THE TRANSALPINE MATHEMATICS RALLY IN PRIMARY AND LOW SECONDARY SCHOOL: A PROBLEM-SOLVING AND A MATHS EDUCATION EXPERIENCE

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Abstract

The aim of this paper is that of considering some didactical aspects of a problem solving approach in mathematics education by using a class competition named *Rallye mathématique transalpin* (Transalpine Mathematics Rally). This mathematical competition has stimulated increased interest both from the students called upon to resolve problems collectively and on the part of teachers and researchers in mathematics education who can analyse the resolving strategies adopted. In particular, for what concerns students, the main feature of this competition is the creation of learning situations permitting pupils to mobilise and put in play their knowledge, to hypothesize, to argue and at the same time, if possible, to create new skills. For what concerns teachers it is a useful source of information and suggestions also for teacher training, both at the initial stage and in in-service training. Finally, for what concerns researchers, it permits them to analyse students' strategies, difficulties, obstacles, and misconceptions, and to study the possibility of using "good" problems for constructing mathematical concepts.

INTRODUCTION

In the *Rallye mathématique transalpin* (RMT), the idea is that of a class competition: the whole class has to produce a unique solution for each problem given. As the problems are too many for a single pupil, it is necessary for the pupils to work in groups. The official teacher is not present during the competition, so that the class has the entire responsibility for working out the solutions: a sort of "devolution" (Grugnetti, Jaquet, Tièche-Christinat, 2005). Pupils must make decisions on the answers to be given, which implies the pupils resort to arguing in order to support their statements, and that leads to the validation of the mathematical activity.

In 1992, *Math-Ecole* (a Swiss French-speaking review for teachers of mathematics edited by François Jaquet) proposed an initial confrontation of mathematical problems for 3rd, 4th and 5th grade classes. Interest for this project developed quickly and two years later there was the participation of Swiss-Italian classes too; three years later, Italian classes; and finally, classes from Belgium, France, Israel, Luxemburg and USA. Moreover it was extended until 8th grade and from the last edition until 10th grade. More than 2500 classes are now involved.

SOME EDUCATIONAL AIMS

To solve problems is to do mathematics

Learning mathematics, as we well know, is not merely the mastering of technical calculations or book-learning. Solving problems constitutes the aim and the foundation of learning by experience; it gives a meaning to situations that are to be solved mathematically. The context of the RMT is stimulating; the problems proposed are considerable and original; and the pupils become involved and learn to be responsible. In fact, the problems proposed are not simply applied exercises from the last chapter that was studied, but original situations to be solved mathematically: pupils have to transfer their personal knowledge to different conceptual fields or create new tools.

Today, the ability of working in a group is essential

In today's society, it is more and more important to work in a group; it means being able to divide up the work, manage time, suggest ideas, accept other ideas and cope with various points of view. In this rally, there are too many problems for a single pupil to solve, but the rules of the game assure the cooperation and development of interaction among the pupils.

Confrontation is a source of renewal

Pupils, teachers and mathematical activities in the classroom are all stimulated by the external contribution of problems. There are new ideas, new paths to explore, exchanges, comparisons, challenges, group analysis, etc. The RMT is not only a competition, it is also the occasion to analyse in detail the results, and to give prominence to different procedures, representations, and difficulties.

WORKING OUT PROBLEMS

In this kind of rally, the first thing to consider is the selection of problems which have to satisfy certain criteria of quality (Jaquet, 1999).

The language of the terms of the problem must be clear and rigorous in order to avoid misunderstandings due to local habits; the style must be adapted to the pupils' language, but differ from the traditional stereotyped school exercises. From the mathematical point of view, we must be sure of the existence of one or more solutions and that the required knowledge matches the pupils' development. From the didactical point of view, it should be possible for the problems to be done within the school curriculum, using different strategies and representations, with difficulties at different levels (everybody can do something). It is important to be able to interpret the pupils' different procedures, especially procedures with typical mistakes that reveal the pupils' conceptions and models. The problems must be completely original.

'A-PRIORI' AND 'A POSTERIORI' ANALYSIS

The a-priori (or previous) analysis (Charnay, 2003) is one of teachers' professional tools in order to help them in anticipating pupils' reactions and, therefore, for orienting some teachers' choices.

In particular, the a-priori analysis of a «problematic situation» is a work on hypothesis concerning, at least:

a) procedures, strategies, reasoning, solutions which a pupil could use in the proposed situation taking into account the supposed knowledge: may she/he start to work out the problem?

b) difficulties she/he can have and errors she/he could make: in particular, does the situation allow the pupil to put into evidence his/her wrong conceptions?

