THE LABORATORY OF MATHEMATICAL MACHINES OF MODENA

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WHAT IS A MATHEMATICAL MACHINE?

A mathematical machine (in geometry) is an artefact that has no practical purpose and is designed to force a point, a line segment or a plane figure (supported in a way as to make it visible and touchable) to move according to a mathematical law determined by the designer.

The most well-known mathematical machine is the pair of compasses (to draw circles) that is part of the iconography of mathematicians (Fig. 1). It is the ancestor of many curve drawing devices and pantographs. Another class of mathematical machines is given by perspectographs (e.g. Fig. 2), which are related to the ancient three-dimensional theory of conics, on one hand, and the roots of projective geometry on the other.

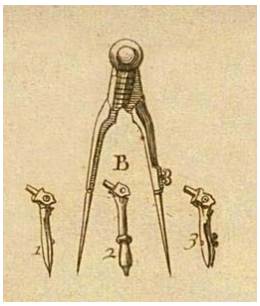

Figure 1: compass

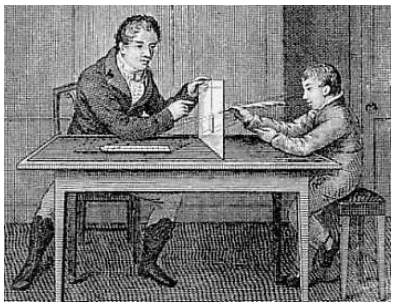

Figure 2: perspectograph

THE LABORATORY OF MATHEMATICAL MACHINES

The Laboratory of Mathematical Machines (the MMLab), in the Department of Mathematics in Modena, contains a large collection of mathematical machines. They have been reconstructed with a didactical aim, according to the design described in historical texts from classical Greece (linked to the theory of conic sections) to the 20th century. In the MMLab there are different kinds of instruments: curve drawing devices, pantographs for geometric transformations, models of conic sections, models of singular point of algebraic curves, models of the 3D genesis of plane geometric transformations, perspectographs, anamorphoses and so on.

The MMLab works for both didactical research (Bartolini Bussi, 2000, 2005; Maschietto & Bartolini Bussi, 2005) and the popularisation of mathematics

(European Mathematical Society – September 2005[1]). The activities of the MMLAB take place both inside the Department of Mathematics and outside. In the MMLAB, teachers and pupils are received for visits (which are described later); whereas outside the MMLAB staff is involved in both teaching experiments and organisation of exhibitions, presenting collections of mathematical machines. For example, the MMLAB has participated in the Science Festival (Genoa 2004, 2005), Bergamoscienza (Bergamo, 2005) and the V Salon de la culture et de jeux mathématiques (Paris, 2004). The MMLab staff includes academic researchers, university students, teachers and members of the "Macchine Matematiche" association[2].

The aim of this paper is to present the activity at the MMLab.

Figure 3: the MMLab

THE VISITS AT THE MMLAB

The MMLab is open to classes during the whole school year. When teachers decide to organise visits to the MMLab, they book it and specify a topic, choosing between conic sections or geometrical transformations. The topic of conic sections is often chosen by high school teachers, whilst the topic of geometrical transformations can be also proposed to middle school pupils. A visit lasts an hour and a half.

Three stages form each visit: an introduction to the chosen topic, group work and the presentation of the results found by each working group.

Stage 1. As regards the conic sections, at the beginning, historical elements concerning the development of Menecmo-Euclid and Apollonius's theories are presented; then models of a perfect compass and Descartes' instrument for hyperbolic lenses are considered; at the end an ellipse is studied as a cylindrical section (right cylinder). As regards the geometric transformations, the introduction stage starts with the description of some instruments concerning the genesis of geometric transformations in 3D space (homothety, translation) and the instrument for translation in 2D space (Kempe's translator). This first stage is conducted by the MMLab personnel, using both physical instruments present at the MMLab and interactive/non interactive simulations (Cabri II Plus, Cinema4D).

