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MATHEMATICS COMPETITIONS

JOURNAL OF THE
WORLD FEDERATION OF NATIONAL MATHEMATICS COMPETITIONS

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The aims of the Federation are:–

1. ***to promote excellence in, and research associated with, mathematics education through the use of school mathematics competitions;***
2. ***to promote meetings and conferences where persons interested in mathematics contests can exchange and develop ideas for use in their countries;***
3. ***to provide opportunities for the exchanging of information for mathematics education through published material, notably through the Journal of the Federation;***
4. ***to recognize through the WFNMC Awards system persons who have made notable contributions to mathematics education through mathematical challenge around the world;***
5. ***to organize assistance provided by countries with developed systems for competitions for countries attempting to develop competitions;***
6. ***to promote mathematics and to encourage young mathematicians.***

From the President

It is a great pleasure to announce that the next recipients of the Paul Erdös Award are now identified. The Awards Committee guided by Prof. Peter Taylor collected and assessed the nominations and recommended the candidates to the Executive Committee of WFNMC. The Executive Committee of WFNMC approved the candidates and the Paul Erdös Awards for 2006 will go to (given in alphabetic order):

Simon Chua (Philippines)
Ali Rejali (Iran)
Alexander Soifer (USA)

Congratulations to our colleagues for their outstanding achievements and well deserved Award!

Let me remind you that the Paul Erdös Award was established to recognise contributions of persons which have played a significant role in the development of mathematical challenges with essential impact on mathematics learning. The awarded persons completely (and with abundance!) satisfy the requirements. What follows is a brief (and very incomplete!) description of the main contributions of our colleagues. It reflects partially the report of the Awards Committee.

Alexander Soifer is well-known to WFNMC community. His engagement with mathematics competitions is strong and decades old. It started with his school years in Russia when he participated successfully in the Russian mathematics competitions and culminated in the Colorado Springs Mathematical Olympiad which he founded in 1984 and has guided ever since. The Colorado Mathematical Olympiad is an essay-type mathematical competition (rather unique for USA) in which 600 to 1,000 participants compete annually for different prizes.

Alexander Soifer authored several books and many articles devoted to problem solving. He is well-known for his contributions to Geometry and Combinatorics. In a recent list of top 25 articles, most frequently downloaded from the *Journal of Combinatorial Theory*, Series A, there are three papers where Alexander Soifer is an author or co-author. They are ranked first, third and twenty first! He also has a considerable lecturing record at the Federations meetings over the years. Last, but not least, his service as a Secretary of the Executive Committee contributed significantly to the well-being of WFNMC.

Ali Rejali is also well-known to members of WFNMC. He is the founder of the Iranian national mathematics competitions and has a very strong record of establishing enrichment activities in his country including the mathematics houses which he has established in difficult and unusual conditions. He has had considerable influence in setting the scene for the national mathematics syllabus in mathematics and statistics via lectures at national conferences. He has also had considerable influence in supporting teachers. Ali Rejali is well-known for his efforts on an international level. He contributes to the work of several organizations with regional and international importance and takes part in projects aimed at providing more challenging mathematics in classroom and beyond the school.

Simon Chua is a principal of a school in Zamboanga, the southern province of Mindanao. In difficult conditions he has established the Mathematics Trainers Guild of the Philippines (MTG) which organizes the identification of talented students and trains them for events such as international competitions. Simon still takes a leading role in these activities and the MTG celebrated its 10^{th} anniversary in October 2005. Simon is an academic contributor, composing problems and working on small juries in events such as the International Junior Olympiad, to which he takes teams from the Philippines.

More comprehensive biographies of the Erdös winners you can find at:
http://www.amt.edu.au/wfnmcann06.html

The official ceremony with the presentation of the awards will take place during the fifth Conference of WFNMC in Cambridge, 22–28 July 2006.

This reminds me once again that it is time to collect bids for hosting the sixth Conference of WFNMC which is to take place in 2010. The decision regarding where the next Conference of WFNMC will be held has to be taken during our meeting in Cambridge in July 2006. Those interested in organizing and hosting the event are encouraged to contact me as soon as possible so that there is enough time to discuss the matter in more detail. The easiest way to contact me is:

via e-mail:
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by ordinary mail:
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Petar S. Kenderov
President of WFNMC
November, 2005, Sofia, Bulgaria

From the Editor

Welcome to *Mathematics Competitions* Vol. 18, No 2.

I would like to thank again the Australian Mathematics Trust for continued support, without which each issue of the journal could not be published, and in particular Heather Sommariva, Bernadette Webster and Pavel Calábek for their assistance in the preparation of this issue.

