FROM THE IDEA OF MATHEMATICS LABORATORY TO A CHALLENGING PROPOSAL FOR THIS ICMI STUDY:

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INTRODUCTION

I shall start from an Italian story. In recent year, the Italian Ministry of Education, has issued the new curricula for all pre-University grades. The curricula have been prepared by commission appointed by the Italian Mathematical Union – Italian Committee for Mathematical Instruction (UMI – CIIM) and are available – in Italian – in the website http://umi.dm.unibo.it with the titles *Matematica 2001, 2003, 2004.* An English abridged version has been published (http://umi.dm.unibo.it/italiano/Didattica/ICME10.pdf) for the ICME 10 Conference in Copenhagen-Lyngby. The new curricula are the results of a longstanding research effort in the field of mathematics education. In this short contribution I wish to elaborate one of the most innovative idea – in my opinion – that draws, on the one hand, on the epistemological discussion about the genesis of mathematical ideas carried out by mathematicians in the last two centuries and, on the other hand on the longstanding tradition of active methods in education. The challenging issues, in this elaboration, consist mainly:

- in the contrast against the traditional image of mathematics lessons as ‘a professor facing a blackboard full of formulas’;
- in the (sometimes ignored) potential of the laboratory activities with tools for the construction of mathematical meanings;
- in the strong involvement of students – even low attainers – in the solution of challenging problems rooted in the laboratory activities.

The above issues are related to three interconnected aspects addressed by this study:

- public awareness of mathematics;
- teaching,
- learning.

(further elaboration on these ideas may be found in Arzarello, 2004; Cannizzaro, Pesci & Robutti, 2004; Maschietto, 2006

THE MATHS LABORATORY IN ITALIAN CURRICULA

A wide excerpt from the text of the Italian curricula helps to set the stage for further discussion (http://umi.dm.unibo.it/italiano/Matematica2003/prima/premessa2.pdf, p. 26-8):

> A mathematics laboratory is not intended as opposed to a classroom, but rather a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematical laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments).
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> We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing.
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> In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students).
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> It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning can not be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities.
>
> **The tools of a Mathematics laboratory**
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> The tools can be traditional, or advanced technological: here we will describe a few of these.
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> *Poor materials […]*
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> *The Mathematical machines […]*
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> *The DGE: Dynamic Geometry Software […]*
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> *The CAS: Computer Algebra Systems […]*
>
> *The spreadsheets […]*
>
> *The symbolic- graphic calculators […]*
>
> The construction of meaning with a methodology based on the Mathematics laboratory is strictly connected with the social interaction of the students, during an activity, carried on in small groups work. During the group activity, the students can share the process of conceptualisation, through a collaborative or cooperative interaction. After the group activity, it is hopeful that a class discussion, led by the teacher, permits the students to share the results of the groups. A mathematical discussion consists in a social interaction aimed at the construction of a common knowledge in the classroom, shared by all the students.

THE MATHEMATICAL ROOTS

In the above quote, tools from ICT are mentioned. The plentiful supply of low cost computers and softwares has shifted the attention of mathematicians and of mathematics educators to activities with them. In most cases what is meant as a Maths Laboratory (or mathlab in short, try to surf the web!) is a room with PCs. The roots of the idea of a mathematical laboratory, however, go back a long time ago when computers did not exist yet. Let us see, for instance, what Emile Borel (1871-1956) said in 1904, in a lecture given at the Musée Pédagogique (http://www2b.ac-lille.fr/apmep/labo.htm):