An 'a priori' analysis is controlled by the teachers' and researchers' teams who work out the problems: the choice of the activity within a conceptual field (knowledge base), the description of the task and expected strategies, and the definition of the "assessing" criteria for grading. The contrast between the 'a priori' analysis and the 'a posteriori' one is fundamental: i.e. the interpretation of unexpected strategies, the analysis of all the strategies and arguments of the pupils, the statistical comparison, etc.

This activity also helps to spotlight the pupils' difficulties and misconceptions.

The teacher knows where he/she stands concerning the development of classroom mathematics.

AN EXAMPLE OF AN RMT PROBLEM FROM A DIDACTICAL POINT OF VIEW

In RMT some problems are given to different age pupils. The more consistently made mistakes reveal the pupils' conceptions and models.

Also important could be “a didactical variable”, which is defined as “a problem’s datum, the modifications of which can lead to a change in the resolutions’ procedures”.

A similar word statement for different age pupils’ groups with different didactical variables: an example

We give here an example of a problem situation with a similar word statement for different age pupils’ groups with different didactical variables (Grugnetti, Jaquet, 2005):

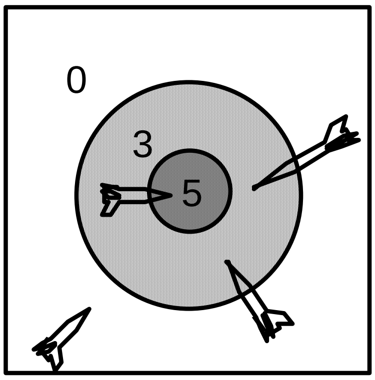
THE TARGET (Cat 3, 4: 3rd and 4th graders)

Saverio has reached a total of 11 points by throwing his four little arrows at this target.

He asserts that by throwing each time four little arrows he can obtain all the possible scores from 3 to 20.

What do you think about this claim?

For each score you find indicate your calculations.



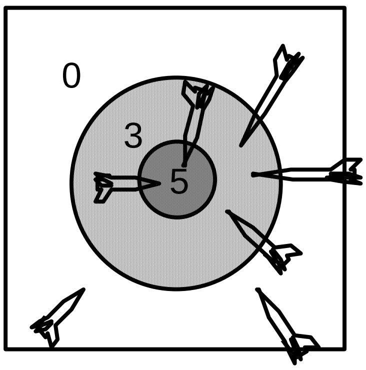
THE TARGET (Cat 5, 6)

Saverio has reached a total of 19 points by throwing his seven little arrows at this target.

He asserts that by throwing each time seven little arrows he can obtain all the possible scores from 3 to 35.

Is he right?

For each score you find indicate your calculations.



As we can see, in the second version of the problem, for 5th and 6th graders, the arrows are seven, instead of four, and the research on the totals goes from 3 to 35, instead of going from 3 to 20.

This didactical variable has been very interesting to analyse. It allowed the older pupils to use more economical expressions and therefore to apply multiplicative procedures given that the terms are 7, and not only 4, as in the first version of the problem, and therefore a list would be too long to carry out.

The young pupils used, in general, additive procedures, which did enable them to answer the first question.

Anyway we could observe in some multiplicative procedures a loss of the meaning of the activity, which leads to formal “decontextualized” expressions of some “traditional” exercises. For example the pupils of a lot of classes gave an answer with multiples of 3 and 5, but without their linear combinations. The influence of school practice and of the moment in which the multiplication is worked, plays an important role in the pupils’ strategies, while the influence of the different regional or national contexts and curricula is not clearly perceptible.

THE IMPACT OF RMT ON STUDENTS

Beatrice, 8 years old, writes:

My impressions about the Rally. I like the mathematical rally because, according to me, it is nice to work in group and to try several solutions, to know the opinion of all the members of the group and to help each other. In this way we work in mathematics and we learn to help each other. I like Rally’s problems because they are not the routine problems we have to solve by calculating, but here we have to use logic and we can find several different solutions. By working in group we can compare our ideas and in this way we solve problems being alone, without the help of the teacher.