Submission of articles:
The journal *Mathematics Competitions* is interested in receiving articles dealing with mathematics competitions, not only at national and international level, but also at regional and primary school level. There are many readers in different countries interested in these different levels of competitions.

- The journal traditionally contains many different kinds of articles, including reports, analyses of competition problems and the presentation of interesting mathematics arising from competition problems. Potential authors are encouraged to submit articles of all kinds.
- To maintain and improve the quality of the journal and its usefulness to those involved in mathematics competitions, all articles are subject to review and comment by one or more competent referees. The precise criteria used will depend on the type of article, but can be summarised by saying that an article accepted must be correct and appropriate, the content accurate and interesting, and, where the focus is mathematical, the mathematics fresh and well presented. This editorial and refereeing process is designed to help improve those articles which deserve to be published.

At the outset, the most important thing is that if you have anything to contribute on any aspect of mathematics competitions at any level, local, regional or national, we would welcome your contribution.

Articles should be submitted in English, with a black and white photograph and a short profile of the author. Alternatively, the article can be submitted on an IBM PC compatible disk or a Macintosh disk. We prefere LaTeX or TeX format of contributions, but any text file will be helpful.

Articles, and correspondence, can also be forwarded to the editor by mail to

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Jaroslav Švrček,
June 2005

46$^{\text{th}}$ International Mathematical Olympiad 8–19 July 2005 Mérida, Mexico

The 46$^{\text{th}}$ International Mathematical Olympiad (IMO) was held from 8–19 July in the city of Mérida on the Yucatán Peninsula, Mexico. This was the third time a Latin American country has hosted the IMO, the other occasions being in Cuba (1987) and Argentina (1997). Teams from 91 countries making up a total of 513 contestants participated in this event making it the largest IMO to date.

Before the contestants arrived, their Team Leaders were already busy putting together the exam paper in the paradisaic surroundings of 'Hotel A'. There is almost a feeling of espionage the way that secrecy surrounds the location of the Leaders before the competition. There is good reason for this. It helps preserve the security of the paper. As it turned out Hotel A was the Reef Yucatán in Tel Chuc, a five star hotel right on the shores of the Gulf of Mexico. Within the confines of the hotel there were palm trees and swimming pools everywhere. Outside was nothing except wetlands.

Prior to the Leaders' arrival over 100 problem proposals had been sent from all around the world to the Problem Selection Committee in Mexico. This list had been reduced to a shortlist of 27 problems considered highly suitable as candidate problems for the IMO papers. The task of the Team Leaders (collectively called the 'Jury') is to choose six problems from this list which form the two $4\frac{1}{2}$ hour long exam papers, each with three questions. Nowadays it seems that there is an unofficial procedure that is followed. Firstly, the Jury members try the problems without solutions. Then they are given solutions to study. This is followed by some discussion of the merits of various problems in the Jury meetings. Finally, there are a number of rounds of voting. The least popular problem combinations are removed and this culminates in an exam paper. In the final product for this year there were two problems each from geometry and number theory and one problem each from algebra and combinatorics. Two of the problems came from Poland as did two from Romania, while a further one each came from Korea and The Netherlands respectively. While most of the Jury's discussion is carried out in English, at various important stages, points for consideration are translated into French, Russian and Spanish. This also carries over in an extended way onto the exam papers where the papers are translated into the languages needed by the contestants. At this stage marking schemes are also discussed.

The first event for the contestants after settling in was the opening ceremony. Welcoming speeches were made by the President of the 2005 IMO Organizing Committee, Radmila Bulajich Manfrino and by the Chairman of the IMO Advisory Board, Jozsef Pelikan. Both spoke about the passion and challenge for studying interesting and tough problems in mathematics as a reason for being at the IMO. However not to be overlooked was also the opportunity to meet like minded young people, to experience some of the Mexican culture and to make new friends. The Mayor of Mérida and the Governor of Yucatán also welcomed the contestants and their Leaders, after which there was a parade of the teams. The very next day the contestants took the first day's exam.

After the exams the Leaders and their Deputies assess the work of the students from their own countries. However sometimes it is easy to view the work of one's own students with rose coloured glasses. This is where the Coordinators take their place. They are the standardizers of the IMO. Using a marking scheme they endeavour, with some flexibility, to make sure that the marks awarded to contestants for their work are fair and consistent. Perhaps another highlight for some of the Leaders during their time at the Reef Yucatán hotel were the soccer matches between the Leaders and the Coordinators. With more Leaders than coordinators, the Leaders' team always had fresh legs and of course they won on the soccer field. When it came to coordination, however, I am pleased to say that despite the Coordinators being outnumbered, it was my experience and that of many other Team Leaders that the coordination process was of high quality. There was a feeling of security that the marks were indeed fair and consistent.

