IDENTIFICATION, MOTIVATION AND SUPPORT OF SCHOOL STUDENTS WITH HIGHER MATHEMATICAL ABILITIES (THE PROJECT "MATHEU")

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Abstract

In many countries the standard education in mathematics is mainly oriented to serve the average ability students. This is natural because such students form a majority in schools. Special care is usually assigned to lower ability students so that they could cover the educational standards. Less attention is paid however to students with higher than average abilities in mathematics. The standard curriculum and syllabus requirements are no challenge for these students. As a result, a lot of mathematical abilities (and even mathematical talents) remain undiscovered and undeveloped. On the other hand, the higher ability individuals provide a critical resource for the development of the society, if they are properly educated, motivated to work and supported. With this in mind a project named MATHEU was started aimed at developing tools for identification, motivation and support of students with higher ability (and perhaps talent) in mathematics. The project has been carried out with the support of the European Union within the framework of the Socrates Programme (Comenius 2.1 action). It intends to network Mathematical Societies, Universities and other relevant organizations in their efforts to improve the identification, motivation and support of mathematically able students and to enhance learning of mathematics in general by providing challenging educational materials to teachers and students. On behalf of the partners and participants in this project (see the bottom of the paper) we present here the philosophy, the aims, the difficulties and the results obtained so far in the realization of the Project MATHEU.

Introduction

The school educational system in every country is shaped under the influence of mutually contradictory factors. In order to be prosperous, the contemporary society needs highly educated professionals as well as good educational level for general public. On the other hand, the costs for education increase exponentially, if a higher level of education is aimed for the majority of the population. The trade-off usually results in a system designed to fit well the average ability students, which constitute a majority in school. The educational requirements and standards (especially in mathematics) are set up in such a way that even students with lower abilities could cover them, albeit with significant efforts. As a consequence technical and boring aspects of classical mathematics dominate in school which leaves wrong impressions about the nature of contemporary mathematics. As a rule, the higher ability students are not given enough attention in the general educational systems. The standard curriculum and syllabus requirements do not represent a real obstacle or challenge for such students. They are not

prompted to apply more efforts during the educational process and, as a result, their abilities (and even talents) often remain undiscovered and undeveloped. On the other hand, people of higher ability and talented people constitute a critical resource for the development of the society. Unlike other resources, like mineral deposits, which disappear once discovered and used, the abilities and the talent disappear forever, if not identified and not used.

This problem is well recognized today and in many countries one can find a diversity of measures and activities designed to serve as a remedy for the situation. Most popular and with confirmed positive effect on the learning of mathematics are the different competition-like activities. They allow the students to exhibit their abilities and talents and prompt them to work hard while preparing for the events. There is a kind of misbalance here however. What usually attract the public attention in this area are the golden medals and the medalists from the International Mathematical Olympiad as well as from other international competitions. Behind this "emanation" of few winners however are hidden the efforts of hundreds of thousands (even millions!) of students preparing for earlier stages of the corresponding competitions. It is the preparation for competitions where the main positive effect on the learning of mathematics is coming from. This is why the process of preparation for competitions deserves special attention and care. It has to be studied, supported and improved. Generally speaking, it is a process of mathematical enrichment for the involved students but also a time of efforts and hard work for them. To motivate the students for additional efforts is not an easy task. Even if the student wants to learn more and to participate in competitions, it might still not be the case that there is a proper source of information around to learn from. Hence, attractive and challenging educational materials are needed that would allow and motivate the students to undertake the venture of mathematical enrichment.
The role of the teacher in this process is extremely important too. In every country there is a small number of very special teachers that are much more successful than the majority of other colleagues in identifying and motivating the students with higher abilities for hard work. These are teachers that encourage their students, follow their development and finally bring them to highly successful participation in competition-like activities. What characterizes these teachers? Is it the bigger volume of mathematical knowledge in their heads that matters or some other skills and special abilities are more important? Is it possible to increase the number of such teachers and how to do so? The questions are more than the answers, as usual, however one thing is clear. Teachers also need preparation, training and support in order to increase their skills in identifying and motivating higher ability students. Hence, once again, attractive educational materials are needed in order to teach the teachers how to identify and motivate higher ability students.

