ICMI Study 16: Current Perspectives

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Abstract

ICMI Study 16: "Challenging Mathematics in and beyond the Classroom" has the potential to place a significant aspect of mathematics education on a much higher level. There has been little research on challenge as a device to improve the learning process. The Study is now in its concluding stages and most of the Study Volume is complete, just subject to editing changes. In this lecture I will review what we have done, indicate where we are at, and discuss the likely impact of the Study.

Background

ICMI Studies first started in the middle 1980s when the International Commission on Mathematics Instruction (ICMI) when a study was commissioned on the influence of technology and informatics on the learning of mathematics.

Studies are designed to thoroughly explore the contemporary thought on the subject, usually one of current interest, particularly identifying issues connected with the Study, and having gathered a range of experts together, to publish a considered document, a book called a Study Volume, defining the state of the art.

Studies have embraced a range of issues, some central to the subject, such as addressing the teaching of algebra, some closely related, such as those looking into the impact of history and psychology, and some looking at the subject from a perspective, such as comparing cultural differences between East and West.

A number of Studies have been commissioned at an approximate rate of one per year. They commence with the decision to commission the Study, then identification of one or two chairs, then the formation of an International Programme Committee (IPC) of approximately 12 experts. This is followed an IPC conference which substantially leads to the writing of a Discussion Document and subsequent call for papers from interested parties. From the papers some are invited to attend a Study Conference, in which the essential material of the conference is gathered after much discussion. In the ensuing period the material is written up and finally published in a book called a Study Volume. The entire process can take about 6 to 8 years, typically.

The 16th Study was apparently decided by the ICMI Executive in 2002 and Ed Barbeau, of the University of Toronto, and I, were appointed joint chairs. An IPC was appointed in early 2003 and the IPC Conference was held in Modena, Italy in November 2003. The Discussion

Document was generated in draft form there and finalised at an IPC meeting in Copenhagen, at ICME-10, in July 2004. The call for papers was issued and finally about 50 people were invited to attend the Study Conference, which was held in Trondheim, Norway over June/July of 2006.
Much of the material was decided there with broad groupings identified and then sub-groupings for individual chapters. Much of this material was formally generated in the latter part of 2006, while 2007 has largely been taken up with editing and improvements. At the time of this Creativity Conference in Haifa, February 2007, it is likely that the material will all have been prepared in camera ready form in readiness for publication later in 2008.

Competitions and the possible origins of the Study

Not having been an Executive member of ICMI when the Study was mooted and commissioned I cannot explain the full background of the reasoning behind the Study. However there were two major factors which were present and which were able to be blended in the process.

The most obvious one was the existence of competitions. The World Federation of National Mathematics Competitions (WFNMC) was the only Affiliated Study Group (ASG) of ICMI whose area of interest had not been the subject of a Study.

Competitions are a controversial activity in the eyes of some. There are some who view them as elitist even though there has been a growth in participation in inclusive competitions, such as the European *Kangaroo*, in recent years, based on the model of a successful Australian competition. There are also gender and psychological issues in some eyes. For example there is evidence that boys are more successful than girls (even though a closer examination shows only small differences in mean scores), and some would argue that competitions provide pressure, even though competition results cannot adversely affect normal assessment results.

However there is a significant body of committed competition supporters, who can point to many advantages. An example is the huge catalysing effect the Hungarian competitions had on creating generations of leading mathematicians there through the 20th century. Colombia, for example, provides another nice example. Having attended an impressive Colombian Mathematical Society meeting in Bogota, I observed that the former Olympians now comprise a significant proportion of Colombia's research community (not all based at home) and it is debatable if this would happen without the Olympiads.

The other background factor could be described as related to the increased definition of school syllabi. Over the last decades material taught in schools generally, and certainly in Western countries, has been increasingly defined by syllabi. Assessment is also strictly defined to the syllabi. The outcome is that assessment tasks can become more and more predictable. Classroom tests also will be testing material which is fresh in the mind of students as having recently been taught them in the classroom.

