MATHEMATICAL CHALLENGE
TO WIDEN EXCLUDED STUDENTS' FOREGROUND

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Abstract
This paper presents an ongoing research that studies the mathematics teaching and learning as it takes place in a reinsertion school in the neighborhood of the City of Buenos Aires. The 'back-to-school' teaching project, as it is established by the educational administration, aims to youngsters that have abandoned secondary school and are living in the margins of society. It intends to offer them with an education that widens those students' foreground for, on the one side, it allows for the necessary official certificates and grades to get a dignified job, and on the other side, it regains them for society being the school the social link. Mathematics is one of the subject matters that the students must follow, and our study has to do with the teaching and learning of algebra on the basis that mathematical challenge can constitute an initial motivation for those students' participation in the mathematics classroom as a first step for their learning mathematics.

THE CONTEXT OF THE STUDY

As a result of the 2001 crisis, there has been in Argentina a social segmentation reflected, among other things, in dropouts from school. Several figures show this situation: 35% of youngsters between 15 and 24 years old neither study nor work, 13% of teenagers abandon school, the global unemployment rate among those under 29 years old is 13%, out of which 54% live in poor households.

In this scenario, the government of Buenos Aires started, in year 2004, the Back-to-School program for secondary students in line with the Zero Dropout Plan[1], which aims to youngsters that have abandoned secondary school and are living in the margins of society. It intends to provide for them a curriculum equivalent to that of compulsory secondary education[2] that will allow them to obtain the necessary official certificates and grades to get a dignified job.

The plan aims at meeting the demands of an important sector of the young population. The students involved in the Back-to-School are, at least, 19 year old when they join the school and have for varied reasons interrupted the education for at least one year, but are interested in completing the secondary education and show commitment towards it. An important number of these students have undergone failure experiences in the educational system, both at the primary and secondary levels, and many of them share family and work responsibilities together with education.

[1] More information in http://www.buenosaires.gov.ar/areas/educacion/desercioncero
[2] Since 2002, in the City of Buenos Aires, education is compulsory until the students are 16. (Law n. 898 on Compulsory Secondary Education)

Some students drop the school many years ago and others have poor primary education, many of them do not even know the multiplication and division algorithm or use basic procedures to subtract. For what regards their place in the society, some students have penal charges or suffer from drugs addiction. The attendance levels are low, high absenteeism rate endangers the regularity. Another characteristic of these students that affects their education is their lack of schooling abilities, which is showed in the poor fitting to the school culture, especially among those in the first year.

However, for many of them, school seems to be the only space where to construct social networks that have been denied to them in other contexts, school becoming for them a possible link to feel included in society again.

The following sketch shows how much they value school. During one of the school breaks, two students left school and went to the drugstore and, armed with guns, tried to rob the newspaper seller. At that moment their tutor entered the drugstore to buy cigarettes and seeing them exclaimed: *'¿What are you doing here? Go back to school immediately!'* Facing their teacher intervention, the students left the drugstore, and even if they did no go back to school that day, abandoned their intention to robe the newspaperman. This is only a story, but it shows the symbolic value that these students attach to school.

THE RESEARCH STUDY

The first author, working for the Secretariat of Education, Government of the city of Buenos Aires, has taken part in the establishing of the mathematics curriculum for the teaching project[3] and is leading a group of mathematics teachers involved in it. It seemed to us that the enormous administrative, educational and personal efforts that the project meant could be paralleled with a research project. Therefore, the first author, being also a university researcher in Buenos Aires, has decided to develop her research with the collaboration of the second author.

The research study is developed in one of the reinsertion schools in Villa Lugano, a school in the neighbourhood of the City of Buenos Aires. We are working together with one of the teachers involved in the project, and with her class. She has showed a strong commitment towards the project, has positive expectations of their students, and has a sound mathematical education. The three of us have worked together to put up the problems that are expected to be 'a mathematical challenge' for her students.