With the above ideas in mind, a group of participants in the International Conference "Give Talent a Chance", held in Sofia in connection with the 20th anniversary of the International Foundation "St. Cyril and St. Methodius" (2002), decided to launch a Project that would network the efforts and the experience of the different countries in the work with higher ability students in mathematics and their teachers. The project was named "MATHEU" and got support from the European Union within the framework of the Socrates Programme (Comenius 2.1 action). The latter seems to be a natural corollary of the policies adopted by the European Union in the last years. According to these policies, mathematics belongs to the so called "Basic Skills" and has to play an essential role in the creation of the European Knowledge-Based Society. It is also identified as one of the major priority subjects in the educational policies of the European Union.

Partner Institutions and Organizations

There are nine partner institutions. Namely Intercollege (Coordinating institution) and University of Cyprus from Cyprus, Academy of Sciences – Institute of Mathematics from Bulgaria, North University in Baia Mare from Romania, Charles University from Czech Republic, University of Palermo from Italy, University of Crete from Greece, University of Duisburg-Essen from Germany and University of Miskolc from Hungary.

The project envisages the participation of mathematical societies in the respective countries, both in the process of developing the project and in the dissemination of the results. So far Mathematical Societies that were represented and contributed to the project were the Cyprus Mathematical Society, Cyprus Union of Mathematicians, the Union of Mathematicians in Bulgaria, MASSEE (MAthematical Society of South Eastern Europe), European Mathematical Society, Romanian Mathematical Society, Hungarian Mathematical Society, Czech Mathematical Society.

Objectives and Tools of Project MATHEU

The objectives of Project MATHEU are stipulated in more detail in its web-site (www.matheu.org). The main aim is the development of methods and supporting materials for the identification, motivation and development of students with higher abilities in mathematics.

In the kernel of these supporting materials is a set of "ladders" that had to be created during the working period. Each "ladder" is a self-contained mathematical text, focused on a specific mathematical topic, which could be used by teachers or by students in their work in and beyond the classroom. In essence the ladder is a sequence of mathematical problems, explanations and questions for self-testing, ordered in such a way that the degree of difficulty increases slowly. By working on the ladder-text, the student (and also the teacher) could elevate his/her mathematical knowledge to essentially higher levels. This is where the name "ladder" is derived from: a device for climbing to a higher level, an instrument facilitating the process of overcoming different difficulties. Using the ladder, the students (but also their teachers) could enrich, deepen and test their knowledge on a specific mathematical topic. The lower part of the ladder is rooted in the normal curriculum material studied in class. As "ladder steps", one has the mathematical problems, definitions and explanations, pieces of information and other challenges that the learner has to master in order to acquire the higher level of understanding the material. Depending on their individual abilities the students will advance i.e. "climb" to different heights on the ladder. The degree of advancement will single out higher ability students. Therefore the ladders will help identify talented students too.

If the ladder is well designed and consists of interesting and challenging mathematics, it will attract and motivate the students to apply more time and energy in studying mathematics.

It is required to design the ladders in such a way that the level of difficulty increases slowly (a small distance between two consecutive steps) so that the students are capable of climbing the steps even without the help of the teacher. The ladder however should present challenges as well. For example, among the ladders already crated there is a ladder named "Invariants". It introduces the use of invariants for solving different problems. The ladder contains five sections (A. Invariants, B. Games, C. Semi-invariants, D. Problems for individual work and Solutions to the problems from Section D.). Section A. starts with the following simple problem

Problem A1.
The Brave Knight met the Three-Headed Dragon and decided to show how brave he is by cutting all its heads. In fact he proved his incompetence: it turned out that after cutting one head, three new heads appeared on the place of the cut one. However the Knight insisted on cutting heads. At the end he decided to count the heads of the upgraded Dragon. He counted 2006. Did he count correctly?