Anecdotally, I am increasingly becoming aware of students who receive the top scores in the classroom and who cannot solve problems which I set them in mathematical circles. Also, on the other hand, I see students who do solve problems in the mathematical circles, but do not have the same classroom esteem. In saying this, there is still obviously a high correlation between the students who perform well in each environment.

One might ask which environment is more meaningful? Whereas there are reasons why the classroom is a necessary part of the student's life in any case, there is a compelling case for the student to be exposed to challenging situations, and competitions and similar activities are there able to provide this challenge.

It can be well argued in support of this that it is important to be exposed to the unforeseen situations which competitions and other challenges provide, because in real life workers have fresh challenges each day.

So there is a case as such for these challenges, which traditionally exist outside the classroom. But if the challenges outside the classroom have a role why should not the teacher find ways of teaching inside the classroom in a way which exposes students to challenge?

So this is essentially what the study is about, to explore what value there is in exposing student to challenge, and if the value is seen to be positive, to explore ways of providing it.

A secondary mathematical issue also arises here. In mathematics classes, particularly in the Western classes with which I am most familiar, there is less time to teach mathematics. As a result the syllabus can be contracted to mathematical skills only, not using the skills in various ways to solve problems in everyday life. It is difficult for the teacher to find time to teach problem solving, which is in itself in any case a very broad subject. External challenges such as competitions do tend to expose the student to problem solving as part of the challenge process, and this augments their role from just being another test.

A final note on the background is that this is not one of the most commonly trod path in mathematical education. Many of the ICMI Studies have focused on subjects such as teacher training and other topics on which there are many experts. However this is one of those topics on which there are not so many experts, particularly from in the education community, and very little has been published. As a result this has proved to be one of the smaller ICMI Studies in terms of participation. But this might result in an impact larger than one on one of the more extensively researched topics.

Tying in with education and lack of published material: joining of cultures

The last point above has produced an interesting mix of competition people and those from general education disciplines interested in the topic. When the Study was first announced I expected that many WFNMC people would submit proposals and there was a danger of them dominating the Study. Indeed the IPC contains some WFNMC but was carefully chosen to include people definitely outside the sphere, even for outside the classroom activities. Areas here included exhibitions, mathematical museums and project work.

As it eventuated probably about two thirds of the participants were well outside the WFNMC orbit and almost all of these had interests in the mainstreams of education, particularly mathematics education.

The coming together of the two groups proved a useful blend of different academic cultures and it seemed to work well. However one of the outcomes has been, even among the "educationists" difficulty in finding sufficient reference material, particularly in some chapters of what will be the Study.

When the Study was announced I expected that two thirds of the chapters would be on "Beyond the Classroom" with one third inside. As it happens these ratios were reversed. So the Study is actually going to be reported in about three equal parts, with the first third on "Beyond" and the other two thirds split about equally on the student and on the teacher perspective. These will be described in more detail below.

The definition of Challenge

I should note that the Study has a web site

www.amt.edu.au/icmis16.html

to which I will refer the reader for matters of detail. The site has some components secured for the use of participants writing the material (which is now substantially complete). It does contain certain formal details such as the names of IPC members and participants, and it does contain a publication of all submissions which were accepted after a refereeing process.

The web site also contains the discussion document, in four languages. In Chapter 2 we describe one of the most fundamental questions we first faced: What is Challenge? As this is a fundamental term in this paper I will quote here from the discussion document (Section 2).

"One answer is that a challenge occurs when people are faced with a problem whose resolution is not apparent and for which there seems to be no standard method of solution. So they are required to engage in some kind of reflection and analysis of the situation, possibly putting together diverse factors. Those meeting challenges have to take the initiative and respond to unforeseen eventualities with flexibility and imagination.