One of the aims of the research is to establish the meaning for 'mathematical challenge', something that we plan to do from a theoretical perspective as well as from the perspective of the teacher and the students involved in this particular situation. We will study, together with them, and from a socio-cultural perspective, how mathematically challenging activities can constitute an initial motivation for the students to participate in the mathematics classroom and how a particular way of dealing with the interactions among the participants can contribute to create a certain classroom culture that facilitates all students participation as a first step for their learning mathematics.

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[3] Curriculum Bureau. Secretariat of Education, Government of the city of Buenos Aires. Mathematics Syllabus. First year, Buenos Aires. 2002.

Curriculum Bureau. Secretariat of Education, Government of the city of Buenos Aires. Mathematics Syllabus. *Second year,* Buenos Aires. 2003.

In http://www.buenosaires.gov.ar/educacion/

Algebra is understood as a tool to model and deal with a specific problem. Thus, for instance, the process to obtain a formula to count the number of elements of a collection, whatever their number is, allows making known the structure of the underlying calculation algorithm. At the same time, this process can give some meaning to a first use of a 'letter' as a variable and also to the need for a correct use of algebraic expressions. Moreover, the different ways to deal with a same problem may give meaning to the discussion about the equivalence of the different expressions that represent it and to the realization of the transformation of some algebraic expressions into other equivalent ones.

From this perspective we consider that for the students involved in our project, a challenging activity could be the production and validation of formulas using natural numbers. The intention is that students look for regularities that allow them to find formulas and produce arguments to validate them. The teacher is not expected to "teach" formulas and students are not expected to "apply" them, but it is expected that students have the chance to speculate, create, test and validate their proposals. We are giving the students problems that, by admitting different ways to deal with them, lead to the production of different writings to represent the same process. As it was said before, this diversity is considered as a support to work on the equivalence between expressions.

Why have we decided to work on challenging problems to facilitate the students learning of algebra? There is a significant number of important research studies about collaborative work and project work that show that students from socially deprived contexts do learn mathematics when working in activities evoking real life practices relevant to students. However, the learned knowledge in most of these situations does not necessarily correspond with or cover the curriculum's content. Nevertheless, the students' disposition of the curricular knowledge is a necessary condition to find a path to be inserted or included in the society. To find a job, to become a cop for instance, students need to have succeeded in their secondary school and have to pass a mathematics examination that will include algebra. Moreover, it is difficult to find mathematical activities from these students 'real life' that could motivate them, since their 'real life' is precisely what they want to overcome!

Most probably, algebra is not 'the essential content' to achieve the mathematical enculturation of citizens. However, while we try to change the social representation of which mathematical knowledge and which mathematical education school must provide for the society, we could at least try to find ways to put the means for these students to be included in society. It is our goal to try to know more about which are the possible paths for mathematics education to facilitate these students' access to qualified jobs and to allow them to decide their future in equal conditions with those that have followed the standard secondary school.

This kind of thinking, together with the experimental study, leads us to take as a starting point the idea that it must be possible to facilitate the arousal within the mathematics classroom of a classroom culture that allows the students' assimilation into it. It is our goal to characterise the conditions of such a classroom culture, focussing in its socio-cultural aspects but also deepening on the mathematical ones. We will study the discourse, the valorisations, the construction of norms as they take place in the mathematics classroom where the change in the ways teacher and students deal with problem solving is the medium to achieve assimilation into the mathematics classroom, because the 'mathematical challenge' has become the reason why the students wish to study. Our main assumption is that the 'mathematical challenge' can be as valid as the everyday context to helping to create students' foreground (Skovsmose, 2005).

INITIAL STEPS

An initial questionnaire has been administered to the 13 students involved in the mathematics classroom we are studying with the aim to come to know their reasons for their coming back to school, their own positioning towards mathematics and their previous experiences with it. Following, we present some of the questions and the answers to them.

- *Why did you decided to come back to school? Which school were you attending before? Why did you leave it?*

- *Do you believe that you will manage to succeed in mathematics? What do you feel that you need to succeed?*

The first question tries to collect the reasons both of the desertion from and of the coming back to school; as per the second one we wanted to come to know the students' own positioning towards and their representation of school mathematics.

