

Will mathematics online renew the teaching of math¹?

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Abstract

ICT have lead to deep changes among a group of French teachers of mathematics, but their number is still scarce. Those who have adopted ICT are no longer solitary teachers facing their classes but productive collaborative workers with colleagues from all horizons. Elaborating new online resources in common reveals new and unsuspected skills. It follows that those who are implied undergo an intense continuing training. New enthusiasms enlighten everyday difficulties and make them bearable... Numerous new sites are a testimony to the on-going renewal.

Those who have integrated ICT into their classes, at the very heart of the process of learning mathematics, have discovered that their job was profoundly changed. From dispensers of knowledge, they have been changed into experienced companions who are aware of their students' efforts in the learning process. They are now able to use their students' discoveries and their unforeseen questions. They resist their ability to divert an item from its normal use. They refuse the non-critical “copy-stick”, which often stands instead of work. They are ever so pleased to see the students who were so far demotivated getting newly interested in what school can bring them. They measure the profound novelty of teaching in a multimedia environment. They claim for help and training from their supervising authorities, who often merely give orders and recommendations, without measuring what they demand from their agents. Those pioneers are probably inventing the school of to-morrow. They measure the cost but also the newness and the thrill. In the long run, ICT could bring some new magic to the teachers' job, and of course to the students'.

Key-words

Collaborative work, collective intelligence, ICT, continuing training, integration of ICT, “Mathenpoche”, “Wims”, “Publirem”, “La Main   la P te” (hands on), scientific debate, open problem, research narrative, digital working space, data processing, solving collaborative problems, profound changes to the teaching job, self-sufficiency, responsibility.

Within the last few years, numerous new online resources of mathematics have appeared. Their attentive study has lead many observers to the conviction that the renewal of the teaching of mathematics is being prepared and sketched out on the Internet. All the more so because the creativity of the authors comes with debates, exchanges of experiences, and joint remote work. Thousands of teachers are engaged in collaborative work on a scale, which was so far unknown. The repercussions on classes of that effervescence are still timid and even scarce if we consider the integration of ICT in the math learning process by students. (pupils)

The renewal of professional practices of teachers: from solitude to exchange and to collaborative work.

A French teacher is the only master in his class. That privilege, which he claims, leads to solitude. In these troubled times, the privilege often becomes a trap. The creation of sites and the remote exchange about emblematic software (dynamic geometry, computer algebra, spreadsheets etc.) help the teacher to come out from his loneliness and from the doubts linked to the difficulties of the teaching job. The information gets round, training is permanent, experiments, whether successful or disappointing are circulated. Everyone's ideas are made available for all. The individualistic teacher discovers...collective intelligence.

¹ A MS Word file of that paper (**with active links**) is available. Please mail to g.kuntz@libertysurf.fr

² **R**esearch **I**nstitutes for the **T**eaching of **M**athematics.

Four examples will illustrate the evolution of mentalities.

The collaborative work about **MATHENPOCHE**³ speaks for itself. A small number of teachers, forming an association, have given themselves the mission of creating online resources so as to cover all of the mathematics taught in “Collège” (11-16yo). Those resources are to be available to teachers costless⁴. They asked many colleagues, over various sites, to suggest ideas for exercises. Scripts were written and put online. They were tested in classes on a large scale, and improved according to the criticisms that were received. A vast exchange was installed between hundreds of teachers, leading to the evolution from basic Multiple Choice Items to more delicate and elaborate forms, down to open problems.

Groups were constituted in various IREMs so as to create resources, which integrate into their exercise basis all the ideas and experiences accumulated over thirty years of educational and didactic research. Supported by its numerous participants, the basis evolves, becomes richer and diversifies. No doubt, it is the one which is mostly used in French “Collèges” (secondary school, 11-16yo). An official experiment was lead by the Ministry of Education in the “département” of Seine et Marne⁵. Some 80% of the teachers in charge of “sixième” (11-12yo) participated on a volunteer basis. Teachers, pupils and parents unanimously approved the experience. It is to be carried further in 5ème (12-13yo) in the following year. Mathenpoche will probably expand itself considerably in the coming years and become a standard for the teaching of mathematics in France (extensions to primary schools and “lycée” (16-18yo) are being studied.).

