundred years ago. In our formally very democratic time there aroused the real problem how to give a suitable mathematical education or how to teach the students to think under the circumstances when they are to forced to study more subjects spending for each less time.

Naturally because we are starting from the Lithuanian background, let us make a short historical discourse. For better understanding about the (mathematical) education during the first period of Lithuanian independence (or separate existence) we ought to recall some historical circumstances.

Grand Dutch of Lithuania was in personal union with Polish Kingdom since 1387. This common state existed till 1795 when it was divided between three neighboring empires. Lithuania came to Russia. In 1831 and 1863 in Russian part of divided state there were 2-armed revolts against Russian power. After the first one the Vilnius University (founded in 1579 and provided by Jesuits) was closed. After the second revolt the publication books in Lithuanian using Latin alphabet was prohibited. This lasted till 1904. It could no talk about the schools in Lithuanian.

So after the First World War it was necessary to create the national education system. University in Lithuania was (re)founded in Kaunas in 1922. First mathematical graduations works was written under guidance of German professor Otto Volk. He worked in Kaunas till 1930.

The possibility to become an education in mother tongue gave strong impulse to the whole system of mathematical education. These 20 years of Independence remarkable influenced

the state of education in Lithuania also after World War II when Lithuania it existed as Soviet Socialist Republic. The similar state was in other Baltic Republics too.

It ought to be told that in the Soviet Union the system of education was a strictly authoritarian system with the only announced and for everybody known possible truth and with the corresponding education. In the same time such education system served rather well for spreading the concrete information among others and may be first of all in the area of exact sciences. This could serve also as a possible explanation e.g., for the Sputnik and J.A.Gagarin flight.

In Lithuania (in that period being a part of the Soviet Union as well as in many others so-called Soviet Republics) the mathematical Olympiads for high school students started in the year 1951. The first Olympiad in Lithuania was being organized under guidance of Jonas Kubilius This very first Olympiad organized in Lithuania was organized as Vilnius - the capital city of Lithuania - Olympiad but all following mathematical Olympiads were already organized as whole Lithuanian Olympiads.

The mathematical Olympiads in Lithuania with not much more than 3 Mio of population were provided in three levels – first one being school level. The second was regional one – Lithuania is divided into approximately 50 regions. At regional level local staff provides the correction of works. In the first period central jury consisting mainly from University teachers and other scientific workers controlled the marking of regional staff.

In the third final stage between 100 and 150 high school students are participating. All materials of these Olympiads was prepared and published in two volumes by dr. J.J.Mačys (the first one together with dr. A.Grincevičius).

In that period or later the specialized boarding mathematical schools on the base and by support of Universities were organized in many Republics in Soviet Union; first of all in the leading centers such as Moscow, Sankt-Petersburg (former Leningrad), Kiev, Novosibirsk. From that point of view Lithuania unfortunately was an exception.

The idea for several persons to work together is as old as the world. In mathematical education and especially in training of gifted students this idea found its realization as team-contests.

In 1986 this idea in Lithuania was practically carried into life mainly by dr. Algirdas Zabulionis organizing efforts. This contest was named after mentioned above in that period already famous Lithuanian mathematician in probabilistic number theory Jonas Kubilius, who founded the cup. By the way, Professor Jonas Kubilius for more than 30 years was a rector of the Vilnius University.

As it was told the idea was to gather the students and to propose them to work "in corpore" as if they were one person. The invitation of teams is based on the regional principle - that means if the corresponding region has some achievements in the "usual" individual Olympiad then they are invited to participate. The team in Vilnius contest consists from 5 persons and is working in the separate room for 4 hours. Immediately after that the estimation of their papers follows and is provided under guidance of the mathematicians mainly from the Department of Mathematics and Informatics of Vilnius University and leading students. Afterwards in few hours the papers are marked, the winners announced and awards distributed.

This Olympiad proved itself useful in many aspects.