> Mais pour amener, non seulement les élèves, mais aussi les professeurs, mais surtout l'esprit public à une notion plus exacte de ce que sont les Mathématiques et du rôle qu'elles jouent réellement dans la vie moderne, il sera nécessaire de faire plus et de créer de vrais *laboratoires de Mathématiques*. […]
>
> On a déjà deviné quel pourrait être, à mon sens, l'idéal du laboratoire de Mathématiques : ce serait, par exemple, un atelier de menuiserie ; le *préparateur* serait un ouvrier menuisier qui, dans les petits établissements, viendrait seulement quelques heures par semaine, tandis que, dans les grands lycées, il serait présent presque constamment. Sous la haute direction du professeur de Mathématiques, et suivant ses instructions, les élèves, aidés et conseillés par l'ouvrier préparateur, travailleraient par petits groupes à la confection de modèles et d'appareils simples. Si l'on possédait un tour, ils pourraient construire des surfaces de révolution ; avec des poulies et des ficelles, ils feraient les expériences de Mécanique que nous décrivait M. Henri Poincaré, vérifieraient d'une manière concrète le parallélogramme des forces, etc. Il y aurait dans un coin une balance d'épicier ; de l'eau et quelques récipients permettraient, par exemple, de faire faire aux élèves, sur des données concrètes, les problèmes classiques sur les bassins que l'on remplit à l'aide d'un robinet et que l'on vide en même temps à l'aide d'un autre robinet, etc.

Surely this quotation is challenging (at least it challenges the general idea of mathematics): at the time when ‘virtual’ objects were not yet available, a carpenter workshop was an example of mathematical laboratory. The need of seeing, handling and constructing was commonly shared at that time. Twenty years before, an Italian mathematician (Giuseppe Veronese (1854-1917), the founder of projective geometry of hyperspaces, who was advisor of the Ministry of Education), had written (my translation):

> Insight in consists in representing in our mind space figures, in order that our thinking may enter into them, by linking each other and separating each other and discovering their intimate relationships. This very space insight must be developed in the young mind, from the very early age: hence it is useful to add to every geometrical proof

> drawings and models, by means of which the student may understand and see the geometrical properties of bodies without mind efforts (Veronese, 1883).

About twenty years later, Guido Castelnuovo (1865-1952), one of the founders of the Italian school of algebraic geometry, said in the lecture he gave at the International Congress of Mathematicians in Rome (1928):

> We have built, abstractly I mean, a big amount of models of surfaces of either our or higher spaces; we had divided them, so to speak, into two windows. The former contained the regular surfaces, for which everything worked in the best way; analogy allowed to transport to them the salient properties of plane curves. Yet when we tried to verify the above properties on the surfaces of the other window, the irregular ones, we get into trouble and found all sorts of exceptions. At last the sedulous study of our models, had lead us to guess some properties that could be valid, with suitable modifications, for the models of both windows; we tested these properties with the building of new models. If they standed the test of this verification, we, the vary last phase, looked for logical justification (in Campedelli, 1965, p. 168).

Campedelli sharply commented that “Castelnuovo mentions ‘models’ and ‘windows’, and almost only accidentally precises that they were not material objects, but only mental constructions, yet living in front of eyes, as if they had a physical existence”.

This emphasis on the object manipulation (either real or mental) was common until the mid of XX century in the community of mathematicians, both when they spoke about their process of research and when they addressed, from the mathematician perspective, educational issues. There is no room here to go into details, but it is enough to quote the methodological books on problem solving written (1945, 1954) by George Polya (1887-1985) and the treasure house of instructions for building material models and instruments written by Cundy & Rollet (1961).

THE EDUCATIONAL ROOTS AND SOME SCHOOL APPLICATIONS

Comenius, Pestalozzi, Montessori, Dewey, Decroly, Bruner are just a few names of educators from different ages and different parts of the world who emphasized the need of active methods in teaching. The importance of children action put by some scholars of developmental psychology in the process of teaching and learning produced longstanding research in mathematics classrooms. Just to quote one, in the sixties, the Nuffield curricular reform was launched with the famous slogan: *I hear and I forget; I see and I remember; I do and I understand*. A kind of misunderstanding was, however, widespread among mathematics teachers: practical and manipulative activity was considered to be important especially for young learners and less meaningful for secondary school students.

Some exceptions occurred: in Italy the so called Roma group (lead by Lucio Lombardo Radice) supported the production of teaching experiments and textbooks that covered the secondary school: for grades 6-8, the experiments of Emma Castelnuovo were well known all over the world. They were coordinated with the

Realistic Mathematics Education project initiated by Hans Freudenthal (1905-1990) and continued by the Istitute of Utrecht (http://www.fi.uu.nl/en/projects/realme.html), that gets its name from him. Also my research group has studied extensively this issue (Bartolini, 2000; Bartolini & Maschietto, 2006; Maschietto, 2006).