WIMS⁶, a virtual tool for sharing, has many points in common with Mathenpoche. It was created by one single man, Gang Xiao⁷, and it is now used and fed by many colleagues of various departments. Among those are a majority of teachers of Mathematics belonging to Collège, Lycée or University. Though it was created in order to teach maths, there is no obstacle to its being used to teach chemistry, electricity, French, English etc. That interesting prospective also concerns Mathenpoche which could be diverted to fields other than mathematics. The powerful tools in the field of ICT have a universal call: even the Ministry of Education could find interesting means for economies of scale.

But unlike Mathenpoche, where those who conceive are also those who suggest contents, WIMS is a system in which you get out what you put in. Each user is invited to become a creator of exercises and activities in his turn. Being tested and improved by use, online resources little by little gain in importance. But the fields covered by that basis depend on the conceivers’ centers of interest. A data basis of exercises of mathematics is implemented⁸ and offered, free of charge, to everyone. That work is collective and non exhaustive.

Like Mathenpoche, WIMS allows the creation of *virtual classes*, which makes it a tool well adapted to the work of a teacher with his class or a group of students.

PUBLIREM⁹ is a search engine to the various online resources of the French IREM. Usually they were elaborated in various research groups over several years. They stage rich and complex situations. Those resources are not directly meant for the pupils (students). They require the mediation of a teacher who draws his inspiration there and adapts them to the needs of his class. Important ahead

³ <http://mathenpoche.sesamath.net/index.php> (see under the heading “Logiciel”)

⁴ Pupils and parents also have free access.

⁵ “Département” here means one of the 90 French administrative districts

⁶ Web Interactive Mathematics (now Multipurpose) Server. <http://wims.univ-mrs.fr/>. The English version is available at <http://wims.univ-mrs.fr/wims/wims.cgi?lang=en&+session=69F0DBD8AC.1&+module=home>

⁷ Professor at Nice University

⁸ Unlike Mathenpoche, which tries to cover all the fields included in the programs of secondary schools, WIMS only deals with the themes suggested by its creators.

⁹ <http://www.univ-irem.fr/index.php?module=Publirem&func=view>

work is indispensable¹⁰. Is that the reason why PUBLIREM is so scarcely used in comparison with Mathenpoche¹¹?

Many other sites could be mentioned, which are close in spirit to the three sites that were rapidly described. They are a collective work and they leave, in their trail, an important work of exchange, criticisms, and improvements. The teachers who take part benefit from the intense intellectual activity thus generated and in which they eventually take part. In France this is a considerable revolution.

It is with the site “**LA MAIN A LA PATE**¹²” (Lamap) that we can fully measure the power of transformation brought by a quality site in the conception of the teaching of mathematics and in the teachers’ practices. Like PUBLIREM and unlike MATHENPOCHE, LAMAP is not directly meant for student use. It offers resources in great quantity and quality to teachers of the elementary school (6-11) for them to adapt to the level of their class. They can complete their own scientific training, which may be insufficient. They can find scientists and educationalists with whom they can exchange virtually or by word of mouth.

Concurrently with the teaching of science, Lamap insists on the importance of language for the purposes of description, debate or explanation: each pupil expresses his scientific observations and conclusions *in his own words*, in his notebook of experiences.

An internal Lamap seminar took place in September 2005 to re-evaluate the place of mathematics on the site. Can they be included in the experimental process, which characterizes Lamap? No doubt they can, if, however, teams of teachers adapt to that spirit some situations that were created within the frame of mathematics¹³. That approach will have to be completed. Teams must be set to work in the Lamap spirit to imagine new situations using primary schools mathematics¹⁴.

