The Role of Challenge in Engaging Lower Secondary Students in Investigating Real World Tasks

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Data from a class of Year 9 students solving two extended investigation tasks set in real-world contexts are analysed to gauge what they perceived as challenges during the tasks. Some students took up expected challenges but for others these did not eventuate as the significance of task requirements were missed, or the mathematical implications of results produced during the task which should have generated challenge were not realised. At other times unforeseen challenges arose for students who discovered different complexities in their unanticipated interpretation of the tasks.

1. Background

A challenge for lower secondary mathematics teachers is to design learning experiences to manage the level of cognitive demand of tasks to ensure students are challenged, willing to engage with these tasks, and learn mathematically from the experience. Recent curriculum documents (e.g., VCAA, 2005) advocate students at this level of schooling being given opportunities to "engage in investigative tasks and problems set in a wide range of practical, theoretical and historical contexts" (p. 36). Student use of electronic technologies is seen as an essential learning at this level "to support analysis in mathematical inquiry" (p. 40). As part of an Australian research project[1] how project teachers engineer learning environments in their classrooms to manage increased cognitive demand of lessons where task contexts involve real-world applications and how students negotiate this challenge are being investigated. The project involves design-based research (Collins, Joseph, & Bielaczyc, 2004) where iterative cycles of design, implementation, evaluation, and refinement are used to improve educational practice. Researchers and teachers work collaboratively to test theories in everyday classroom settings. Both theory and practice inform the design phases and are informed by what transpires during each teaching experiment. Some results from the first two years of the project are reported here.

2. Managing Cognitive Demand of Extended Investigative Tasks

For practitioners to value participation in a design-based research project, "practitioners' issues" should be the starting point (Dede, 2004, p. 113). The design and sequencing of extended investigative tasks so the cognitive demand matches students' needs at a particular stage in the development of their mathematical, technological, and investigative procedure knowledge are issues of interest to teachers in the project. At the beginning of the project it was hypothesised that management of cognitive demand of teaching tasks in technology-rich teaching and learning environments is mediated through careful tuning by the teacher of the interplay between (a) task scaffolding, (b) task complexity, and (c) complexity of technology use (Stillman, Edwards, & Brown, 2004).

Task scaffolding is the degree of cognitive processing support provided by the task setter enabling task solvers to solve complex tasks beyond their capabilities if they depended on their cognitive resources alone. Task structure (e.g., carefully sequenced steps or a bald task statement), type of technology chosen (e.g., a real world interface tool such as a data logger or a mathematical analysis tool such as a calculator), and whether technological assistance rather than by-hand calculation is privileged, all contribute to task scaffolding. Whose choice it is to decide all of these also contributes to the level of task scaffolding. The *complexity* of a real world task can be characterised by identifying and assessing the level of those attributes of the task that contribute to its overall complexity. These are potentially numerous contributing via the mathematical, linguistic, intellectual, representational, conceptual, or contextual complexities of the task (Stillman & Galbraith, 2003). For example, one property of conceptual complexity is pedagogical development where required concepts can be anywhere along a continuum from early to complete development. Overall task complexity also varies along a continuum from simple to complex with the latter presenting a challenge for many students. For a particular task, students focus on only a subset of

[1] RITEMATHS is a collaborative research project, funded by the Australian Research Council Linkage Scheme, involving the Universities of Melbourne and Ballarat, six schools and Texas Instruments as industry partners.

attributes when assessing overall task complexity (Stillman & Galbraith, 2003) but these indicative cues contribute to their sense of challenge with the task.

Use of electronic technologies such as calculators and image digitisers can reduce the cognitive demand of tasks through "supplementation" and/or "reorganisation" of human thought (Borba & Villarreal, 2005) by carrying out routine arithmetic calculations, algebraic manipulations, or graph sketching; acting as an external store of interim results; or overlaying visual images within an interactive coordinate system to facilitate analysis. However, these technologies also have potential to influence the complexity of what students do as they transform classroom activity and allow new forms of activity to occur. Regulation of this complexity is a further opportunity for teachers to mediate cognitive demand, and therefore the challenge, of tasks through careful crafting of tasks and management during implementation. In particular, use of multiple representations, easily accessible with graphing calculators and tasks amenable to electronic technology use, harness opportunities for students to use technology to stimulate higher order thinking in investigating real-world situations. Within tasks diagrammatic, numerical, symbolic, graphical, and algebraic representations can be intentionally employed to support bridge making from one representation to another and to provide opportunities for interpretation across representations as well as from each representation back to the situation being investigated.