[1] http://www.ems-ph.org/newsletter/news.php
[2] http://associazioni.monet.modena.it/macmatem/

Stage 2. Pupils are invited to form small groups (at most five pupils), which receive a mathematical machine and a worksheet to guide their exploration. A worksheet contains several questions: the first questions concern the structure of the given instrument, the following questions request explanations of the relationships between its components and drawn curves or corresponding points in limited plan region.

Stage 3. Each group presents the studied mathematical machine to their fellow pupils. It is an important institutionalisation moment (Brousseau, 1997), because the results of each group work session are shared and acknowledged by both the class and the teacher.

Figure 4: Stage 1

Figure 5: Stage 2

Figure 6: Stage 3

As regards what is described above, we would like to draw attention (and also the discussion) to some aspects which could mark out this MMLab activity in terms of challenge for pupils and teachers.

1. A visit to the MMLab is structured in a different way from a visit to an exhibition: just the first stage can be compared to a “standard” visit, where an animator introduces the exhibits (we will return to this point later).

2. In the MMLab, the “rules of the game” are known a little to teachers (even if it is the first time that they accompany the classes), but they are quite unknown to pupils. Nevertheless, the chosen topic of the visit is known. This situation is different from, for example: a “standard” visit; a visit to a science centre, when there are interactive exhibits that pupils “have to” touch, play and handle; a mathematical competition, where proposed problems are coherent to training, based on the knowledge of the “rules of the game”. An aspect that can be observed is the pupils’ surprise when they come into the MMLab hall.

3. From one stage to another, the three subjects involved in the MMLab visit (i.e. the animator, pupils and teacher) play different roles. The animator is the voice of the history during stage 1, he/she supports and controls the working groups in stage 2, he/she guides the pupils’ presentations (i.e. he/she chooses the sequence of their presentations) and validates their answers in the last stage. The pupils are visitors and manipulators; they sketch, formulate conjectures and discuss; they present their work. Generally, the pupils do not have so many different roles during a mathematical course. Only the teacher’s role is not well defined in our visit: he/she is a kind of “joker”. In fact the teacher is responsible for teaching in a classroom situation, but

he/she is relieved of the responsibility of teaching during a MMLab visit. He/she takes part in the introduction of his pupils, then he/she can follow and participate in working groups without having any time management to take care of. Moreover, the teacher can, on one hand, be an observer of pupils' exploration processes and conjecture formulation, on the other he can experiment a different kind of mathematical session without being directly involved. Generally, this is new situation for the teacher and it is different from a mathematical course.

This new situation can determine different relationships among the pupils, between the pupils and teacher and between pupils and mathematics.

INFORMAL, NON-FORMAL OR FORMAL LEARNING IN THE MMLAB

Observations of the MMLab visit are a starting point for our reflection on the MMLab activities, that has to be studied in depth with the analysis of what happens during visits in terms of pupils' cognitive processes and learning with respect to other situations. A "tool" to begin the study of the mathematics laboratory format is the distinction among informal, non-formal and formal learning. The EC communication (2001) specifies as follows.

> **Formal learning**: learning typically provided by an education or training institution, structured (in terms of learning objectives, learning time or learning support) and leading to certification. Formal learning is intentional from the learner's perspective.
>
> **Non-formal learning**: learning that is not provided by an education or training institution and typically does not lead to certification. It is, however, structured (in terms of learning objectives, learning time or learning support). Non-formal learning is intentional from the learner's perspective.
>
> **Informal learning**: learning resulting from daily life activities related to work, family or leisure. It is not structured (in terms of learning objectives, learning time or learning support) and typically does not lead to certification. Informal learning may be intentional but in most cases it is non-intentional (or "incidental"/ random).