First of all, in the period of regaining the Independence of Lithuania and other (but not only) Baltic states the Lithuanian team-contest gave perhaps the main impulse – thanks activities of Latvian professor Agnis Andžans as well as by already mentioned dr. Algirdas Zabulionis - to the team-contest of the whole Baltic region. This contest was named “Baltic Way” in order to honor the chain of peoples connecting hand by hand all three capital cities of Baltic republics from Vilnius via Riga till Tallinn (600 km). We believe that it also raised the collective-work of young and promising students to a higher level.

In first phase in that already international team-contest only Baltic States (Estonia, Latvia and Lithuania) participated. The next country that joined this already International contest was Finland. Very soon other countries of Baltic region and also Iceland followed Finland. As a "Super Baltic country" was invited because Iceland was a first country who recognized the Independence of all Baltic States. In Lithuania we are proud to remember that in the Millennium year 2000 Professor Peter Taylor who visited our country opened the Lithuanian team-contest.

Another offspring of the Lithuanian team-contest was the individual Lithuanian Olympiad for youngsters. It serves as trainings base for the invited teams and for engagement of youngsters. In this year 2006 we will enjoy already the 7th edition of that Olympiad which is meanwhile provided in two levels – one for the grades 5-6 and another for the grades 7-8.In this period the net of regional Olympiads in many places of Lithuania was created. Some of them are individual, some are organized as team-contests and some are individual while the third are mixed. The grades, which are involved, also differ. So for example in Raseiniai region Olympiad is mixed, had already 6 editions and the grades involved are from 7 to 10. The secondary education in Lithuania lasts meanwhile 12 years. Another regional Olympiad, which has already 7 editions, takes place in Pasvalys at Latvian border. The winner is awarded with Bronius Grigelionis cup, which is another famous Lithuanian mathematician and well known specialist in probability theory. This Olympiad is individual and provided in two levels. The third local Olympiad worth mentioning is Olympiad in Rietavas – small but in Lithuanian well-known historical and cultural center almost at Baltic Sea. About the power of tradition in Rietavas is enough to mention that in this small city the first telegraph line in whole Russian Empire was provided. There are more regional Olympiads for high school students in Lithuania – for example Olympiads in Alytus, Kretinga and other places.

The Universities (in Lithuania is almost 10 of them) was always eager to attract clever students. That is one of reasons why some Universities provide their own contest for gifted high school students.

Firstly we would like to mention Kaunas Technological University, who realized project that was not realized in Lithuania in Soviet time – we have mentioned this already –namely at the base of this university gymnasium with the boarding school and so can take high-school students from the whole Lithuania. This gymnasium together with lyceum in Vilnius is since many years without any doubt two strongest schools. Both are taking about a 100 newcomers every year.

Another thing worth mentioning when we speak about Kaunas Technological University and gifted students is that the University since 15 years provides contest in four age's groups. In last contest about 400 high-school students participated.

It ought to be mentioned that all Universities providing these contests remain in touch with Ministry of Education – the winners can take part in the third - final stage of the Lithuanian Mathematical Olympiad.

Another University in Vilnius – the Pedagogical University – since years provides his own competition too – this year 350 persons participated. Šiauliai University has its own contest too.

Such contest are not so easy to provide and in the time they involve not only high school and University teachers but also their students – that is they widen the base of education of gifted students from both sides. We will further speak about the delicate matter of how we could estimate the influence of various kinds of Olympiads or efforts of good teachers.

Another thing is how to reach the students who are spread in the whole country. Main for that reason the Correspondence school for Mathematics was founded. This school is addressed to high-school students.

There was two periods in existence of that school.

The first period began 1969 and lasted till 1989 – that means it functioned twenty years in the Soviet time. The mathematicians of the Vilnius University provided this school. The

problems were published in the Lithuanian daily newspaper. For the students of that school some theoretical materials were presented, control works proposed and the answers marked. This school was renewed – so the second period begun - in the year 1998 on the wider basis under cooperation of several universities, teachers, students. The Lithuanian Mathematical Society is naturally also involved.

The leading force in this renewal was dr. Antanas Apynis, who became the head of the Advisory Board of Lithuanian School for young Mathematicians. The deputy director is the first author of this contribution.