As Dede (2004) points out, several projects implementing well-formulated technology-based designs such as the SimCalc project (Roschelle, Kaput, & Stroup, 2000) have demonstrated that "typical middle years students [are capable of] mastering science and mathematics previously thought appropriate to teach only" to students at higher schooling levels (p. 111). However, two challenges middle years students face when engaging in extended investigations for the first time (Loh et al., 2001), are inability to recognise when to keep records and failure to plan and monitor progress effectively. It is thus prudent for teachers designing extended tasks for the lower secondary years, initially at least, to provide timely instructions throughout task statements supporting recording of key information, a planned solution, checking and verification of results. As student task expertise and familiarity with technology grow, some "fading" of this scaffolding (Guzdial, 1994) should occur, particularly that related to task structuring and technological tool selection and instructions. This is not to say mathematical analysis tools need be withdrawn. On the contrary, "learning to 'work smart'" in a technology-rich learning environment may involve "learning to establish one's own scaffolds for performance, and fading these may be beside the point" (Pea, 2004, p. 443).

3. Context for the Study

One project school is developing a lower secondary mathematics curriculum (Years 8–10) providing opportunities for engagement in extended investigation and problem solving tasks set in real-world contexts considered meaningful for students by the teachers. A major focus has been in Year 9 (14-15 year olds), the first time students at the school are required to have laptop computers and graphing calculators. Both are used frequently in mathematics lessons, always being available. During the Year 9 program, in keeping with local curriculum requirements (VCAA, 2005, p. 36), students are introduced to a mathematical model being used to describe the relationship between variables in a real situation and then being used to predict an outcome in terms of a response variable when a control variable is altered. A series of extended real-world tasks designed by one teacher, Peter (a pseudonym), and the implementation and refinement of these tasks are being studied in depth.

One problem of design-based research is lack of attention to "scalability and sustainability" (Dede, 2004, p. 113). Adoption by other classroom teachers with different motivations for the use of real-world tasks and/or electronic technologies in the lower secondary years is not guaranteed even if the design can be shown to be "generalisable and transferable". Some of the tasks from this first school have been modified by members of the research team and teachers at another project school where they have been implemented to fit the different conditions existing at that school.

4. The Teaching Experiments

Teaching experiments related to task implementation occurred in one of Peter's Year 9 classes in two successive years of the project. This paper will deal with two tasks, *Cunning Running* and *Shot on Goal*, and their implementation in the second year of the project in Peter's class of 28 Year 9 students (11 male, 17 female). Data collected during these teaching experiments include task sheets, audio-taped teacher interviews and reflections, field notes of lesson observations for task implementations, videotapes of two focus groups for both tasks, audio-taped student interviews (8 and 4, respectively) and written reports from students.

Research questions currently subject to on-going investigation include:

1. What do students perceive as challenges during the solution of extended investigation tasks set in real-world contexts?

2. How can tasks be altered for implementation in different contexts (e.g., shorter time frame and teachers and students with less technological expertise) but the level of challenge and engagement retained?

This paper focuses on the first research question. Analysis of data from the implementation of a shortened version of *Shot on Goal* at the second school collected to address the second research question appears in Galbraith and Stillman (2006).

4.1 Task design considerations

These tasks follow the investigative cycle proposed by Kader and Perry (1994)—pose a problem, collect data, analyse the data, interpret and communicate results. Explicit promotion of multiple representations as advocated by Friedlander and Tabach (2001) was to be facilitated by use of electronic technologies in teacher demonstrations supporting development of student understanding of a real-world situation and in the requirement to produce and interpret multiple representations of the task situation and data produced during the investigation both with and without technology. The real-world situations chosen had to be able to be mathematised using mathematical techniques studied at Year 9 level and amenable to a level of mathematical analysis commensurate with the curriculum requirements for this schooling level.