Studies on this theme propose different approaches with several nuances, but they agree on the statement that

> Boundaries or relationships between informal, non-formal and formal learning can only be understood within particular contexts. They conclude that it is often more helpful to examine dimensions of formality and informality, and ways in which they inter-relate with each other; and that attention should be paid to the wider historical, social, political and economic contexts of learning, and to the theoretical view of learning that is held by the writer.[3]

As the SEEQUEL project (2004) emphasizes "all three learning formats can be integrated and seen as a means of providing a unique path/strategy for individual personal/organisational/societal development" (Fig. 7).

[3] http://www.infed.org/archives/e-texts/colley_informal_learning.htm#preface

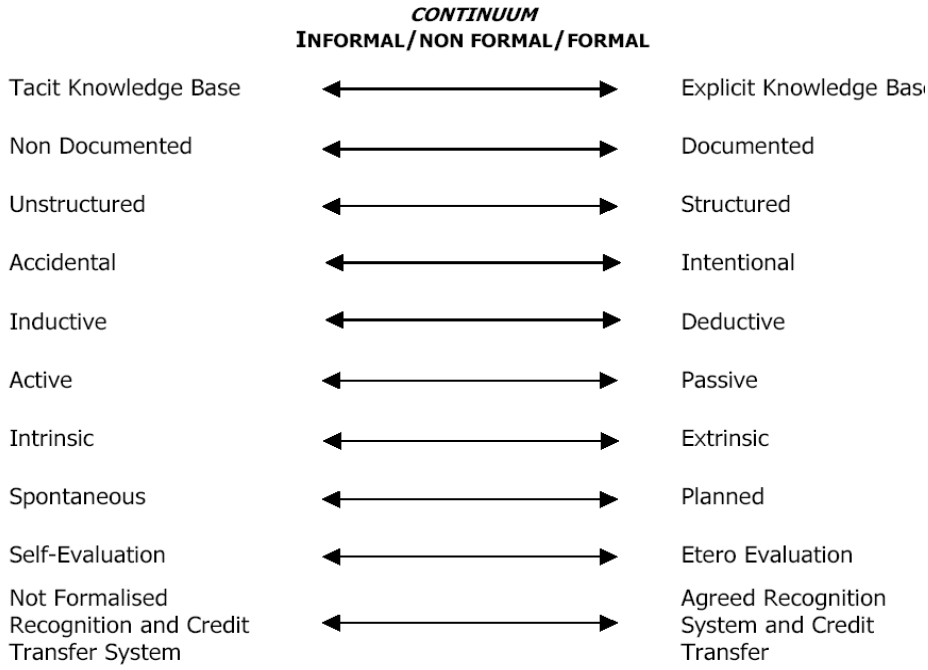

Figure 7: Continuum informal/non-formal/formal

With the distinction above, we can try to classify the MMLab activities, exhibitions and visits to the Laboratory. During the exhibitions, two kinds of visit are present: free visits for the public and guided visits for classes. Even if the aim of these activities is the popularisation of mathematics and to foster a positive attitude towards mathematics, showing the beauty and wonder of the instruments, during the guided visit there is intention of learning.

We could assert that the first way concerns informal learning, whilst the second one concerns non-formal learning. But as regards to a guided visit, as we have already mentioned, a MMLab visit is different. We have tried to summarise the two situations as follows.

	Guided visit	**Visit to the MMLab**
Structure	Presentation of approx. 20 machines	Presentation of 5 machines at most, then group work on one machine
Time management	Few minutes for each instrument	Three quarters of an hour at least for each instrument
Exploration	First with the animator, then free	First with the animator, then guided by worksheets on different instruments
Pupils' involvement	Listeners, then manipulators	Listeners, manipulators, writers, commentators
Teacher's role	Listens, and intervenes if necessary	Listens during stage 1, follows the group work, intervening when requested by pupils, listens and intervenes during the presentation of the group work
Pupils' position	Standing	Sitting down

Table 1: guided visit vs visit to the MMLab

Nevertheless, a MMLab visit does not entirely correspond to mathematics classroom, that is, to formal learning. The use of working copies of historical instruments has the potential to address cultural and affective issues, on the other hand, they match the need to construct the meaning of mathematical objects and practice mathematical processes, such as conjecturing and proving. Can we conclude that the activity with the MMLab instruments may effectively interface both non-formal and formal learning?