Now the time of that correspondence-education last also two years, they are expected to solve 8 series of problems with 10 problems in each. All these problems as well the entering test and all theoretical materials and explanations they can find in corresponding Internet site (http://www.mif.vu.lt/ljmm). If the entering test is done successfully student gets individual password and at the end of education these who succeed are invited to come to Vilnius for the final test consisting of 4 problems for the solving of which 1 hour is given. We know that many students are applying to their teachers how to catch the idea of solution or what to do with the problem. Naturally they are also discussing the problems between themselves. When the gifted students apply to their teachers it is a challenge for both sides.

After reasonable result also in the final test the participants get the special individual certification, which is the same time also a recommendation for studying of exact sciences and give some advantages by entering exact science studies in some Lithuanian universities.

In this site the theoretical materials together with solving of examples and proposed problems as well as the information concerning the school can be found.

In the very next future site will also provide the English version with most important information and with the texts of all proposed problems.

The corresponding materials – all proposed problems with the exact solutions - after every cycle of two years are published year after year so that they are acceptable for everyone who is interested in it.

After the renewal of this Correspondence school in 6 issues approximately 2300 students graduated this school. Also 6 books (each containing more than 100 pages) were published [1].

The thematic of part of these tests is partly similar to what the students have in school and partly extends it e.g., Elements of graph theory, Concerning optimizations problems, Notion of complex numbers etc.

The important thing for non-formal mathematical education is also to possess an approach to the media where it would be possible to present, discuss and first of all to show how to solve some nice, simple and useful mathematical problem. In Lithuania the second author enjoys such a possibility since 1998 writing a series of articles on these subjects in the magazine for informatics (Computer and PC magazine). The very interesting thing is ability to provide the correspondence with the reader and propose the problems and discuss their solutions.

As an example we can take the famous Russian problem that the second author proposed also in International Conference on Creativity in Mathematical Education in Riga 2002 [2].

A 10-digit number is said to be interesting if all its digits are different and it a multiple of 11 111. How many interesting 10-digit numbers there are?

When proposed to the reader of Magazine we got the answer of the reader who using the computer found all 3456 an interesting numbers and listed them all.

HOW TO MEASURE THE INFLUENCE OF OLYMPIADS, CONTESTS? HOW TO ESTIMATE THE INFLUENCE OF GIFTED PERSONS IN THE LEARNING PROCESS?

There is no doubt that the various kinds of competitions, contests and other similar activities are of great importance and influence for every one who came in touch with this.

First of all sometimes the opinion is expressed that the top Olympiads e.g., IMO, are devoted for too few persons.

Professor Peter Taylor during the conference devoted to the gifted education in Riga 2002 answered that question telling that the pyramid can't exist without the top.

Thinking about similar things one could ask how to measure the height of such pyramid or how one could build the higher pyramid? As far as we understand the movement of gifted education is a reaction against the average state in education. In terms of pyramids one could tell that the movement of gifted education tries to push towards the top of pyramid as many person as possible.

The second author is the University teacher in the same time since years is teaching in usual (frankly speaking, quiet normal) gymnasium and he sees that the school education naturally tends to average state of existence. It's clear and understandable.

Firstly, one can't expect great wonders from the public education but otherwise everyone is expected to do his best. The process of education is rather irrational by his nature.

The explanation of this is quite natural – the teacher is explaining in one way and student (very often) understands it in another and explains it in the third way.

These are common truths. What could be done? Can we achieve remarkable success? Can we have noticeable achievements? What if being not so far from the top of the pyramid we begin to loose the sight of the bottom of that pyramid?

What kind of brainstorm could be undertaken to avoid such a situation?

Is it right that the democracy in education expects that the height of pyramid of education wouldn't be too high?

So again: how to attract and how to make the things interesting?

Some of the means for this are clear and classical while the others are relatively new. This for example concerns the processes, which take place in mind of solvers when he is dealing with the problem. What to do when we have to deal with not very easy problem? This can became and indeed always is a psychological problem.

The second author became interested and involved in psychological problems and has written a book about this "How to solve the problem when you do not know how?"[3].

The classical mean could be for example to come your listener curious? You can present him two quiet similar situations and ask him why the one situation is possible while the other not?