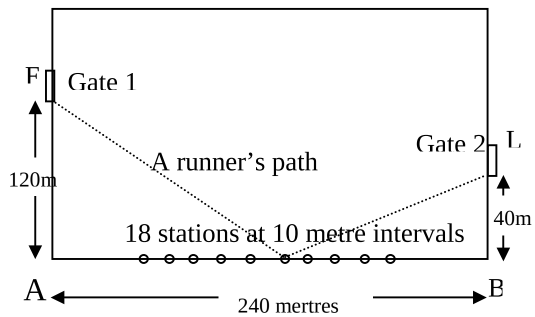
In designing such tasks Peter was aware of the need to make trade-off decisions about the competing needs of students. There was little point in setting task challenge too high especially in the lower secondary years where negative experiences in mathematics have the potential to foster long term negative attitudes towards mathematics because "if we are not careful but, some of our projects fatally wound kids because they got the first line wrong." On the other hand, some students are quite capable of producing work beyond that expected for the majority of students especially when allowed to use electronic technologies. Peter said he had "underestimated for years what these technologies are and I've actually held them back. So, for some students I think we've got to make sure it's big enough to allow some students to run further than we actually envisaged." Peter saw a further use of "technology might be to alleviate … the endless repetitiveness after they've engaged in the process of 'This is what I understand the process to be'". Above all, he was aware he was to create tasks which would engage students and develop their expertise as learners "who love learning and who know how to find things out for themselves" (Collins, Joseph, & Bielaczyc, 2004, p. 18). So, decisions about the level of task scaffolding provided in task sheets and on an individual basis by peers and teacher are critical, as too are decisions about when and to what extent that scaffolding should fade as the year progresses. For Peter, "the engagement process is: 'Yes I'm confident enough to present this to you, not as a total solution but as a representation of what I know'" and this occurs in a collaborative learning environment. When asked what images came to mind when he thought of engagement in the context of a mathematics class he said:

> Peter: Two students in dialogue, I've got a visual picture, it's on my wall of two students with a graphics calculator in hand looking down at the calculator, just punching numbers and discussing it between them and I'm no longer needed to scaffold. I can move out. I can move to another group so the engagement is that their knowledge is important, saying, "What do I need next?" Engagement to me is that the task of my expertness has moved out. I've facilitated the change. I can move out to the side, come back and ask questions.

4.2. The Tasks

The first task for the first 2 years of the project was *Cunning Running* (Figure 1) which came at the end of the first unit for the year on Pythagoras' Theorem. It served as both the culmination of this unit and the transition into a later unit on trigonometry. During the teaching of Pythagoras' Theorem an investigation into the patterns and algebra of square numbers was used on several occasions. Learning experiences included a first introduction to operations on graphing calculator LISTs. Also, as part of this investigation, students worked as a class through the solution of the task, *Elma's Poster*, where they had to investigate the area and dimensions of various sized square posters that could be hung in a square shop window when restricted to hanging the poster by its corners from existing hooks placed at a regular fixed interval around the window frame. In this task numerical, graphical and algebraic methods were used. Prior to *Cunning Running*, students undertook a test set by Peter of use of Pythagoras' Theorem in single triangles, a rectangle and compound figures to find various lengths. Results for the 40 mark test (M = 36.2, SD = 2.16) indicated that the class had demonstrated a high level of proficiency in the topic.

In the Annual College Orienteering event, competitors choose a course that allows them to *run the shortest possible distance,* while *visiting a prescribed number of check point stations.* At one stage, the runners enter the top gate of a field and leave by a bottom gate. To cross the field**,** *they must go to one of the stations* on the bottom fence. Runners claim a station by reaching there first and remove the ribbon on the station to say it has been used. Other runners must go elsewhere. There are 18 stations along the fence line at 10m intervals. The station closest to Corner A is 50 metres from A. The distances of the gates from the fence with the stations are marked on the diagram.



THE TASK

Investigate the changes in the total path length travelled as a runner goes from gate 1 to gate 2 after visiting one of the drink stations. To which station would the runner travel, if they wished to travel the shortest path length?