Figure 8

Some features of non-formal learning have been borrowed and introduced in schools. This is not completely new: the practice of using tangible instruments in mathematics (and in geometry too) was present in the work of mathematicians until the early decades of the 20th century, when the Bourbakist programme shifted focus to formal and symbolic aspects. The enactive mode of knowledge (according to J.Bruner) remained in mathematics education, usually limited to young pupils, as if the importance of handling objects and exploring space decreased with age. In some cases the confidence of the power of the concrete experience itself was surely excessive, as if it were transparent for the mathematical meanings or procedures embodied in it.

RESEARCH ON THE LABORATORY

With reference to what has been described above, we have a good example of what has been called a mathematics laboratory by the Teaching Commission of the Italian Mathematical Society[4]:

[4] http://www.dm.unibo.it/umi/italiano/didattica/2003/secondaria.pdf

> A mathematics laboratory is (...) rather a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. (...) We can imagine the laboratory environment as a renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in one word: practicing. In the laboratory activities, the construction of meanings is strictly bounded, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students).

This description represents a meaning of the expression "mathematics laboratory". In fact, in the debate on this theme promoted by the congress *Mathématiques : des laboratoires pour le primaire et le secondaire?* (Maubeuge, France, 2006) other positions are appeared:

- "mathematics laboratory" as a room where pupils can study mathematics in a different way compared to mathematical classrooms;
- "mathematics laboratory" as a way to organise mathematical activities (working groups, manipulation, games) outside the classroom (and often outside the school) in contrast with mathematical sessions; the contents may or may not be related to class mathematical curricula.

However, reflection and/or analysis of processes (cognitive, ...) during a mathematics laboratory session does not always appear clearly, beyond the enthusiasm towards the response of the class participation (a part some research project, as example Rodari et al., 2005). From our point of view, the laboratory activities put forward several questions:

1. What is the degree of reinvestment of what is done in a MMLab visit?
2. Is there really an influence (and change) in the attitude toward mathematics?
3. What are the effects on the teaching practice of teachers who see their pupils work with instruments?

On the basis of some observations on the reactions of the teachers with respect to the pupils' involvement and the fact that every teacher that has visited us has also come back, we have planned a research project to investigate the third question mentioned above. This choice depends on the fact that pupils come to the MMLab only once, whereas teachers come again and again. We have structured two questionnaires to find some aspects of their teaching practice before and after the visit. In particular, in the pre-visit questionnaire, we would like to obtain: some ideas on teaching, the mathematics contents of their mathematics courses before and after the visit and their expectations of the visit. With the post-visit questionnaire we would like to find out: if the visit was up to their expectations, if some elements of the visit topic have been revised and what kind of reaction pupils had to the visit. Since this is an ongoing research project, we will present some results during the conference.

CONCLUDING REMARKS

In this paper we have proposed our contribution to the discussion on mathematics laboratories, with reference to our experience in the Laboratory of Mathematical Machines. We have tried to point out how an MMLab visit can be considered a challenge for pupils and teachers and placed in the continuum informal/non-formal and formal learning. Finally, we have presented an ongoing research project concerning teachers who accompany their classes to the MMLab. We do not discuss the other activities of the MMLab, i.e. exhibitions and didactical research projects, besides the MMLab visits, in great detail. In fact, the main aim of our permanent Laboratory at the Department of Mathematics is didactical research, i.e. research into the teaching and learning of mathematics. Many examples are discussed in detail, in the book by Bartolini Bussi and Maschietto (2006).

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