The answers show a genuine interest to conclude secondary school and most of the students explicitly say that it is a tool to improve their future or that of their children –three of the students are to become parents in the near future. The reasons for desertion are associated to the need of working or to family problems, all of them reflecting complex social and economical situations. Another reason for desertion is linked to the school system, their previous schools being blamed of segmentation, and segregation *'I was thrown away from the school because I failed twice'*

It is important to know that among the answers it is frequently mentioned the word 'study' together with willingness and effort. It is less frequent their claiming for the need to be helped by the teacher, possibly because the students consider that the teachers in the present school are 'different': *'with the teachers that I have, I think I will manage'* or *'I count on the teacher helping us when there is something we do not know how to do, but, of course, I will put all of myself to succeed'.*

The following two questions try to gain information about the relationship the students have established with the learning of mathematics in the school.

- *Tell us a good experience, a moment where you have felt well in the mathematics classroom*

- *Tell us a bad experience, a moment where you have felt bad in the mathematics classroom*

The students' explanations show that the good experiences are linked, mostly, to pass the examinations or to non-traditional forms of teaching like to visit a museum or to participate in a mathematical competition but also, in fewest cases, to not work.

Some answers show that failing the examinations or a poor understanding of a particular topic is lived as a negative experience. The students are conscious of their failures or what they lack and this consciousness leads them to feel frustrated. It is worthy to note that a situation commonly associated to bad feelings is to be called to the blackboard in a coercive way. They feel this situation as a degrading self exposition, a way on the side of the teacher to make evident what the student does not know. However, we want to point out that in the previous question one of the students said that it considered a positive experience to be called to the blackboard when the goal is to discuss what the student has done with the whole class.

Having seen the answers to the questionnaire, the teacher felt the need for a feedback to the problematic around being called to the blackboard. She makes explicit that her intention when calling a student to the blackboard will only be to give the opportunity to each group to show what they have done and to discuss with them all. After discussing under which criteria the students should go to the blackboard, the students suggest that they should volunteer, and there is agreement upon that.

In another session, the teacher organizes a discussion around the following questions in order to establish a didactical contract.

1. How would you like to be the mathematics class?
2. And the working with your mates within the mathematics class?
3. How would you like to be your relationship with the mathematics teacher?

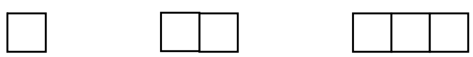
As a synthesis of this activity the students write down on a poster to be hang on the wall the ‘Ten Commandments of the Mathematics’ that define both teachers´ and students´ responsibilities

SOME CHALLENGING PROBLEMS

The problems initially posed to the students allow generating the elaboration of formulas to calculate the “n” step of a process that fulfils a certain regularity (addition of “n” first natural numbers, calculation of the number of elements of a certain geometrical configuration, etc.). The geometrical context serves as a possible support to validate the equivalence of different writings.

The problem

The teacher shows the following series of figures built with matches and explains how they should be further assembled.



a) Calculate the necessary number of matches to build the figure in the sixth order.

b) How many matches would it be necessary to build the figure in the 100 order in the series?

c) Find a formula for the number of matches in the “n” order.

d) Could it be possible that somewhere in the order the figure has 1549 matches? And 1500?

THE STUDENTS SOLVE THE PROBLEM

In the outputs it may be seen the work carried out by students in the elaboration of formulas, work that supports the equivalence of algebraic expressions and their use to solve new problems. For instance, when solving the problem, some students counted the total amount of matches required by considering that every square meant the addition of 3 matches to a first match. This way they produced the formula $n.3 + 1$. Another possible formula that was produced was $2.n + n + 1$. The students that came out with this formula have counted in pairs the matches of the parallel sides of the squares, adding to them the third match that repeats “n” times and finally adding the first one. Other students’ output was the formula $4+3(n-1)$ resulting from adding to the 4 matches corresponding to the first square the 3 necessary matches to be added each time (n-1) that a new square was produced. Finally, another possible solution that appeared (as shown in figure 1) was $4.n – (n-1)$. In this case, the students multiply the number of sides of the square by the number of required squares subtracting from that n-1, the number of matches repeated on that counting.