The algebra problem, a classical inequality, was slightly unfortunate this year because it had been chosen as the difficult problem on the first day's paper. However, it turned out to have not only the elegant solution of the proposer, but also a rather tedious mechanical solution. Nonetheless there was an unexpected turn of events because one of the contestants managed to find a solution better than any of the Leaders' solutions. This student, from Moldova, who also scored a perfect score for the IMO, was awarded a special prize. The last special prize was awarded at the 1995 IMO in Canada.

There were 248 medals awarded—the distributions being 127 bronze, 79 silver and 42 gold. Of those who did not get a medal, a further 65 contestants received an honourable mention for solving one question perfectly. The remarkable feat of a perfect score was achieved by 16 contestants.

It was during the coordination process that it became clear that this IMO was going to be memorable for an unusual reason. It was going to be 'the IMO that had a hurricane'! Indeed Hurricane 'Emily' which was bordering on category five had passed through Jamaica and had the Yucatán peninsula in its sights. The IMO had originally been planned for the city of Cancun. Fortunately for us this was changed as elections were scheduled there at the same time. Emily hit Cancun quite severely, damaging the airport and leaving many stranded there for some days. It then marched towards us in Mérida. For this reason the trip to the Mayan archaeological site of Chichén Itzá had to be shortened. Although causing anxiety for the adults, many of the contestants thought it was exciting. Indeed it was somewhat like a pyjama party in that many of us had to be crowded into a safe-room for the night while Emily passed by. Thankfully, by the time Emily neared Mérida she had weakened somewhat and her eye deviated around us. Nevertheless the closing banquet had to be cancelled. However when it was clear that Emily had sufficiently passed by there was time for a last minute closing ceremony to award the medals.

Our thanks for this highly successful IMO go to the IMO 2005 Mexico Organizing Committee along with Olimpiada Mexicana de Matemáticas, Sociedad Matemática Mexicana and the many institutions who sponsored the event.

Next year the IMO will be held in Ljubljana, Slovenia.

1 IMO Paper

First Day

1. Six points are chosen on the sides of an equilateral triangle ABC: A_1, A_2 on BC; B_1, B_2 on AC; and C_1, C_2 on AB. These points are the vertices of a convex hexagon $A_1A_2B_1B_2C_1C_2$ with equal side lengths.

Prove that the lines A_1B_2, B_1C_2 and C_1A_2 are concurrent.

2. Let $a_1, a_2, \ldots$ be a sequence of integers with infinitely many positive terms and infinitely many negative terms. Suppose that for each positive integer n, the numbers $a_1, a_2, \ldots, a_n$ leave different remainders on division by n.

Prove that each integer occurs exactly once in the sequence.

3. Let x, y and z be positive real numbers such that $xyz \geq 1$. Prove that

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} + \frac{y^5 - y^2}{y^5 + z^2 + x^2} + \frac{z^5 - z^2}{z^5 + x^2 + y^2} \geq 0.$$

Second Day

4. Consider the sequence $a_1, a_2, \ldots$ defined by

$$a_n = 2^n + 3^n + 6^n - 1 \qquad (n = 1, 2, \ldots).$$

Determine all positive integers that are relatively prime to every term of the sequence.

5. Let $ABCD$ be a given convex quadrilateral with sides BC and AD equal in length and not parallel. Let points E and F lie on the sides BC and AD respectively and satisfy $BE = DF$. The lines AC and BD meet at P, the lines BD and EF meet at Q, the lines EF and AC meet at R. Consider all the triangles PQR as E and F vary.

Show that the circumcircles of these triangles have a common point other than P.

6. In a mathematical competition 6 problems were posed to the contestants. Each pair of problems was solved by more that $\frac{2}{5}$ of the contestants. Nobody solved all 6 problems.

Show that there were at least 2 contestants who each solved exactly 5 problems.

2 Statistics

Some Country Scores

Rank	Country	Score
1	China	235
2	United States	213
3	Russia	212
4	Iran	201
5	Korea	200
6	Romania	191
7	Taiwan	190
8	Japan	188
9/10	Hungary	181
9/10	Ukraine	181
11	Bulgaria	173
12	Germany	163
13	United Kingdom	159
14	Singapore	145
15	Vietnam	143

Some Country Scores

Rank	Country	Score
16	Czech Republic	139
17	Hong Kong	138
18	Belarus	136
19	Canada	132
20	Slovakia	131
21/22	Rep. of Moldova	130
21/22	Turkey	130
23	Thailand	128
24	Italy	120
25	Australia	117
26	Kazakhstan	112
27/28	Colombia	105
27/28	Poland	105
29	Peru	104
30	Israel	99