The quality of the site and Georges Chapak’s support have lead the very careful French Ministry of Education to profoundly changing the programmes and methods of teaching science in elementary schools. The way of thinking and process of Lamap have been largely adopted. Within a few years, Lamap has been globalized. Seminars and sites multiply from Brazil to China, from Germany to Africa, thus contributing to improve the access to science to learners from an early age.

But the use of ICT in classes is still reluctant and occasional.

With the multiplication of wide-ranging sites, we might think that ICT forced their way into classrooms. Now their entry is still scarce and occasional. Less than 15% of the French math teachers have **integrated ICT into the work with their pupils**¹⁵ (students). Indeed, after the initial enthusiasm, many questions have arisen. Disillusions have caused doubt and backward movements. Teachers have become aware of the difficulties and of the important cost of a successful conversion to ICT.

¹⁰ It can be done collaboratively as well.

¹¹ At first sight students can be set down to work on Mathenpoche *without any special preparation*. But there is a risk of poor results and of loss of control of the class.

¹² “Hands on”, <http://www.lamap.fr/>?

¹³ There are many brochures meant for primary school teachers to help them to teach mathematics. Unfortunately they were conceived for one branch of teaching and given to teachers in charge of *all the branches of teaching*. Which explains why the creators of those resources are so reluctant when facing Lamaps’s widely interlinked approach.

¹⁴ A site of statistics is in preparation. It will cover a field going from elementary school to «lycée». (end of secondary education). The experimental aspect will be very important in it.

¹⁵ According to a study led by the Montpellier IREM.

The cost of a successful conversion to ICT.

The IREM of Montpellier's initiative, a research about mathematics teachers' remote training is, from the start, a several years' task, requiring partnerships with the University and with the Ministry of Education. The objectives must be clearly defined and valued, according to University criteria. If online resources are to be useful to others than their creators, they must be firmly structured and documented. It is then necessary to restrain oneself to a few themes and to work in depth. If trainees are to *really change their pedagogical practises*, they need to be immersed into collaborative network labour, with a strong personal implication. Numerous trainers are necessary (one for 6 or 7 trainees) and they should be experimented and very available (even though they are themselves on continuing training). Therefore we can say that the creation of resources that can be used on a large scale in classes, has an important cost: the Ministry of Education will find no easy way to save money on the budget of education (some superficial minds had toyed with the idea).

No doubt, this is the main reason why so many teachers resist entering that field unreluctantly, when some experiences have revealed its great complexity. *It is not without inconvenience that you graft some modern computer science on a traditional teaching*. Interconnected work deeply modifies the relations between teachers, students and knowledge. Many are thereby destabilized and they limit the intrusion of ICT into their classes to a minimum¹⁶. The school authorities (institution) are unaware of the profound changes they require when they demand from teachers who were trained in a very hierarchical system (University) that they should spontaneously convert to a network transmission of knowledge. The study carried on in Montpellier¹⁷ (and other IREMs) underlines the complexity: it allows to question the Academic Authorities about their will and their possibilities to mobilize important means for initial and continuing training, means without which no real integration of ICT into classes is conceivable on a large scale.

In a similar way the project initiated by Lamap meets a considerable obstacle: few primary school teachers have an initial scientific training that meets the challenge...Some 15 % entered the new programmes and the new methods easily, but a majority of them experience great difficulties in taking the turn. Here again we find the tormenting question asked by the Montpellier IREM to all those in charge of the educational system: are you ready to invest the considerable means which are necessary so as to train teachers who don't have the minimal scientific competence¹⁸ to the methods of the future? They should be able to evolve with all the riches of the site and not be left behind.

A class that works in a multimedia environment somewhat escapes from the teacher's strict control. The pupils readily engage into original thought processes, they stray from the suggested scheme, they divert the tool from its imposed destination, they ask unforeseen questions that the teacher does not master. It is a factor of enrichment within the sequence. But for many teachers, it is a difficult experience, all the more so because they do not master the techniques of computer-science¹⁹. Facing all that insecurity, many of them give up exposing themselves... The teacher's status evolves from the comforting position of "he who knows" to the uncomfortable position (his pupils'!) of "he who daily pioneers new fields."