The solution is based on the simple observation that after each head cutting two more hads appear. Thus the parity of the number of heads remains invariant. Therefore, at the end, the number of heads should be odd, as it was in the beginning. It cannot be 2006.

Later the degree of difficulty increases. Some problems are given where the invariant is a set, not a characteristic of a number. For instance, Problem A8 (taken from the Tournament of Towns) reads as follows

Problem A8.
On an island there are 13 white, 15 green and 17 red chameleons. When two differently colored chameleons touch, they turn to the third color. Can all chameleons turn white?

One possible way to solve this problem is to realize that the set S of residues modulo 3 of the quantities of white, green and red chameleons does not change after any touch. At the start S={1;0;2} and, regardless of what kind of chameleon touches occur, this set remains unchanged. If all chameleons turn white, the set of residues modulo 3 would be S={0;0;0}. Therefore the answer to the problem is "No"..

Toward the end of the ladder the problems become more and more difficult and challenging. The presence of problems, the solution of which is commented later, allows the student to check whether or not he/she understands what is going on.

The Manual that the Project MATHEU has to produce and present to EU at its final stage will be a set of such "ladders". Offered to students in and beyond the classroom the ladders in the Manual will serve all the purposes of the Project: Identification, Motivation and Support of mathematical abilities and talent.

It was agreed to create ladders for two age groups of students, first group (Level 1) for the ages of 9-14 and the second group (Level 2) for ages 15-18.

A list of ladder-topics was determined and every participant in the project was given the task to write one or more ladders. Ladders for each level are prepared in the following topics.
LEVEL 1: Dirichlet Principle, Discrete Mathematics, Geometry, Graph Theory, Inequalities, Invariants, Games, Number Theory, Polynomials, Sequences
LEVEL 2: Complex numbers and Geometry, Dirichlet Principle, Discrete Mathematics, Geometry, Induction, Invariants, Polynomials, Statistics, Transformation Methods, Linear Algebra, Inequalities, Games, Sequences, Number Theory, Functional Equations Analytical Geometry

All the ladders are produced originally in English. Then they will be made available in the languages of the partner countries, i.e. Greek, German, Bulgarian, Romanian, Italian, Hungarian and Czech.

In order to test some of the ladders, before finalized, the project envisaged an experiment with selected students from the partner countries. The selected students were gathered in Cyprus for a working week and were given the task to work on the ladders. Their reactions, advancements and difficulties were recorded and analysed. The mere selection

process of the students was based on an Identification Tool Box that was created in advance through the project and each partner institution has applied this toolbox (or a slight modification of it) in the process of student selection for the experiment. Basically, the Identification Tool Box is a set of mathematical problems divided into the two age levels and covering different fields of mathematics. The first aim of the experiment was to examine whether the Identification Tool Box was successful in identifying higher ability and talented students in mathematics. The second aim was to examine some of the ladders and to see how the students climb them, what the difficulties are, what is their motivational power etc. To test whether the selected students from partner countries were of high level, the RAVEN test was implemented to this group before the experiment began. As is well-known (see Raven, J.C., Manual for Advance Progressive Matrices: Sets I and II, Section S4. (1983). London: H.K.Lewis and Co,Ltd.), the Raven test tests (in a non-verbal way) the intellectual abilities. It recognizes whether a person is able to form comparisons between figures and develop a logical method of reasoning. It allows children to show pattern-recognition, attention to details, memory, and spatial reasoning. The maximal score possible is 36 (as is the number of problems in the test). The students covering 95% (or more) of the problems are considered "brightest". Altogether 16 students were subjected to the test. With one exception all students ranked in the top 1-3 percent performance. In our opinion, this shows that the Identification Tool Box has served the goals it was designed for.
The experiment has helped the partnership and some of the authors of the ladders, to verify whether the "ladder steps" needed adjustments and improvements. Moreover, it was helpful to see and evaluate the whole process of working with higher ability students: identification, motivation and support. This is important also for the final design of the in-service training course for teachers that is planned for May 2006. The latter course is among the major outputs of the MATHEU project. It is designed as an in-service training course for teachers who teach students in the ages 9-18. It is important that teachers in Europe are trained to identify, motivate and support mathematically able students at different ages. The comparative study conducted in the earlier stages of the Project has shown that most of educational systems in Europe and the partner countries do not have systems for teacher training in this direction. We are expecting that the training course developed through the MATHEU project will be the beginning of a system that will grow, improve and enrich through time.