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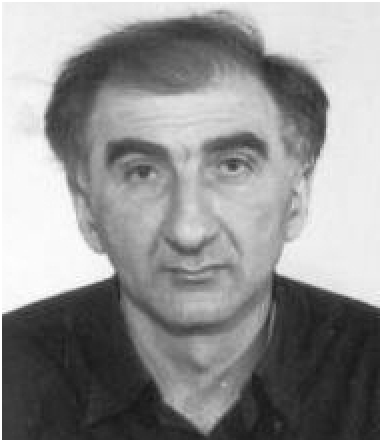
Around an Inequality from the 46th IMO

Nairi M. Sedrakyan



Nairi Sedrakyan teaches mathematics in the Shahinyan High School of Physics and Mathematics in Yerevan, Armenia. From 1986 he has been a jury member of the Armenian Mathematical Olympiads. From 1997 he has been Leader and Deputy leader of the Armenian team at IMO. He is author of books Inequalities: Methods of Proving, *Fizmatlit, Moscow, 2002, 256p;* Geometrical Inequalities, *Edit Print, Yerevan, 2004 (both in Russian) 364p.*

The problem No. 3 proposed at the 46th IMO was to prove the following inequality

$$\frac{x^5-x^2}{x^5+y^2+z^2}+\frac{y^5-y^2}{y^5+x^2+z^2}+\frac{z^5-z^2}{z^5+x^2+y^2}\geq 0, \tag{1}$$

where x, y, z are positive numbers and $xyz \geq 1$.

In this article a solution will be presented which differs from the author's solution which can be found in the Short-listed problems of the 46th IMO.

Note that

$$\frac{x^5-x^2}{x^5+y^2+z^2}=\frac{x^2-\frac{1}{x}}{x^2+\frac{1}{x^3}\left(y^2+z^2\right)}\geq\frac{x^2-\frac{1}{x}}{x^2+y^2+z^2},$$

hence

$$\frac{x^5-x^2}{x^5+y^2+z^2}+\frac{y^5-y^2}{y^5+x^2+z^2}+\frac{z^5-z^2}{z^5+x^2+y^2}$$
$$\geq\frac{x^2+y^2+z^2-\frac{1}{x}-\frac{1}{y}-\frac{1}{z}}{x^2+y^2+z^2}\geq\frac{x^2+y^2+z^2-xy-yz-zx}{x^2+y^2+z^2}\geq 0.$$

Hence the inequality (1) is correct.

This solution allows to make the following generalization. Given

$$x, y, z > 0 \qquad \text{and} \qquad xyz \geq 1$$

prove the following inequalities:

a) $$\frac{x^\alpha - x^2}{x^\alpha + y^2 + z^2} + \frac{y^\alpha - y^2}{y^\alpha + x^2 + z^2} + \frac{z^\alpha - z^2}{z^\alpha + x^2 + y^2} \geq 0, \tag{2}$$

where $2 \leq \alpha \leq 5$.

b) $$\frac{x^\alpha - x^2}{x^\alpha + y^2 + z^2} + \frac{y^\alpha - y^2}{y^\alpha + x^2 + z^2} + \frac{z^\alpha - z^2}{z^\alpha + x^2 + y^2} \leq 0, \tag{3}$$

where $-1 \leq \alpha \leq 2$.

Proof.

a) If $\alpha \geq 2$, then

$$\frac{x^\alpha - x^2}{x^\alpha + y^2 + z^2} = \frac{x^2 - x^{4-\alpha}}{x^2 + \frac{1}{x^{\alpha-2}}\left(y^2 + z^2\right)} \geq \frac{x^2 - x^{4-\alpha}}{x^2 + y^2 + z^2}.$$

Now we will prove, that if $2 \leq \alpha \leq 5$, then $x^2 + y^2 + z^2 \geq x^{4-\alpha} + y^{4-\alpha} + {} + z^{4-\alpha}$, from which we will find that the inequality (2) holds:

Indeed, for $2 \leq \alpha \leq 4$, we have that $x^{4-\alpha}\left(x^{\alpha-2} - 1\right) \geq x^{\alpha-2} - 1$ hence

$$\begin{aligned} x^2 - x^{4-\alpha} + y^2 - y^{4-\alpha} + z^2 - z^{4-\alpha} &\geq x^{\alpha-2} + y^{\alpha-2} + z^{\alpha-2} - 3 \\ &\geq 3\sqrt[3]{(xyz)^{\alpha-2}} - 3 \geq 3 - 3 = 0. \end{aligned}$$