In addition to the feeling of insecurity inherent to the use of ICT, there are doubts as to the efficiency of that form of work, even among those who are convinced of the interest of online mathematics. Some questions come again and again, insistently, and rightly so.

¹⁶ Although they largely use ICT to prepare their classes, their professional and personal communications.

¹⁷ The Montpellier IREM estimates that less than 15% of the mathematics teachers have actually integrated ICT into their classes.

¹⁸ Here arises the question of how to recruit and train Primary School teachers and that of the minimal scientific competence which is necessary to teach at that level

¹⁹ Many of them fear being ridiculous in front of the technical virtuosity of younger generations...

Questions about online resources aimed at pupils (or students).

-*How to enrich and open* the somewhat closed exercises that Mathenpoche or Wims offer to more and more students? How to introduce scientific debates, open problems or research narratives, for instance? How to free those treasures from their confidential aspect and spread them on the Internet for the thousands of colleagues who use those databases? Don't technical limitations restrain the mathematical and educational contents?

-*What are the complementary aspects* of work in a multimedia environment and of learning in a traditional environment? What profitable knowledge can a student gain thanks to ICT?

No one imagines a future where everything could be learned individually with a computer! The exchange in class is an essential moment for the pupils' socialisation. There is no scientific debate without coming up against others. But the application field of the new tools will broaden, when the numeric space of work becomes more consistent. Very soon, Mathenpoche will offer several items in one software package: a spreadsheet, a mathematical word processor, a curve tracer, and dynamic geometry software. Thus, the user will have considerable means to enable himself to learn, to experiment, to conjecture. Those means will be even more efficient if they do not claim to replace all the other ones and if their use is relevant.

-*How to evaluate what pupils learn of mathematics* if they work with online database exercises? A crucial question, for you cannot be content with their mere pleasure, nor with their obvious activity in that environment.

In the meantime, teachers define use procedures of the very rich matter that is at their disposal. Thereby, they want to get control of the software and check the misuses that pupils are so quick to operate. Instead, they will clarify the really pertinent uses.

It is still uncertain, however: the teachers who supervise that approach *are learners themselves*. Nothing indicates that a well-prepared sequence will reach its assigned goal. (no one has the necessary detachment yet). They will have to set at work on it, again and again: give more detailed instructions, help those who get stuck, *prepare individualized sequences* to meet the difficulties that were revealed. Those who expect Mathenpoche or any other learning tool to be a rest cure must change their tune: make it a learning tool for everyone requires *a lot of imagination and a great amount of work*.

Examples of enthusiasm and success.

In the last few years the Montpellier IREM has been testing the use of networks for the collaborative solution of open problems, by classes²⁰. For each problem, classes of different levels work, exchange ideas with other classes, test them, confirm or reject them, suggest solutions, which are assessed by other classes. Procedures are implemented: they help the pupils become aware of the force of collective intelligence. The intelligence of various work groups in a class, the intelligence of the whole class when the synthesis takes place, the intelligence of various classes that exchange, debate and get confronted, by the means of a virtual platform. The leaders of the experiment do not hesitate in putting to work together pupils of very different levels ("Collège", "Lycée" i.e. 11 to 18 yo) and they work on a same problem. Everyone furnishes their own contribution and gets out from it a great number of ideas, methods, approaches, and questions, which nurture everyday life in a class and justify its existence. A wonderful idea developed by IREMs, that of open problems, hereby finds a second life and new developments. No doubt the school of the future will be inspired by it.

One thing is sure: work in a multimedia environment changes the students' attitudes as to work in class.

A University Professor told a significant anecdote. At the beginning *she was sceptical about WIMS* and the like... Being asked to replace a colleague at a moment's notice, she found herself without being warned with a group of students working in... a WIMS environment. *She was extremely surprised*

²⁰ See "A Babylonian problem" in the appendix.

to see them concentrated on their subject and absorbed in solving the given problem together. Before, in a traditional environment, she had known students with a straying attention, and no personal investment. One day *the reasons* for that sort of renewed miracle *will have to be clarified...*

How can we explain many students' interest and attention when they use Mathenpoche, while the same people are demotivated in a traditional environment? Isn't amazing to see some of them pursue their work at home?