For the station on the base line closest to Corner A calculate the total path length for the runner going Gate 1 – Station 1 – Gate 2. Draw a scale diagram of the situation and use this to check your hand calculations for the first four stations. Use LISTs in your calculator to find the total distance across the field as 18 runners in the event go to one of the stations, and plot the points to show how the total distance run changes as you travel to the different stations.

Observe the plot, then answer these questions: Where is the station that has the shortest run total distance? Could a 19^{th} station be entered into the base line to achieve a smaller total run distance? Where would the position of the 19^{th} station be? If you were the sixth runner to reach Gate 1, to which station would you probably need to travel? Use your Lists to find the algebraic equation that represents the graph pattern. Draw the graph of this equation on your plot of the points. If you could put in a 19th station where would you put the station, and why?

(Additional suggestions as to how the work might be set out and for intermediate calculations provided some task scaffolding.)

Figure 1. Major elements of *Cunning Running* Task

The lengthy task statement posed potential problems with linguistic complexity. For the implementation in the first project year, Peter mediated these problems through class discussion and connections with visual mental images from a precursor gym activity. For the second implementation he used an introductory dynamic geometry demonstration and a classroom demonstration instead as this eliminated the need for extra lessons and having to coordinate scheduling of the task with availability of the gym. After the introductory lesson, students worked on the task in groups usually of 3 or 4 for a double lesson (2×50 minutes). Students were then allowed more time to work on the task at home before handing it in.

The second task, *Shot on Goal* (see Figure 2), came two months after the first which was undertaken in the fourth week of the school year. Students had now completed a unit on the trigonometry of right angle triangles. Four class lessons over one week were allowed for the task. It was handed up at the end of the last lesson so students had no time to complete and polish their reports as they had with *Cunning Running* as they were about to go on school camp. Peter selected hockey for the task context as he considered it to be more inclusive in a co-educational school and some class members played hockey. Some caveats apply to this decision. Firstly, a goal in hockey is only allowed from a point within the penalty area, so only some of the run lines are feasible. This aspect can be included at a later stage by first finding the location of the best shooting position in terms of angle as is required by the question, and then checking its position relative to the penalty area. (With soccer there are no such restrictions.)

Many ball games have the task of putting a ball between goal posts. The shot on the goal has only a narrow angle in which to travel if it is to score a goal. In field hockey or soccer when a player is running along a particular line (a run line parallel to the side line) the angle appears to change with the distance from the goal line. At what point on the run line, has the attacking player opened up the goal to maximise the possibility of scoring the goal?
Assume you are not running in the GOAL-to-GOAL corridor. Find the position for the maximum goal opening if the run line is a given distance from the side line As the run line moves closer or further from the side line, how does the location of the position for the widest view of the goal change?

Figure 2 Major elements of *Shot on Goal* Task

5. Student Perception of Challenges

All but two of the 10 students interviewed unequivocally stated they liked doing tasks like *Cunning Running* or *Shot on Goal* with Sandra saying she found "them challenging and once I've done them I find it like rewarding." She saw the purpose of the tasks as making her think. Leo liked them "more than doing the textbook work" as the tasks engaged his interest. Similarly, Pat liked "doing tricky things and learning how to do difficult things and the rules. Writing out what they mean." The latter refers to what engaged and challenged him the most—the development of an algebraic model for the situation. Val, on the other hand, did not "like it when I am doing it but when I get it, when I understand it, I am all happy in myself. Just because, I don't know, usually it is when I finish and I look back and actually see that I could do it!"

Cunning Running required students to vary distances with the purpose of minimising the total distance run, while *Shot on Goal* involved looking at varying angles to maximise the shot angle. Surface similarities tend to obscure different levels of complexity in the respective formulations and thus different levels of challenge in the tasks at different points in the solutions. Similarities in the tasks were noted by Sandra and her comparison of the level of task challenge echoed that of the other students interviewed.

Sandra: I found them pretty similar because you had triangles and stuff. Yeah.

I: But what about difficulty and challenge was it similar, the experience of it?