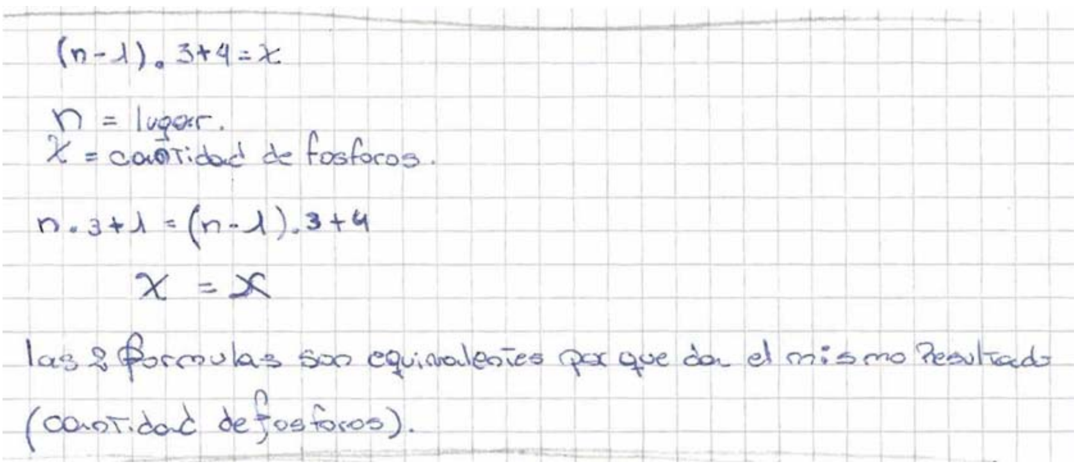
Working with equivalent expressions allowed the introduction of the idea of the common factor and the distributive property. Thus, when solving the second problem, to prove the equivalence between the equations 4+3(n-1) = 3n+1 a great number of students used the concept of multiplication as repeated addition. They considered the expression 3(n-1) as n-1+n-1+n-1 which they wrote following the same pattern as additions with natural numbers, i.e., as a calculation:

$$\begin{array}{c} n-1 \\ n-1 \\ n-1 \\ \hline 3n-3 \end{array}$$

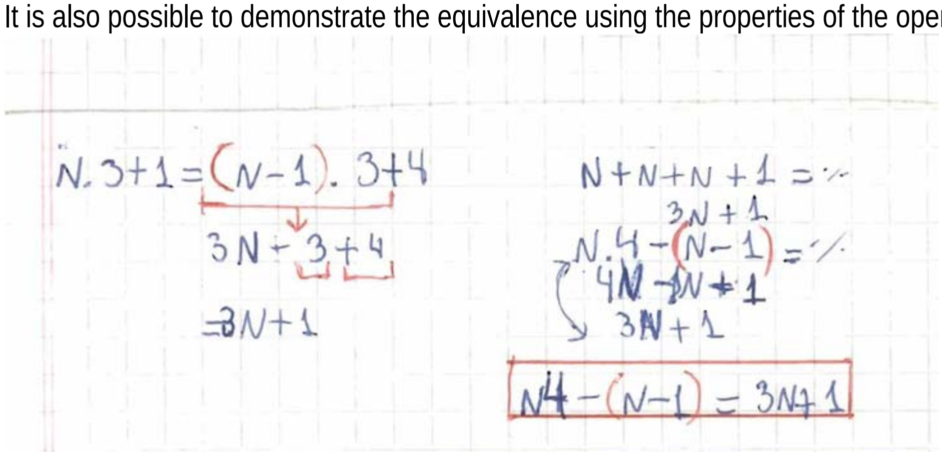
Then they added by associating on the one hand the “n”s and on the other hand the –1 getting to establish the equation 3(n-1)=3n-3. We observed that students implicitly made use of the commutative and associative properties in connection with the addition although they had not learned their symbolic formulation.