Undoubtedly, modernity does not explain everything (videogames are certainly more thrilling than Mathenpoche or Wims!) Don't those students simply make it clear to us that they find it satisfactory *to be stakeholders* of their training? That they find it satisfactory *to rule their own time again*, satisfactory to be shielded from the violence of imposed time, dictated by *those who know* and who have to cover broad fields in too little time? A ZEP²¹ teacher explains: "I prefer it if my pupils do *just a little* mathematics rather than *no mathematics at all*." That remark could be valid in other areas than the ones in which great difficulties are experienced.

Online mathematics have also played an important part in projects called TPE. That type of research work in groups has spread over several months and led to surprising achievements and a considerable commitment on the part of pupils who were otherwise little implied in their own training.

The enthusiasm can be explained by the discovery of new scientific and interdisciplinary fields, autonomy at work, the fact of working collectively. A better benefit should be taken from all that²². We must however admit that for some groups, an Internet research means practising "copy/stick" without any work about the meaning of the documents thus collected.

Conclusion

We have not spoken of many excellent sites where you can simply find quality mathematics²³. They put a question to school in general, regardless of matters taught: how can our pupils access those riches, otherwise than by the "copy/stick" method, which they use without really examining the contents²⁴? Because information that is displayed will only turn into knowledge by hard work, for which they interact with their peers and experienced adults²⁵. Learning from a site, be it excellent, is not granted. It requires the ability to discerningly read the documents which are displayed, to bring them together, to confront them and to make a synthesis. When exactly will the French School take in charge that difficult and essential training? Into which programme will it be integrated? Can we be satisfied with the well-intended official speeches if no new specific means or no new timetable is granted? Can all that learning process (without which ICT are closed to their users) be left to the good will of teachers, in addition to all their other duties?

We have not spoken either of all those sites, which are more or less questionable. Only culture and a critical turn of mind can sort things out. Where can those qualities be acquired without which the errors, which are displayed, *are held for truths*? Those questions are very serious in a school which, now that we have Internet, thinks it is its duty to dispense an exhaustive knowledge (the programmes in our "Lycées" are tremendous) when its essential goal should be to teach its students *to change information into knowledge, and then knowledge into learning*²⁶.

²¹ ZEP: Priority Education Area, with pupils who are in great difficulty (translator's note).

²² Right when everyone was becoming aware of the benefits that could be taken from TPE for the training of pupils (and their teachers' evolution,) the Ministry has suppressed them in "Terminale" (last year of secondary school, ages 17-18) for budgetary reasons.

²³ A remarkable example is the site of St Andrews about the history of mathematics (<http://www-groups.dcs.st-and.ac.uk/~history>)

²⁴ Pupils when using ICT currently do it. Many teachers try to resist the trend but, unfortunately, others are accomplices. Therefore many heavy TPE documents are just an impressive illusion.

²⁵ TN* German proverb : Was du ererbt von deinen Vätern hast, erwirb es um es zu besitzen : If you want to possess what you have inherited from your fathers, you must conquer it.

²⁶ The pupil who has those skills will be able to learn by himself. This includes some mathematics that he was never taught in class. He will find them in books and on the Internet. This is what he will often have to do in his professional life.

ICT offer something new to small and very determined groups (and who are ready to pay dearly for it): the means to be heard far beyond their pre-Internet sphere of influence. The creators of Mathenpoche were some ten people. Numerous sites, which are often visited, were due to one person only (WIMS, for instance). Lamap has weighed much upon the definition, the contents and the learning methods of science in the Primary School. Those very influential sites combine an irreproachable design with quality contents. This meets the expectancies of numerous potential users. It finally takes shape; the critical mass of users and interactions is reached. Without that it cannot survive, however interesting it may be.