Sandra: I found this one [*Shot on Goal*] more simple because the first one [*Cunning Running*] you had to [work out the equation], yeah. I found it a bit difficult.

The cognitive demand of the two tasks as a whole is similar; however, the level varies throughout the task solutions. In *Cunning Running* there is a moderately high level of cognitive demand required throughout but this rises substantially, for students in the early stages of their algebraic development, when translation from symbolic LIST formulae to algebraic formulation is required. At points where cognitive demand rises students can experience a sense of challenge. In *Shot on Go*al, at two junctures in the formulation stage cognitive demand is high. Both give Year 9 students a sense of challenge. The cognitive demand then falls for much of the remainder of the task rising again moderately when an algebraic formulation is required. Producing such an equation is much less challenging than in *Cunning Running*.

5.1 Taking up challenges

The greatest challenge in *Cunning Running* was formulating a two variable algebraic model with only 2 of the 27 students who attempted the task doing this successfully but these were early days in their study of algebraic equations. For most of the double lesson observed by the researchers, Sandra worked firstly on developing her equation and then on using her graphing calculator to verify it. The other student, Ken, had completed the task at home. A third student, Ben, described the challenging process of concatenating LIST formulae to produce such an equation as, "putting them together and then kind of simplifying it so they worked because it is pretty well what it is, just all the LISTs." Unfortunately, Ben's simplification process was flawed, although he did produce an equation in two variables. Mei, who worked in class with Sandra but had not begun her equation by the time the class ended, produced what to her was a one variable expression for total distance $\sqrt{(40^2 + x)} + \sqrt{(120^2 + x)}$ as only one station was involved at any time when her expression was evaluated. She did not see this conflicting with the *x* being the distance from the station to corner A in one part of the formula and from corner B in the other part. She also did not find the task challenging but admitted "the actual algebra bit, the equation to try and get the equation for the actual graph was really hard." The remainder of the task she felt was at an easier but similar level of difficulty but time consuming. Only Sandra was able to verify her equation was correct by using the function window of her calculator to draw a graph through the scatter plot of points from her LIST data (Figure 3). She made several attempts and her delight was obvious when finally the function went through the points.

Mei: You are close.
Sandra: Yes, but why does the line go down the side?
Mei: I am not sure.
Sandra: Maybe you need brackets. You may need brackets before that. There. [Talking to herself and working on her algebraic model in Y=.] Brackets. And then have a bracket right at the end.
Mei: Maybe? [pause] Damn!
Sandra: Wait, it is still going [referring to the calculator]. Nuh.
Mei: Awh, oh my god. Too hard!
Sandra: [She picks up the calculator and looks closely at the graph.] Where is the line? Ohh! There!
Mei: So it does work!

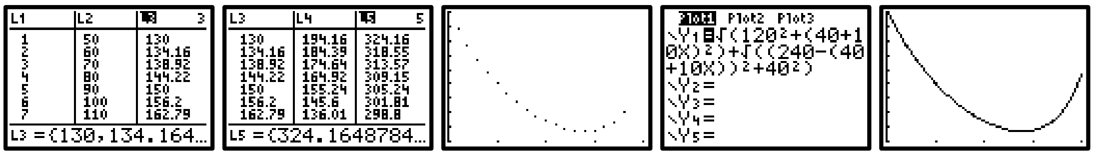


Figure 3. Sandra's verification of her algebraic equation

5.2 Avoiding challenges

Expected challenges built into the task by the task setter often did not eventuate for students who missed the significance of particular task requirements. In *Cunning Running*, for example, several students did not perceive this affordance of the calculator to enact verification even though class discussion highlighted it. Some thought the question was superfluous, stating "the graph will be the same", as they had used a joined scatter plot on a spreadsheet chart or in a by-hand graph. However, Ben, who missed the significance and therefore the challenge of this verification method, perceived an alternative was to substitute into his model on the home screen. Gary emphatically said he liked challenging tasks but thought *Cunning Running* was not challenging at all as he underestimated the mathematical depth required for an appropriate solution. For his algebraic model Gary wrote: *total run distance* $= \sqrt{(h^2 + i^2)} + \sqrt{(j^2 + l^2)}$ with these variables labelled on his scale diagram. This model was similar to the model derived as a class for *Elma's Poster* which he recognised was similar. Other students also did not take up the challenge of the algebraic formulation using the same variable labels as used in *Elma's Poster* and their equation was at an even lower level (e.g., $\sqrt{(S^2 + M^2 = H^2)} + \sqrt{(S^2 + M^2 = H^2)} =$ *answer*).