"Note that the word 'challenge' denotes a relationship between a question or situation and an individual or a group. Finding the dimensions of a rectangle of given perimeter with greatest area is not a challenge for one familiar with the algorithms of the calculus, or with certain inequalities. But it is a challenge for a student who has come upon such a situation for the first time. A challenge has to be calibrated so that the audience is initially puzzled by it but has the resources to see it through. The analysis of a challenging situation may not necessarily be difficult, but it must be interesting and engaging.

"We have some evidence that the process of bringing structure to a challenge situation can lead one to develop new, more powerful solution methods. One may or may not succeed in meeting a challenge, but the very process of grappling with its difficulties can result in fuller understanding. The presentation of mathematical challenges may provide the opportunity to experience independent discovery, through which one can acquire new insights and a sense of personal power. Thus, teaching through challenges can increase the level of the student's understanding of and engagement with mathematics.

"We do note that there are several terms used to sometimes describe similar things, but which really have quite distinct meanings. These terms include the expressions 'challenge', 'problem solving' and 'enrichment'. We have discussed the term `challenge' above. Problem solving would appear to refer to methodology, but problem solving is often associated with a challenging situation. Enrichment would be the process of extending one's mathematical experience beyond the curriculum. This might or might not happen in a challenging context."

The above discussion certainly helped clarify matters for the IPC, and it seemed to survive the Study Conference, so it seems the best way of defining various related terms.

The Participants

The members of the IPC and the participants can be found on the web site above. Not all those who had their papers accepted were finally able to attend. For a variety of reasons, usually personal, a small number had to withdraw at late notice in a couple of cases. There were a couple of IPC members also who did not attend the Study Conference. The best measure of final practical participation is to be gained by looking at the author list, which is not at time of writing publicly listed on the web site but I will give below. I would particularly like to acknowledge IPC members Petar Kenderov (Bulgaria), who chaired the "Beyond the Classroom" group, Mariolina Bartolini Busi (Italy) who with Mark Saul (USA) chaired the Classroom student perspective group and Derek Holton (NZ) who chaired the classroom teacher group.

The IPC also invited two distinguished mathematics educators Jean-Pierre Kahane (France) and Alexei Sossinsky (Russia) as Plenary Lecturers and these both participated very strongly throughout the conference and in the writing stages. Professor Kahane, a former ICMI President for many years who had over-seen many of the Studies from an Executive point of view, noted that this was the first one he had really attended as a participant and he seems to have very much enjoyed his experience.

Broad division into three groups

As indicated above, the Study Volume will be divided into three broad sections. The first will focus on "Beyond the Classroom" perspectives. The second will look at the classroom experience from the student point of view, while the third will look at the classroom experience from the teacher point of view.

The participants divided into three groups and each was expected to divide into three subgroups for writing on a separate theme, although the student classroom group decided that there were two, rather than three natural divisions, both with a strong psychological factor.

As a result the Study Volume will have a significant background introduction and eight chapters. I now briefly outline the chapters (some names are not final at the time of writing this, although they should be by the time of this conference).

Chapter 1: Challenging Problems: Mathematical Content and Sources. As the name of the chapter indicates, this is probably the most mathematical of the chapters. It describes the type of mathematics which is used in challenges, how it relates to classroom material, where problems are sourced or how they are composed, and there is a discussion on context in which problems are set. Authors: Romualdas Kasuba (Lithuania), Mark Applebaum (Israel), Alexei Sossinsky (Russia), Vladimir Protasov (Russia), Alexander Karp (USA), Ed Barbeau (Canada), Peter Taylor (Australia).

Chapter 2: Presentation of Challenges beyond the Classroom – Organisational Issues. This particular chapter surveys the existence of many particular types of challenges from around the world, discusses the value and special features of each type and gives a very large number of examples which indicate the wide variety of types of challenge which successfully

operate around the world. Authors: Petar Kenderov (Bulgaria), Ali Rejali (Iran), Valerie Pandelieva (Canada), Djordje Kadijević (Serbia), Karin Richter (Germany), Mariolna Bartolini Bussi (Italy), Peter Taylor (Australia).