If $4 < \alpha \leq 5$ we have that

$$\begin{aligned} x^{4-\alpha} + y^{4-\alpha} + z^{4-\alpha} &= \frac{1}{x^{\alpha-4}} + \frac{1}{y^{\alpha-4}} + \frac{1}{z^{\alpha-4}} \\ &\leq (yz)^{\alpha-4} + (xz)^{\alpha-4} + (xy)^{\alpha-4} \\ &\leq \frac{y^{2(\alpha-4)} + z^{2(\alpha-4)}}{2} + \frac{x^{2(\alpha-4)} + z^{2(\alpha-4)}}{2} + \frac{x^{2(\alpha-4)} + y^{2(\alpha-4)}}{2} \\ &= x^{2(\alpha-4)} + y^{2(\alpha-4)} + z^{2(\alpha-4)}. \end{aligned}$$

On the other hand $x^2 - x^{2(\alpha-4)} = x^{2(\alpha-4)}\left(x^{10-2\alpha} - 1\right) \geq x^{10-2\alpha} - 1$, consequently

$$x^2 + y^2 + z^2 - x^{2(\alpha-4)} - y^{2(\alpha-4)} - z^{2(\alpha-4)}$$
$$\geq x^{10-2\alpha} + y^{10-2\alpha} z^{10-2\alpha} - 3 \geq 3\sqrt[3]{(xyz)^{10-2\alpha}} - 3 \geq 3 - 3 = 0.$$

Thus $x^2 + y^2 + z^2 \geq x^{2(\alpha-4)} + y^{2(\alpha-4)} + z^{2(\alpha-4)} \geq x^{4-\alpha} + y^{4-\alpha} + z^{4-\alpha}$ and hence the point a) is proven.

b) Note that

$$\frac{x^\alpha - x^2}{x^\alpha + y^2 + z^2} \leq \frac{x^\alpha - x^2}{x^2 + y^2 + z^2},$$

hence

$$\frac{x^\alpha - x^2}{x^\alpha + y^2 + z^2} + \frac{y^\alpha - y^2}{y^\alpha + x^2 + z^2} + \frac{z^\alpha - z^2}{z^\alpha + x^2 + y^2}$$
$$\leq \frac{x^\alpha + y^\alpha + z^\alpha - x^2 - y^2 - z^2}{x^2 + y^2 + z^2}.$$

Now we will prove:

$$-1 \leq \alpha \leq 2, \qquad x^\alpha + y^\alpha + z^\alpha \leq x^2 + y^2 + z^2$$

from which will follow the proof of inequality (3).

Indeed, when $0 \leq \alpha \leq 2$ we have that

$$x^2 + y^2 + z^2 - x^\alpha - y^\alpha - z^\alpha$$
$$= x^\alpha(x^{2-\alpha} - 1) + y^\alpha(y^{2-\alpha} - 1) + z^\alpha(z^{2-\alpha} - 1)$$
$$\geq 1(x^{2-\alpha} - 1) + 1(y^{2-\alpha} - 1) + 1(z^{2-\alpha} - 1)$$
$$\geq 3\sqrt[3]{(xyz)^{2-\alpha}} - 3 \geq 3 - 3 = 0.$$

When $-1 \leq \alpha \leq 0$ we have that

$$x^\alpha + y^\alpha + z^\alpha = \frac{1}{x^{-\alpha}} + \frac{1}{y^{-\alpha}} + \frac{1}{z^{-\alpha}} \leq (yz)^{-\alpha} + (xz)^{-\alpha} + (xy)^{-\alpha}$$
$$\leq \frac{y^{-2\alpha} + z^{-2\alpha}}{2} + \frac{x^{-2\alpha} + z^{-2\alpha}}{2} + \frac{x^{-2\alpha} + y^{-2\alpha}}{2}$$
$$= x^{-2\alpha} + y^{-2\alpha} + z^{-2\alpha} \leq x^2 + y^2 + z^2$$

(see the proof in the case $0 \le \alpha \le 2$).

Thus the proof of b) is completed.

A natural question arises, whether the inequality (2) in the case $\alpha > 5$ (or the inequality (3) in the case $\alpha \le -1$) is still correct. Now we will prove in another way that the inequality (2) holds also in the case $2 \le \alpha \le 6$ (a similar approach was used by Titu Andreescu in the case $\alpha = 5$).

We introduce the following notation: $\beta = \frac{1}{2}\alpha$, $a = x^2$, $b = y^2$, $c = z^2$, then $a, b, c > 0$, $abc \ge 1$ and $1 \le \beta \le 3$.

Now we have to prove, that

$$\frac{a^\beta - a}{a^\beta + b + c} + \frac{b^\beta - b}{b^\beta + a + c} + \frac{c^\beta - c}{c^\beta + a + b} \ge 0. \tag{4}$$

Here we will make use of the following inequality:

$$\frac{a_1^p}{b_1^{p-1}} + \ldots + \frac{a_n^p}{b_n^{p-1}} \ge \frac{(a_1 + \ldots + a_n)^p}{(b_1 + \ldots + b_n)^{p-1}},$$

where $p \ge 1$ and $a_1, \ldots, a_n, b_1, \ldots, b_n$ are positive numbers.