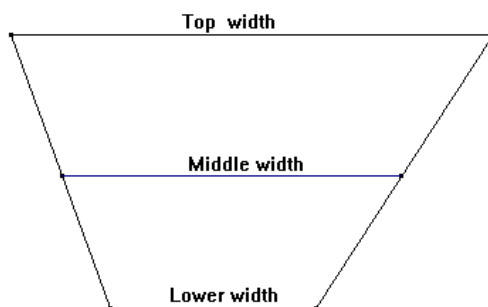
The renewal of the teaching of mathematics in France is undeniably linked to the development and *the use of such sites by teachers*. It also requires a research effort about the changes that such powerful tools bring into classes. It demands *a new definition of the teachers' task*: from dispensers of knowledge, they are now called to be companions for their pupils and students, in their effort to change an element of information (the offer is abundant on numerous sites) *into knowledge and learning*. Before, they were locomotives pulling inert carriages²⁷. Now, under the influence of ICT, they have been turned into mountain guides. That profound change is painful, the more so because the University training of those teachers is still done in the old way. However, a beautiful future lies ahead for those roped parties of mountaineers that are little by little constituted. They blend together youth's impetuosity and imagination, with the experience of those who have stridden the vast, disconcerting, difficult but exalting fields of knowledge. Are they heading for new summits?

Appendix

A Babylonian problem

In Mesopotamia fields are trapezium-shaped.

A land surveyor is to share a field equally between two brothers; the field is a trapezium. The two bases are 7 and 17; the shares are two trapeziums.



Babylonian vocabulary:

- 17 is the "top width"
- 7 is the "lower width"
- The fair share line, which is parallel to the two bases is called "Middle width"

Question: find out the middle width.

²⁷ This explains why so many teachers are tired and discouraged. But the traditional form of teaching necessarily leads to that situation.

References and links.

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3°) Concerning Mathenpoche, see
<http://publimath.irem.univ-mrs.fr/cgi-bin/publimath.pl?r=Mathenpoche&b=biblio>

4°) Concerning Wims, see <http://publimath.irem.univ-mrs.fr/cgi-bin/publimath.pl?r=wims&b=biblio>

5°) Concerning Publirem, see
<http://publimath.irem.univ-mrs.fr/cgi-bin/publimath.pl?r=publirem&b=biblio>

6°) Concerning « la Main à la Pâte » (“Hands on”), see
<http://publimath.irem.univ-mrs.fr/cgi-bin/publimath.pl?r=main+p%E2te&b=biblio>

7°) Concerning Collaborative solution of open problems by classes, see
<http://publimath.irem.univ-mrs.fr/cgi-bin/publimath.pl?r=r%E9solution+collaborative&b=biblio>

8°) Addresses of the main sites mentioned in that paper:
<http://mathenpoche.sesamath.net/index.php>
<http://wims.unice.fr/wims/>
<http://www.univ-irem.fr/index.php?module=Publirem&func=view>
<http://www.lamap.fr/>
<http://www.irem.univ-montp2.fr/>

9°) Different publications of the author of that paper:
<http://publimath.irem.univ-mrs.fr/cgi-bin/publimath.pl?r=kuntz&b=biblio>

A few questions for a debate.

How do ICT change the way of teaching mathematics ? And the way of learning mathematics?

Why are some teachers who master ICT so reluctant to *integrate* them into their actual teaching? (think of the intrusion of *the unforeseen* and of *the loss of control of class activity by the teacher*)

Does the notion of *collective intelligence* get a response in your professional practice?

In how far are work in a multimedia environment and learning in a traditional environment *complementary*? What kinds of learning can pupils increase thanks to ICT? Are there types of learning they cannot get through ICT?

Is it possible to evaluate what pupils really learn about mathematics if they work with an online exercise data basis?

How can we train pupils to take advantage of sites that offer quality mathematics? (please make the distinction between *easiness of access* and *the difficulty of appropriating* knowledge). How can we shape their minds to be critical so as to detect error or fraud?