Each student participating in the experiment was asked to develop the so called "personal portfolio" compiling personal reflections about the work with the ladders. This portfolio turned out useful in several directions. It helped students become more organized in the process of working with ladders. The Project partners were able to assess the progress of the student work. It helped also the final Stage 4 of the Experiment when the students had to present their own reactions to what they have done during the experiment and to what could be improved. The portfolio structure included the following elements:

1. General expressions (personal information, qualifications and activities)
2. Competences
3. Reflections on the three work-days
    3.1. Stage 1 – work without help
    3.2. Stage 2 - work with support throughout full Ladders
    3.3. Stage 3 – work with support by professors (creators of ladders)
4. The Presentation prepared for the Stage 4
5. All the best solutions or examples of work they like to select and to add to their portfolio
6. Last day experiment Evaluation sheet

In Stage 4 the students represented interesting generalizations and/or enlargements of the material they have seen in the ladders. We witnessed and registered very high creativity

on the side of the students during these presentations. The authors of ladders received important feedback.

The external evaluation conducted by Helmut Loidl from Austria has shown that, in general, the quality of ladders was quite high and corresponding to the purpose they were designed for. It was established that, for some ladders, improvements were needed. For example, some easier problems needed to be added to some ladders in order to make them digestible for ordinary students of higher abilities in mathematics. One of the ladders was found inappropriate and had to be replaced or modified. Some ladders were found to be not challenging enough and needing additional more difficult problems. It was recognized also that the structure of ladders should be unified. These improvements have to be made during the summer of 2005 in order to arrive to a final improved English version of the ladders by the end of September 2005. Then the process of ladder translation into the languages of partner countries began.

If the creation of the ladders and the other tools is the first main goal of the MATHEU Project, the second major aim is to establish a network of universities, mathematical societies, foundations and individuals from the partner countries and beyond that would provide sustainable support for the activities related to the identification, motivation and development of higher ability students.

Outputs of MATHEU

- A Tool that identifies higher ability students in mathematics at two different age levels Methods/Activities for motivating potentially talented/competent students in mathematics

- An European Manual and CD-Rom, which will contain the tool above together with the set of ladders needed to support the development of higher abilities. The Manual will be initially published in eight languages, English, German, Greek, Italian, Bulgarian, Czech Hungarian and Romanian.

- A Course Designed in English for teacher trainees and teacher trainers for primary and secondary levels for the target age levels using the tool, methods and the manual mentioned above

- An Information and Dissemination Symposium for administrators/government decision makers, for university enrolment managers/deans/representatives, for presidents/representatives of Foundations and Societies

- The MATHEU Web-site, designed to provide sustainable communication between the partners and to provide support to talented students in mathematics as well as to mathematics educators of different levels. The site will initially support the languages of the partner countries.

Project MATHEU partners and Math Societies participants:

Cyprus
Intercollege (Gr. Makrides, E. Solomou, A. Savva, M. Zembylas, E. Michael)
UNIVERSITY OF CYPRUS (A. Gagatsis, C. Christou)

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Romania
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MATHEU website: http://www.matheu.org