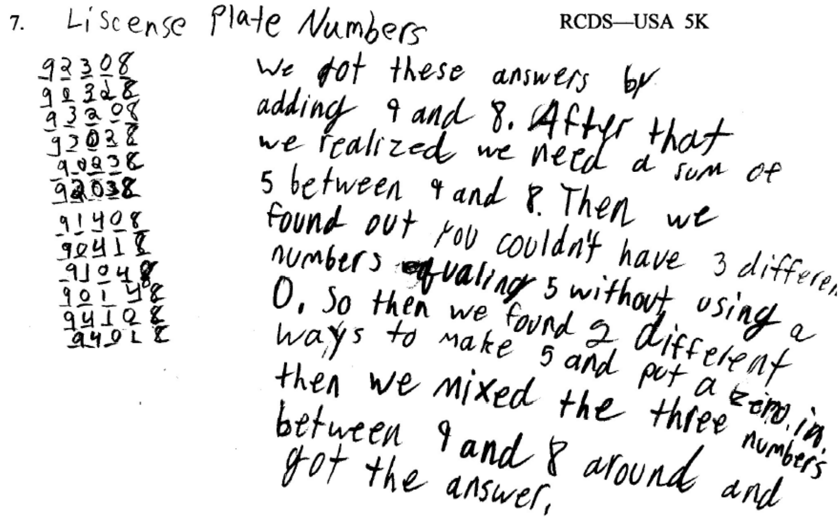
The impressions of Beatrice are quite common in the students participating in RMT, and they put into evidence some fundamental aspects we attach to rally, among them, the importance of working in group and the fact that all the pupils are involved (Grugnetti, Rinaldi, 2003).

Through these metacognitive aspects, RMT influences learning of mathematics. For example, in (Bertazzoni, Marchini, 2005) research, one conclusion of a controlled experiment, is that the time “wasted” with solution in group of RMT problems and the subsequent collective discussions seems to pay with better results than in the routine problem solving.

Moreover, as noted in (Grugnetti et al., in press), RMT contributes also to the development of important attitudes to describe the procedures used for solving the problems.

The value of the habit to describe the solutions’ procedures concerns several cognitive levels. The need (in RMT problems) to explicate the different steps of the research for solving a problem – explication which is done in natural language – is the first important step towards a reflexive attitude on the choices pupils make, as a necessary base for developing argumentative and deductive thinking.

An example of explanation[1] (5th grade)


7. Liscense Plate Numbers RCDS—USA 5K

92308
92328
93208
92038
90238
92038
91408
90418
91048
90148
94108
94018

We got these answers by adding 9 and 8. After that we realized we need a sum of 5 between 9 and 8. Then we found out you couldn't have 3 different numbers equaling 5 without using a 0. So then we found 2 different ways to make 5 and put a zero in. then we mixed the three numbers between 9 and 8 around and got the answer,

THE IMPACT OF RMT ON TEACHERS

In most cases, the teachers who were invited to take part in the RMT with their classes expressed at first their fear about working in groups, which so far had not been much explored as to problem solving. The skills that pupils can develop when working in groups are generally understated; the biggest fear is that some conflict or episodes of overconfidence could take place. Indeed, during the first "training tests", the discussion, which was quite lively within each group, sometimes caused conflicts among the pupils and some stretch of authority by the leaders. On the other hand, as the teachers unanimously state, the rules of the discussion gradually became clearer and the organization of the groups got better during the various "rounds" of the RMT .

Eventually the teachers' fears turned into enthusiasm for going on.

The teachers participating in the various stages of the RMT are able to take advantage of in-service mathematics teacher training and learning (Medici, Rinaldi, 2003). In general they are able to be involved in the following:

- Observation of pupils (in their own or another class) in problem solving activities, in the form of group work;
- The analysis of the work produced by the pupils (of their own or another class) and their organisational abilities, comparing the a-priori and a-posteriori analyses of each problem;
- The discussion of solutions and justifications, developing them further in class;

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[1] **7. THE LICENSE PLATE** (Cat. 4, 5, 6)

The police are looking for the car belonging to a thief.

- a first witness observed that the license plate number had 5 numbers, all different from one another,
- a second witness remembers that the first number was 9,
- a third witness noticed that the last number was 8,
- a fourth witness, who is 22 years old, saw that the sum of the five different numbers of the license plate was the same as his age.

What could be the number of the license plate of the car the police are looking for?
Write down all the possibilities, and explain how you found them.

- The collective discussion of both expected and unexpected errors and possible reasons for their occurrence;
- The receipt of new ideas for their own teaching methods which have been subjected to collective discussion.

Moreover, some research (ex. Bertazzoni, Marchini, 2005, Crociani et al., in press) evidenced the inclination of teachers taking part in the RMT with their classes, to "throw in play" their class practice in teaching mathematics by the light of the potentialities of RMT problems and their didactical analysis. In particular in (Bertazzoni, Marchini, 2005), the research shows in which way the attitude of the teacher (the first author of the quoted paper) changes regarding the use of RMT problems versus the use of "stereotyped" or routine problems; after hands-on experience, she notes the pupils' better attitude towards mathematics and the improvement of their outcomes in problem solving, when they use RMT problems.