The idea of coordinating 'real and physical' tools with virtual ones, that is put forward in the Italian curriculum is rooted in these experiments where the emphasis on social interaction is quite strong and in the development of studies about cognition: recent findings in the field of mathematics education, that take into account also the results of cognitive sciences and neurosciences (see Arzarello, 2004), suggest that the body involvement is really essential also for the construction of very abstract meanings of mathematical objects.

Hence, the 'definition' of Mathematics laboratory given in the Italian curricula draws on many different ideas:

- the historic and epistemologic studies on the features of mathematical thinking;
- the cognitive studies on the individual and social processes in experiencing mathematical activities
- the scientific studies and the descriptions of good practices in the mathematics classrooms.

Yet, a further dimension might be interesting for the purposes of this ICMI Study.

RAISING PUBLIC AWARENESS OF MATHEMATICS AND BEYOND

Both virtual and real-and-physical objects allow an approach that is more friendly than the symbolic approach to mathematics, typical of most educational institutions. They contrast the legend, reported also by Polya (1945) that "a mathematics teacher prefers to face a blackboard and to turn his back on the class". Hence, it is not surprising that the 'same' artefacts and the 'same' hints for exploration appear also in public exhibitions and museum collections, with the aim of catching the attention of visitors and creating a not-so-bad attitude towards mathematics. We have also examples of 'virtual' museums of mathematics (e. g. http://mathmuse.sci.ibaraki.ac.jp/indexE.html). What is to be questioned, in this case, is whether the by-product is not only affective but also cognitive!

In spite of this similarity, activities in different settings are very different, even if the 'same' artefacts are involved. An early elaboration of this point is in the paper by Maschietto (this conference). Although this difference might seem obvious to the researchers in mathematics education, there are some widespread misunderstanding. Sometime popularizers write books or materials for teachers, suggesting interesting and challenging problems, that are, however, not considered with respect to their links (if any) with school curricula. The risk, in this case, is that school is not affected at all by these marvellous ideas (and the challenge remains in authors' good intentions). Sometimes makeshift communicators organize exhibitions without any experience in general audience expectations. Sometimes museum activities are mistaken with classroom didactical activities (without considering that the contract between the museum operator and visitors is very different from the contract between te teacher and students). Last but not least, I do not know any important study of the relationships between the Olympic activity (that is more and more popular all over the world) and school mathematics.

The ‘definition’ of mathematical laboratory given in the Italian curricula, on the one hand, tries to introduce into the standards good practices observed in both classrooms and extra-school settings and (sometimes) studied by mathematics educators. Yet, on the other hand, it opens a list of good questions that are, in my opinion, to the core of this ICMI Study:

- Does it exist a general framework where to study, compare and contrast good practices of challenging experiences in mathematics (in the classroom and beyond)?
- If not, are we able to sketch, if not a theoretical framework, at least a tentative frame where to study, compare and contrast good practices of challenging experiences in mathematics?
- Are we able to collect a data-base of as many as possible relevant experiences from all countries, organized on the base of issues suggested by the framework?

The possibility of trying to answer these questions concerns, in my opinion, the auspices of giving, in this study, a state-of-art image of the field, that does not contain only information but has also the expansive potential to generate new related studies.

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On May 12th 2006 I organized a whole day seminar on *Ricerche in didattica della Matematica: il Laboratorio* (Research in Mathematics Education: the laboratory), at the Domus Galilaeana, Pisa, Italy. I had been appointed by the President of the Domus to produce this meeting on educational issues: it was a novelty with respect to the traditional meetings on the history and epistemology of science, that are held in that place. Galileo’s emphasis on *sensate esperienze* (meaningful experiments) and *certe dimostrazioni* (rigorous demonstrations) created an exceptional environment where to discuss the relationships between practice and theory, between personal manipulation of real and virtual objects and socially shared meanings and processes, between challenging problems and systematic organization. Although, the responsibility of this elaboration is mine, the ideas I have expressed in this paper have been deeply influenced by the contributions to this seminar. I wish to thank the invited speakers Mario Barra, Maria Reggiani, Maria Alessandra Mariotti, Michela Maschietto, Franca Ferri, Michele Cerulli, Domingo Paola and all the participants, for the fruitful discussion.

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