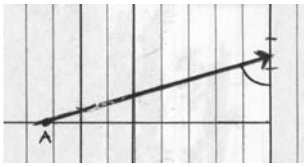
Interpretive aspects of tasks presented challenges for some. In *Cunning Running* determining where to place a 19th station if it could be placed anywhere was a challenge requiring students to perceive previous constraints could be relaxed, such as discontinuing the ordered pattern (i.e., 19th must follow 18th) and the requirement for a 10m distance between stations. Many simply placed an extra station 10m from either the first or last stations. Some, however, searched for a shorter distance but Gary placed his 19th station so the total running distance was the same as for the shortest station as "a good race is a close race so … if you have the first two runners going to the same station round about it is going to be a closer race."

5.3 Challenges in the real world to mathematical problem specification

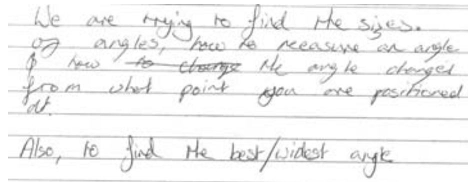
In *Shot on Goal*, the cognitive demand required for task specification was high potentially leading to a blockage in this early solution phase if students found the level of challenge too high to engage with the task. Initially, students had to establish the aim of the task and specify this in terms of a mathematical problem. Ann, for example, saw the aim as "to find out where the best spot on the field is to take a shot on goal". "Best spot" was then translated to mean where the "widest angle" was. However, there was still the dilemma of which angle this was. Peter anticipated specifying the aim of the task and the angle involved would be challenging. He provided a supporting physical demonstration in the classroom which students either participated in or watched. A tennis ball was thrown through two goal posts from various angles.

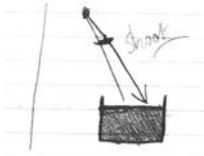
> Sandra: Yeah, uh, it helped to explain like what we hoped to find out. Like I didn't really get what we were trying to do. And that kind of explained what angles we were trying to find.

This was followed by a debate about which angle was the focus by students using diagrams on the front whiteboard. Several boys thought it was various angles made by the ball at the goal mouth as it entered the goal whilst Amy thought it was the angle from the spot where the ball was kicked (Figure 4).



Pat's angle





Amy's angle

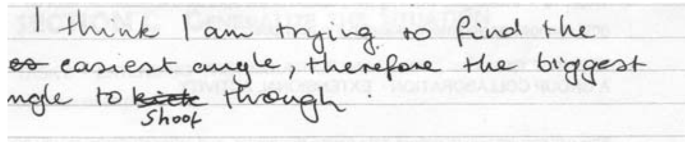


Figure 4**.** Students' diagrams of possible shot angles and their aims for the task.

5.4 Challenge of the geometrical formulation

Once the desired angle was known in *Shot on Goal*, a second challenge came when students had to work out how they might find this angle geometrically in order to apply formulae to carry out their calculations. The latter was considered far less difficult, if not routine.

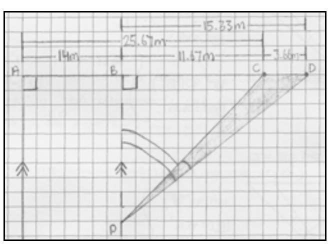
I: How difficult was it to decide which hand calculations you had to do?

Sandra: I found it pretty easy. I thought about it at home. Once I figured it out, I found it pretty simple.

I: How did you know it was an inverse tan?

Sandra: Well, we had been doing it in class just before and we had the side, we had to find out the size of the angle and we had the length, sides and yeah.