In (Crociani et al.) the research puts into evidence the interest of the teachers in preparing an "a-priori" analysis, as for RMT problems, for each problem they give to their own pupils.

These last years the results of the RMT have been more and more useful for the teacher practice in the class work (Grugnetti et al., 2001) and also in teacher training courses (Grugnetti et al., 2003; Medici, Rinaldi, 2003; Grugnetti, Jaquet, in press).

RMT AND DIDACTICAL RESEARCH

RMT is not only a competition; it is also the opportunity for an intensive work on didactical analysis. In preparing the problems for the tasks of the competition, the international group tries, a priori, to consider the procedures and the representations the pupils could adopt and the obstacles they will find. The word statements of the problems become very important as does the adjustment of the didactical variables. Finally, the "a-posteriori" analysis, done taking into account the pupils' protocols, allows us to confirm or to reject the previous hypotheses, to show strategies and representations we didn't consider "a-priori", to calculate the frequency of the kinds of procedures, and to "measure" pupils' difficulties.

Studying several hundreds of pupils' protocols, some didactical phenomena appear with a great regularity in different countries, concerning representations, obstacles, errors, and evolution in procedures according to the pupils' ages.

Here we give two examples:

1. "Chase to three" is a problem which allowed us to observe that positional numeration still presents unsuspected gaps in pupils who are able to write or to read the number series (Jaquet, 2003).

CHASE TO THREE
Isidore is writing the number series, starting from 1:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, …
At a certain moment, Isidore writes the numeral 3 for the twenty-fifth time.
Which number is Isidore writing at that moment ?
Explain how you reached your answer.

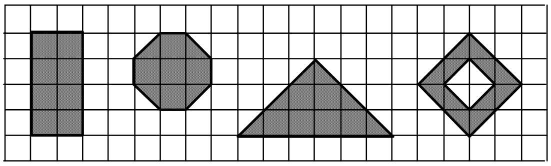
The analysis of the results of 140 classes of two Regions in Italy and in Switzerland evidenced a successful outcome in 25 % of the cases of 8-9 years old pupils and in 50% of the cases of 10-11 years old ones. Evidently there is some incomprehension of the wording of the problem, but also some errors due to an insufficient perception of the regularities of the numeration system: several pupils do not count the two “3’s” of the number 33 and many of them count only the “3” in unities, forgetting those of the tens.

Other problems of RMT, constructed on the same subject, confirmed these gaps in numeration. This set of problems can represent a tool for learning and assessment (Jaquet, 2002).

2. The problem “Decoration”, analysed in detail by Vernex (2001) and Jaquet (2005) has prompted a reflection on the way the pupils used for pointing out the two related magnitudes in a proportionality situation.

DECORATION (Cat. 5, 6, 7)

A painter has painted these four different figures on a wall, each one having the same coat of painting.



He used painting tins of the same magnitude:

- 18 tins for red painting for a figure
- 21 tins for blue painting for another figure
- 27 tins for yellow painting for another figure
- some tins for black painting for the remaining figure.

At the end of his work all the tins were empty.

Indicate the colour of each figure. How many tins for black painting has he used?

Explain how you reached your answer.

The analysis showed that many young pupils did not resort to the ratio of proportionality between the two numbers series, but they made do only with the reproduction of regularities, found by the differences between successive numbers of two series.

Other similar problems have been constructed in order to verify these first remarks and they confirmed the first analysis. A research group of RMT, composed by teachers and researchers in maths education, is at present studying in depth this aspect of proportion: which are the pupils’ spontaneous procedures for completing some proportional series, which are the traps where they fall and which are the situations we have to give them in order to allow them to distinguish proportional situations from the other ones.

CONCLUSION

Mathematics competitions are expanding everywhere in the world inside or outside schools, with different modalities, but having in common the resolution of problems.

Some competitions attract the “strong in mathematics” and are limited to do a winners’ list based on the “good answers”. Others, more and more numerous, have some pedagogical aims:

teachers' agreement, approach to new didactical methods, etc…. . The RMT joins the last trend, putting explicitly into evidence its aims for pupils, for teachers, for teacher-training, and for research in order to further pupils' or classes' discussion, towards a contribution for mathematics education. This singularity of RMT is due, in particular, to the collaboration of its groups of teachers, teacher-training students and researchers, in maths education. Without the "crossing eyes" of these different partners and without pupils' explanations of their solving procedures, the RMT problems couldn't achieve the necessary consistency for having some effects in teaching and learning mathematics.

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