The latter can be obtained by using Hölder's inequality:

$$\left(\sum_{i=1}^n a_i^p\right)^{\frac{1}{p}} \left(\sum_{i=1}^n b_i^q\right)^{\frac{1}{q}} \ge \sum_{i=1}^n a_i b_i,$$

for $a_1, \ldots, a_n, b_1, \ldots, b_n > 0$, $p, q > 0$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Indeed we have

$$\left(\sum_{i=1}^n \left(\frac{a_i}{b_i^{\frac{p-1}{p}}}\right)^p\right)^{\frac{1}{p}} \cdot \left(\sum_{i=1}^n \left(b_i^{\frac{p-1}{p}}\right)^{\frac{p}{p-1}}\right)^{\frac{p-1}{p}} \ge \sum_{i=1}^n a_i,$$

consequently

$$\left(\sum_{i=1}^n \frac{a_i^p}{b_i^{p-1}}\right) \cdot \left(\sum_{i=1}^n b_i\right)^{p-1} \ge \left(\sum_{i=1}^n a_i\right)^p,$$

from which we get

$$\sum_{i=1}^{n} \frac{a_i^p}{b_i^{p-1}} \geq \frac{\left(\sum\limits_{i=1}^{n} a_i\right)^p}{\left(\sum\limits_{i=1}^{n} b_i\right)^{p-1}}.$$

Note that

$$a^\beta + b + c = \frac{a^\beta}{1^{\beta-1}} + \frac{b^\beta}{b^{\beta-1}} + \frac{c^\beta}{c^{\beta-1}} \geq \frac{(a+b+c)^\beta}{(1+b+c)^{\beta-1}},$$

hence

$$\frac{1}{a^\beta + b + c} + \frac{1}{b^\beta + a + c} + \frac{1}{c^\beta + a + b}$$

$$\leq \frac{(1+b+c)^{\beta-1} + (1+a+c)^{\beta-1} + (1+a+b)^{\beta-1}}{(a+b+c)^\beta}.$$

Let us prove now, that if $0 \leq \gamma = \beta - 1 \leq 2$, then

$$(1+b+c)^\gamma + (1+a+c)^\gamma + (1+a+b)^\gamma \leq 3\,(a+b+c)^\gamma. \qquad (5)$$

First prove the inequality (5) in the case $\gamma = 2$. Indeed for $\gamma = 2$ the inequality (5) can be presented in the following form:

$$(a+b+c-2)^2 + 2\,(ab+bc+ca) \geq 7,$$

which is correct since

$$a+b+c \geq 3\sqrt[3]{abc} \geq 3 \qquad \text{and} \qquad ab+bc+ac \geq 3\sqrt[3]{(abc)^2} \geq 3.$$

When $0 \leq \gamma < 2$ then according to Jensen's inequality we have

$$(1+b+c)^\gamma + (1+a+c)^\gamma + (1+a+b)^\gamma$$
$$\leq \left((1+b+c)^2\right)^{\frac{\gamma}{2}} + \left((1+a+c)^2\right)^{\frac{\gamma}{2}} + \left((1+a+b)^2\right)^{\frac{\gamma}{2}}$$
$$\leq 3\left(\frac{(1+b+c)^2 + (1+a+c)^2 + (1+a+b)^2}{3}\right)^{\frac{\gamma}{2}}$$
$$\leq 3\left((a+b+c)^2\right)^{\frac{\gamma}{2}} = 3\,(a+b+c)^\gamma.$$

Thus using the inequality (5) we get

$$\begin{aligned}
&\frac{1}{a^{\beta}+b+c}+\frac{1}{b^{\beta}+a+c}+\frac{1}{c^{\beta}+a+b}\\
&\leq \frac{(1+b+c)^{\beta-1}+(1+a+c)^{\beta-1}+(1+a+b)^{\beta-1}}{(a+b+c)^{\beta}}\\
&\leq \frac{3\,(a+b+c)^{\beta-1}}{(a+b+c)^{\beta}}=\frac{3}{a+b+c},
\end{aligned}$$

consequently

$$\frac{1}{a^{\beta}+b+c}+\frac{1}{b^{\beta}+a+c}+\frac{1}{c^{\beta}+a+b}\leq\frac{3}{a+b+c},$$

which is just (4) written in another way.

I think that (2) is correct for all $\alpha \geq 2$. To end I will prove it for $\alpha = 8$. We must prove, that

$$\frac{1}{a^{4}+b+c}+\frac{1}{b^{4}+a+c}+\frac{1}{c^{4}+a+b}\leq\frac{3}{a+b+c},$$

where $a, b, c > 0$ and $abc \geq 1$.

