

A CURRICULUM FOR MATHEMATICS CONTESTS

Samuel Greitzer



Professor Samuel Greitzer's experience as a teacher of talented and gifted students extends from 1940 to the present. He initiated the USA Mathematical Olympiad in 1972 and took a team of students to the International Mathematical Olympiad in 1974.

It is our contention that the average mathematics curriculum is constructed for average students. This is not enough for the talented and gifted.

Now both can profit from rapid advancement - the talented student with an excellent memory, and the gifted student needs more, and few schools make provision for this.

Naturally, both types should have algebra, geometry and trigonometry. The gifted student should have more of each - say, the theory of equations, Newton's formulas and the like in algebra, such tidbits as Ptolemy's theorem, the theorem of Ceva, Menelaus, Pappus, Desargues, Pascal, Brianchon and Stewart, as well as some solid geometry, including Euler's formula, the Prismatoid formula, and rules for finding areas of volumes (Pappus-Guldin), as well as more than the smattering that is presently called Trigonometry.

We would also expect a curriculum for gifted students to include complex numbers and vectors, at least to triple vector products and some at least of the standard inequalities.

It has been our experience that the gifted student can master so much in less time than is spent on average courses for average students. In fact he or she can do much more in the same time. Number theory through Fermat's and Euler's theorems, as well as Diophantine Equations, appear to need mere introduction to be understood and used.

We have been able to cover Finite Differences, Difference Equations and Functional Equations, Combinatorics and Probability theory.

A good deal of this has been presented out of class. Some is acquired in a mathematics club, which permits interaction among members. More is acquired in outside reading. In connection with this, we are fortunate in having a series, called the Dolciani set, edited by Professor Honsberger, which has seemingly difficult problems having short, ingenious solutions, some of which are actually beautiful - to a gifted student. Finally, we have a variety of contests, local state and interstate, which always draw many students into more intensive works in mathematics.

This brings me to a most important facet in the development of a workable curriculum - the school and the teacher. The school must be willing to allocate facilities for one or more special classes for the gifted. The teacher must be well prepared, willing and able to motivate and encourage students.

This last is most difficult to supply. Nowadays, we have much discussion about teaching problem solving. This cannot be done. If a student is merely talented, nothing will teach him problem solving, since this is not a matter merely of memory. The ability of the gifted student

can be improved by contact with ingenious problems, because his ingenuity may enable him to apply a method from one problem to another with an entirely different context.

We have differentiated between the talented student and the gifted one. We supply an example to illustrate this difference.

Solve the Diophantine equations:

$$a^3 - b^3 - c^3 = 3abc \quad a, b, c > 0$$

$$a^2 = 2(b + c)$$

The talented student will eliminate one letter and may solve the problem after one or more hours of complicated algebra.

The gifted student will proceed as follows:

$$(1) \ a \text{ is even} \quad (2) \ a > b \text{ and } a > c \quad (3) \ 2a > (b + c)$$

$$(4) \ 4a > 2(b + c) \quad (5) \ 4a > a^2 \quad (6) \ 4 > a$$

$$(7) \ a = 2, \ b = 1, \ c = 1.$$

We have seen this happen.

Finally, there are periodicals for gifted students. Many nations have these and they can be made available. They provide good problems and are a means for gifted students to interact. Among these are *Crux Mathematicorum* (Canada), *Kvant* (USSR), *KoMal* (Hungary), *Mathematical Digest* (South Africa), and many others.

In summary, a curriculum for talented and gifted students consists of the average curriculum enhanced by a great many extra-curricular activities including reading, problem solving, club activities and participation, with other students in local and national contests.

*Samuel L. Greitzer,
Mathematics Department,
Rutgers University,
New Brunswick, N.J. 08903
U.S.A.*

* * *