Based on this production, the common factor and the distributive property, which the students had not yet worked with, could be institutionalized. At the end of the activity, the students could write expressions such as 4+3(n-1) = 4+3n-3 = (4-3)+3n = 1+3n= 3n+1.

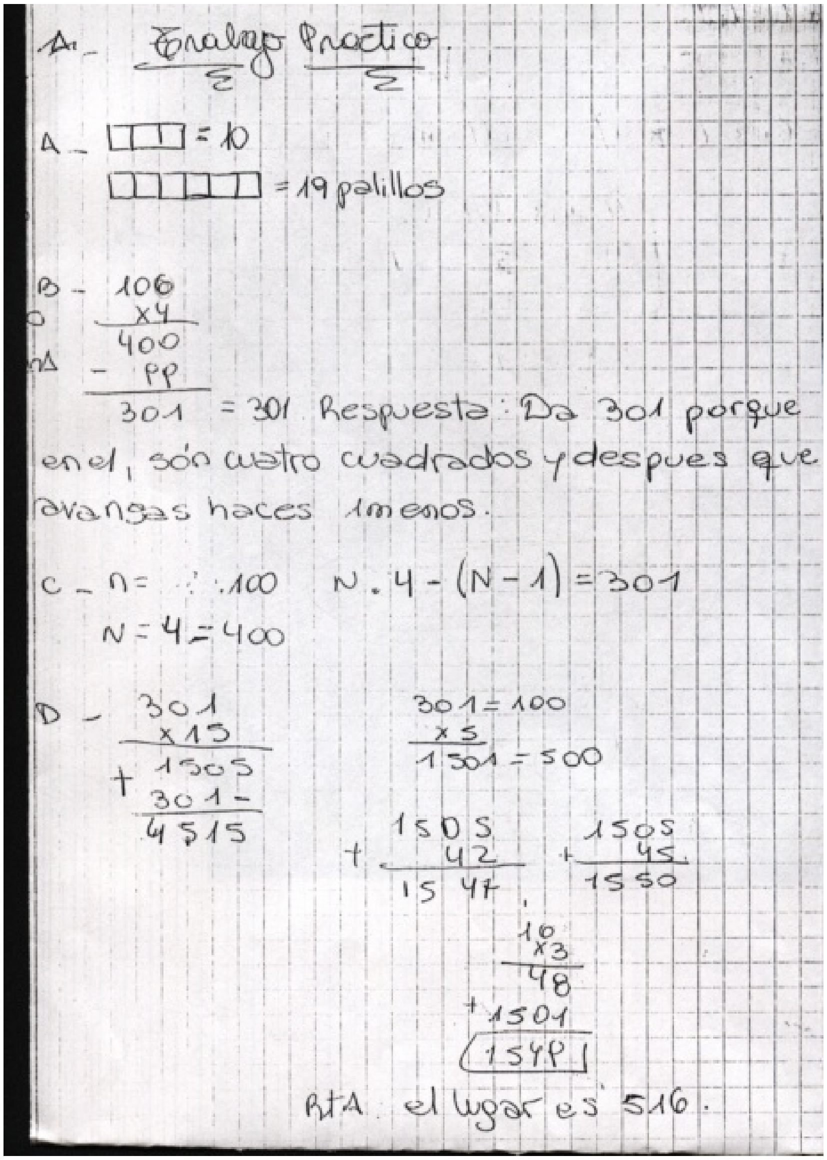
Proving that 2 formulas are equivalent generates algebraic work with regard to writings. Producing a formula based on the use of the variable and on the context, incites the inverse validation process (you can verify it with numbers), as it is shown in the following writings of the students.



It is also possible to demonstrate the equivalence using the properties of the operations.