We have

$$a^4+b+c=\frac{\left(a^2\right)^2}{1}+\frac{\left(b^2\right)^2}{b^3}+\frac{\left(c^2\right)^2}{c^3}\geq\frac{\left(a^2+b^2+c^2\right)^2}{1+b^3+c^3},$$

hence it will be sufficient to prove that

$$\left(3+2\left(a^3+b^3+c^3\right)\right)(a+b+c)\leq 3\left(a^2+b^2+c^2\right)^2.$$

Introduce the following notations $a+b+c=s$, $ab+bc+ac=p$. We have

$$\begin{aligned}
a^2+b^2+c^2&=(a+b+c)^2-2\,(ab+bc+ac)=s^2-2p,\\
a^3+b^3+c^3&=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)+3abc\\
&=s\left(s^2-3p\right)+3abc,
\end{aligned}$$

consequently we must prove that

$$\left(3+2\left(s^3-3sp+3abc\right)\right)s \le 3\left(s^2-2p\right)^2,$$
$$6sabc+3s+6s^2p \le s^4+12p^2,$$
$$2\left(p^2-3sabc\right)+\left(s^2-3p\right)^2+\left(p^2-3s\right) \ge 0.$$

The latter is true since

$$p^2=(ab+bc+ac)^2 \ge 3\,(ab\cdot bc+ab\cdot ac+bc\cdot ac)=3sabc.$$

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Tournament of Towns Corner

Andrei Storozhev

Andrei Storozhev is a Research Officer at the Australian Mathematics Trust. He gained his Ph.D. at Moscow State University specializing in combinatorial group theory. He is a member of the Australian Mathematics Competition Problems Committee, Australian Mathematics Olympiad Committee and one of the editors of Mathematics Contests—The Australian Scene *journal.*

1 Selected Problems from the Second Round of Tournament 26

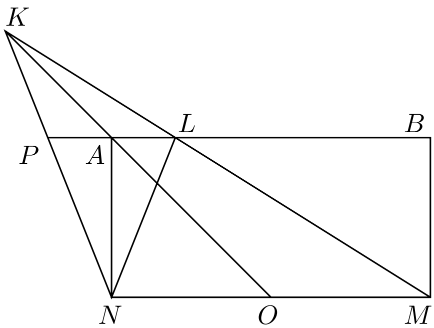
In the second round of Tournament 26, students were challenged with five problems in each of Junior and Senior O Level papers, and with seven problems in each of Junior and Senior A Level papers. Below are selected questions with solutions from this round.

1. Prove that one of the digits 1, 2 and 9 must appear in the decimal representation of n or $3n$ for any positive integer n.

Solution. If the leading digit of n is 1, 2 or 9, there is nothing to prove. If it is 3, then the leading digit of $3n$ is either 9 or 1. If the leading digit of n is 4 or 5, the leading digit of $3n$ will be 1. If it is 6, then the leading digit of $3n$ is either 1 or 2. If the leading digit of n is 7 or 8, the leading digit of $3n$ will be 2. All cases have been covered, and the desired conclusion follows.

2. M and N are the midpoints of sides BC and AD, respectively, of a square $ABCD$. K is an arbitrary point on the extension of the diagonal AC beyond A. The segment KM intersects the side AB at some point L. Prove that $\angle KNA = \angle LNA$.

Solution. Let AC cut MN at O, and extend BA to cut KN at P. Since PL is parallel to NM and O is the midpoint of NM, A is the midpoint of PL. Hence triangles PAN and LAN are congruent to each other, so that $\angle KNA = \angle LNA$.



3. The decimal representations of all the positive integers are written on an infinite ribbon without spacing: 123456789101112.... Then the ribbon is cut up into strips seven digits long. Prove that any seven digit integer will:

(a) appear on at least one of the strips;

(b) appear on an infinite number of strips.

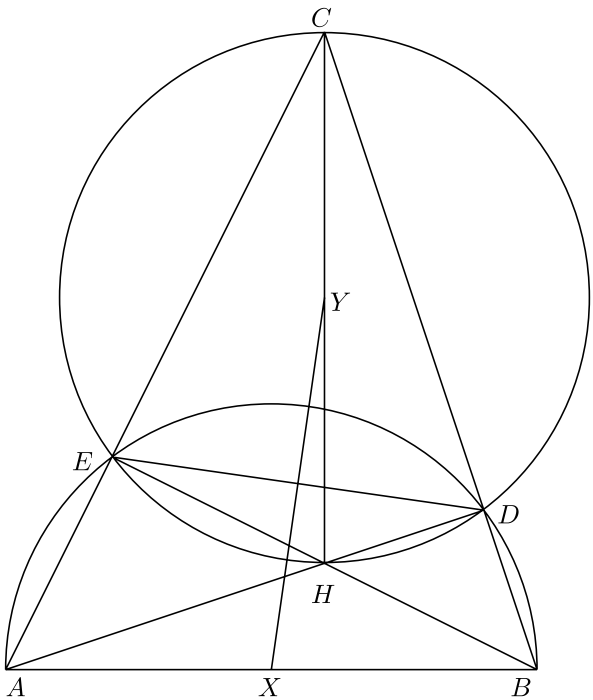
Solution.