As an example we present an easier version of problem of Prof. Agnis Andžans, which was presented in Consortium, that is, in The Newsletter of the Consortium for Mathematics and its application, Number 45, Spring 1993. The idea remains the same. Here is the text:

Is it possible to rearrange the numbers 1, 2, 3, 4 and 5 as $a(1)$, $a(2)$, $a(3)$, $a(4)$, $a(5)$, so that all the numbers $|a(1)-1|, |a(2)-2|, |a(3)-3|, |a(4)-4|, |a(5)-5|$ are different? Prove your assertion.

The answer is easy „yes" as shown in table below. First line presents the numbers, the second – their rearrangement and the third - absolutes values of the corresponding difference.

1	2	3	4	5
5	2	4	1	3
4	0	1	3	2

Now we would like to ask you to take all natural number 1, 2, 3, 4, 5, 6 to rearrange them and again to achieve that all absolute values of corresponding differences would be different again?

The second possible traditional advantage of mathematical problem is to find for it possibly nice formulation. That's not ably easy or even possible but again worth trying.

As a not difficult but a pretty example we could propose such problem "all is 11". Find the number the binary representation of which contain 11 0's and 11 1's and which is divisible by 11.

In British Mathematical Olympiad Anno Domini 2004, Round 2, there was similar problem "all is 2004".

We would like now to say some words about **the Kangaroo Competition, which we regard as a revolutionary movement upwards, or "from below".**

We'll try to explain what we have in mind. But at first we would like to add some words about the local Kangaroo flavor in Lithuania. The Lithuanian Kangaroo version started in 1999. Main organizing efforts of creating this Lithuanian version of that competition were undertaken by dr. Juozas Juvencijus Mačys and by the Ministry of Education and Science, who was till nowadays is represented by main specialist Mrs. Marytė Stričkienė. It ought to be mentioned that in Polish and Russian schools of Lithuania competition started earlier - with the Polish version got from Warsaw and Russian – from Sankt-Petersburg.

Mainly thanks the support of Ministry the number of participants grew rapidly and after some years it reached the mark of 60 000. There are 5 usual groups of age. Again in each region there are local organizing committee or at least main responsible person.

Perhaps main advantage of that competition from the points of view from these who participate is that the student have possibility to see nice formulated problems with the unexpected moments in solution of it and with some mathematical ideas who are represented in such a clear form that even the schoolboy in age of 10 may have some use from it.

For example, we know that it happens rather often that after the competition the student comes to his teacher and asks:

What is combinatorial multiply principle?

Why about the pigeonhole principle I do not hear in school during the usual lectures?

These and other similar questions is a part of this "upward movement".

But the main advantage is that many of these problems are common sense problems – you may solve them, you may try to guess the answer – you'll move forward anyway. These problems are also good and suitable for the training of the university students too especially those who will became teachers.

It happened once that the Kangaroo test were proposed to some student who was silver medallist in mathematics as well as in informatics. He wasn't able to get all points and his comment was following: „Oh, these problem is not so easy as a whole". By the way, in Lithuania the best students in mathematics are winners of Kangaroo too.

In the same time some words could be added about the University students Olympiads and about – and is perhaps even more important about the influence of these participants.

Participating since 1999 in IMC (that is International Mathematical Competition for University students) and similar competitions our University students always proved themselves well. Sometimes their results were astonishing: the 5th place in Hungary in 1999 of Giedrius Alkauskas or 4th place of Andrius Stankevičius in Ostrava 2006 could be mentioned.

Such students are first who help us to provide various activities, local competitions, etc. E.g., the Jury of usual individual Lithuanian Olympiad of the last years consists already mainly from University students, not seldom it happens that the student who is now in Jury a year ago was a participant himself.

So despite the fact that the influence of different mathematical activities to the general process of education is not easy to measure this influence is remarkable. We wish very much that this influence could increase and grow.

We believe that this is the main strategical aim of our efforts.

Conclusions. *The problem of how to influence and improve the learning process in mathematics is a very honorable, difficult and challenging and every responsible teacher has to deal with this task. In this direction is extremely important to try researching each possibility and investigating every way.*

References

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