To finish with the analysis of the outputs to this problem, we want to note what the students whose worksheet is given below have answered to the question: *Could it be possible that somewhere in the order the figure has 1549 matches? And 1500?*


A.- Trabajo Práctico
A.- = 10
= 19 palillos
B - 106
x4
400
- 99
301 = 301 Respuesta: Da 301 porque
en el, són cuatro cuadrados y despues que
avangas haces 1 menos.
C - n = 100 N . 4 - (N - 1) = 301
N = 4 = 400
D - 301
x15
1505
+ 301 -
4515
301 = 100
x5
1501 = 500
1505
+ 42
1547
1505
+ 45
1550
16
x3
48
+ 1501
1549
RtA el lugar es 516.

Some of the calculations they produced were tentative. Those that lead them to a result are:

301 = 100

x 5

1501 = 500

By this writing this they want to express that if 301 matches are needed to be in the 100 order, then for the 500 order they would need 301 x 5 minus 4, which are the matches that repeat when adding 4 series of 301. They multiply 16 by 3 to know the matches required to build up 16 more squares, and they add up those 48 matches to the 1501 to obtain 1549, therefore they answer “the order is 516”.

Until now we have interviewed the teacher to come to know, among other issues, about her understanding of what constitutes teaching and learning mathematics in such a situation, her expectations on the students, and her ideas of what ‘mathematical challenge’ can mean for her students. We have asked the students about the reasons for their coming back to school, their relationship with mathematics, their previous experiences... and we have elaborated a sequence of what we believe to be ‘challenging activities’ to introduce the students to algebra.

What we have presented above is only a small part of the beginning of a project lead by two essential interests. First, as teachers, we are searching for tools to help some students who until now were ‘the excluded’ to regain their interest for the learning of mathematics –we count on that, once they regain their willingness because of feeling challenged, they will take advantage of the opportunities we offer them, since we never accepted the idea of a cognitive deficit model. Secondly, as researchers we want to come to know more about what does it mean ‘mathematical challenge’ to the teacher and the students, which are the conditions for the classroom culture if the mathematical challenge has to go beyond than that in order to foster real learning.

There is much to be done before we may answer the smallest of the questions we ask ourselves. In the near future, we plan to systematically observe the lessons, focussing both on its socio-cultural and mathematical aspects, in order to come to know more about a classroom culture that is expected to be non-threatening to students that, feeling challenged, take the risk to contribute to the mathematical discussion. We will also ‘follow’ some students that show involvement in the given tasks by analyzing their written products, observing them and the groups they are part of within the classroom in order to study the interactions that take place while working collaboratively and by interviewing them to try to figure out the reasons for their involvement.

We would be content if in some years we had evidence to claim that mathematical challenge is as good as a mean for students from socially complex and deprived contexts to learn mathematics as it has proved to be project work or real life activities. The challenge for us is to show that it is not only the good ones or the wealthy ones that will benefit from the challenge of mathematics.

CONCLUSIONS

We want to point out that an important number of students were interested and enthusiastic, especially when coming up with their own formulas. The production of several formulas made it possible to work on their validation and on the equivalence of the different writings made by students based on the geometrical context. Students were able to validate their formulas since they have obtained them from the observation of the arithmetical regularity. Producing a formula using a variable made easier the inverse process –to validate them through verification with numbers. We are ready to assess that students assume their responsibility in their mathematical learning facing a task that sets an intellectual challenge and which they can control.
Once more an episode from a lecture helps to support what we have stated. After the students have solved the problems, the teacher, as part of the learning arrangement, provokes a discussion about the strategies and makes a synthesis of the knowledge that has been used, informing the formal aspects. Once, a student answered “it is boring, teacher” to her proposal. Anyhow, the task was

finished but we observed that the students loose the focus, don't pay attention. Afterwards the teacher gives new problems with regards a new issue; immediately her students focus on it. Later the teacher comments: "problems are like a pacifier for these students".

It is not possible to state that mathematical education or the education in general is the main reason for students' social exclusion or inclusion; there are many reasons involved. However mathematics lectures may play an important role making the students gain their self-esteem; this way mathematics education becomes a tool for social reinsertion.

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