Chapter 3: Challenging mathematics beyond the classroom enhanced by technological environments. As the title suggests this chapter describes ways in which various forms of technology, including the internet, are used to provide challenge in mathematics. The chapter also addresses the issue of training teachers to use the technology. Authors: Viktor Freiman (Canada), Djordje Kadijević (Serbia), Gerard Kuntz (France), Sergey Pozdnyakov (Russia), Ingvill Stedoy (Norway).

Chapter 4: Challenging Tasks and Mathematics Learning. This look at the role of the student in the classroom environment focuses on challenging problems. Authors: Arthur B. Powell (USA), Inger Christin Borge (Norway), Gema Inés Fioriti (Argentina), Margo Kondratieva (Canada), Elena Koublanova (USA), and Neela Sukthankar (Canada).

Chapter 5: Mathematics in Context – Focusing on Students. This look at the role of the student in the classroom environment focuses on challenging environments. Authors: Maria G. Bartolini Bussi (Italy), Sharada Gade (India), Martine Janvier (France), Jean-Pierre Kahane (France), Vince Matsko (USA), Michela Maschietto (Italy), Cécile Ouvrier-Buffet (France), Mark Saul (USA).

Chapter 6: Teacher Development and Mathematical Challenge. This looks at the issues around teacher professional development as they relate to teaching using mathematical challenges. It looks at what mathematics is; discusses why challenging mathematics problems are important in school classrooms; gives some examples of problems that can provide a challenge in a classroom situation; and suggests some barriers that might inhibit the use of challenging problems. It also looks at the mathematics education research that is relevant to the theme of the chapter, and follows with effective pedagogy and teacher preparation, which includes both theoretical and practical aspects. Authors: Derek Holton (NZ), Kwok-Cheung Cheung (Macau), Sesutho Kesianye (Botswana), Maria de Losada (Colombia), Roza Leikin (Israel), Gregory Makrides (Cyprus), Hartwig Meissner (Germany), Linda Sheffield (USA), Ban Har Yeap (Singapore).

Chapter 7: Classroom Practice – Challenging Mathematics Classroom Practices. In this chapter are discussed issues such as designing challenging mathematics for classrooms, designing classrooms for challenging mathematics and research issues are also addressed. Authors: Gloria Stillman (Australia), Cheung Kwok-cheung (Macau), Ralph Mason (Canada), Linda Sheffield (USA), Kenji Ueno (Japan).

Chapter 8: Curriculum and Assessment that provide Challenge in Mathematics. Here selected case studies of assessment that provide challenge in mathematics are used to frame a discussion of assessment issues in relation to the provision of mathematical challenge. In addition, the relationship between the conception of the role of assessment, the features of assessment tasks and the provision of challenge, as well as how this relationship may affect issues of curriculum and may vary under different conditions, are discussed. Possible research questions in this area are included in the final part of the chapter. Authors: Ban-Har Yeap (Singapore), Maria Falk de Losada (Colombia), Gunnar Gjone (Norway), Mohammad Hossein Pourkazemi (Iran).

Conclusion: Possible effects of the study

Obviously we are still on the verge of publishing the Study Volume, and so we have so far no formal reaction from the wider community. However the Study Volume will most significantly highlight challenge, in whatever form it takes, as a field of interest for mathematics education researchers as a vast relatively unknown or understood field which potentially has considerable potential, properly used, to enhance the learning process, which ultimately the main outcome to achieve.

Irrespective of the content of the main outcome of the Study, the Study Volume, the Study Volume will act as one of the main references on this topic for many years to come. However, even though the Study has involved a relatively small number of participants, it has been an interesting cross-section of people who have blended well, worked cohesively and diligently to prepare as good and broad a contribution to the Study as could be anticipated.

I am confident that the outcome will be something that many people should read and will want to read.