Finding the angle involved a decomposition of the angles from the shot spot on the runline to the near and far goal posts into component parts. Most students subtracted the angles from the shot spot, that is, ∠BPD - ∠BPC in Di's diagram (Figure 5a). Sandra, however, who had worked on this before the class discussion, described her method to resolve this second challenge as finding "the extra two". "I did it the other way of finding two different triangles and then taking them away from the other". She saw the task as partitioning the rectangle made by the run line, goal line, and line segments parallel to these (Figure 5b) into three triangles, two containing the extra parts of the angle which would not result in a successful shot. These extra angles were then added together and subtracted from 90° to find the shot angle.



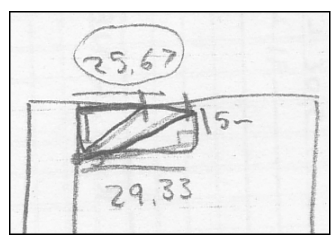


Figure 5 a) Di's diagram and b) Sandra's diagram to find the shot angle.

5.5 Challenges arising from discovered complexities

Other challenges arose for individual students as they discovered different complexities in the task or their interpretation of the task. Mathematising the run line introduced an unexpected challenge for Ben and Ken. The class viewed a Power Point which included two demonstrations of the view of the goal as a player approached on a run line parallel to the side line. Interviewed students found the more dynamic of these helpful in clarifying their thoughts. However, Ben and Ken faced an unanticipated challenge with the transition to a mathematical problem. Instead of advancing down their specified run line in 1 m intervals, they took a stepped trajectory towards the goal.

Expected challenges built into the task by the task setter often did not eventuate for students who missed the subtleties of the implications of generated interim results. Leo's calculated interim results for *Shot on Goal* were not flawed but contrary to what he expected causing him some concern and he had to ask for help to resolve this further challenge. He expected the angle of the shot to increase as the player moved along the run line (Figure 6). Not only was this not the case for his allocated run line of 10 m from the side line, but two of his results were identical. This conflicted with his conception of the situation. After checking his results several times, he sought help from his group. Ascertaining that no others in the group had any identical values, Pat assured Leo his results must be in error but Leo was certain his calculations were correct. Only after viewing a physical demonstration involving string lines was he finally convinced his results were sensible. The same set of results did not provide a challenge for Cate, allocated the same distance for her run line, as she missed the implications claiming: “I thought the angle would increase as you get further from the Goal Line. The calculation has <u>confirmed</u> my initial beliefs.”

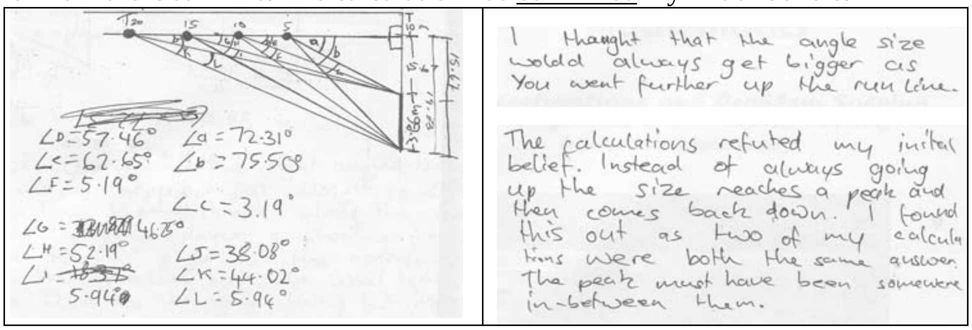


Figure 6. Leo’s challenge

6. Conclusion

Designing extended real-world investigative tasks that present manageable and engaging challenges for lower secondary students is not without challenges. Despite thoughtful considerations by the teacher in both designing the tasks and providing timely task scaffolding at points during task implementation when students were expected to be challenged by the cognitive demand of tasks, there are always differences between the expected student moves and challenges and what transpires. Some students take up the challenges as expected but for others these same challenges do not eventuate as the significance of particular requirements of the tasks is missed, or the mathematical implications of results produced during the task which should generate challenge are not realised. At other times unforeseen challenges arise for individual students as they discover different complexities in their unanticipated interpretation of tasks.

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