(a) Suppose n is a seven-digit number. Consider the seven consecutive eight-digit numbers $10n, 10n + 1, \ldots, 10n + 6$. Since 7 and 8 are relatively prime, some strip will start with one of these numbers and n appears on it.

(b) As in (a), we can consider the seven consecutive nine-digit numbers $100n, 100n + 1, \ldots, 100n + 6$, the seven consecutive ten-digit numbers $1000n, 1000n + 1, \ldots, 1000n + 6$, and so on. For each number of digits not divisible by 7, we get a strip on which n appears.

4. The altitudes AD and BE of triangle ABC meet at its orthocentre H. The midpoints of AB and CH are X and Y, respectively. Prove that XY is perpendicular to DE.

Solution. Since $\angle ADB = 90° = \angle AEB$, D and E lie on a circle with diameter AB, and hence with centre X. Since $\angle CDH = 90° = \angle CEH$, D and E lie on a circle with diameter CH, and hence with centre Y. The common chord DE of the two circles is therefore perpendicular to the line of centres XY.



5. The sum of several positive numbers is equal to 10, and the sum of their squares is greater than 20. Prove that the sum of the cubes of these numbers is greater than 40.

Solution. Suppose $a_1+a_2+\ldots+a_n = 10$ and $a_1^2+a_2^2+\ldots+a_n^2 > 20$.

By Cauchy's Inequality,

$$\begin{aligned}
&10(a_1^3 + a_2^3 + \ldots + a_n^3)\\
&= (a_1 + a_2 + \ldots + a_n)(a_1^3 + a_2^3 + \ldots + a_n^3)\\
&\geq (\sqrt{a_1}\sqrt{a_1^3} + \sqrt{a_2}\sqrt{a_2^3} + \ldots + \sqrt{a_n}\sqrt{a_n^3})^2\\
&= (a_1^2 + a_2^2 + \ldots + a_n^2)^2\\
&> 400.
\end{aligned}$$

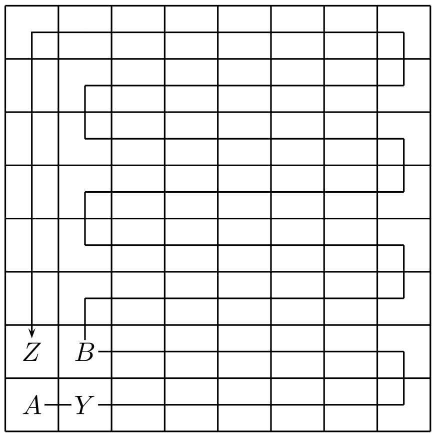
Hence $a_1^3 + a_2^3 + \ldots + a_n^3 > 40$.

6. Prove that if a regular icosahedron and a regular dodecahedron have a common circumsphere, then they have a common insphere.

Solution. Let O be the circumcentre of the icosahedron, C the centre of one of its faces and A a vertex of that face. Its circumradius is OA, and its inradius is OC. Construct a dual dodecahedron by joining the centres of adjacent faces of the icosahedron. Now C is a vertex of three faces of this dodecahedron, and the centre B of one of these faces lies on OA. Its circumradius is OC and its inradius is OB. Note that in triangles OAC and OCB, $\angle AOC = \angle COB$ and $\angle OCA = 90° = \angle OBA$. Hence they are similar to each other, so that $\frac{OA}{OC} = \frac{OC}{OB}$. If we rescale the two solids so that their circumradii are equal, then so are their inradii.

7. A *lazy* rook can only move from a square to a vertical or a horizontal neighbour. It follows a path which visits each square of an 8×8 chessboard exactly once. Prove that the number of such paths starting at a corner square is greater than the number of such paths starting at a diagonal neighbour of a corner square.

Solution. The diagram below shows a path from A to Z along which a lazy rook visits every square of the 8×8 chessboard once and only once, where A, B, Y and Z are as labelled. Note that A and B have the same colour in the usual chessboard pattern. Since the squares visited by the lazy rook must alternate in colour, no path can start from A and end at B, or vice versa. We claim that there are more such paths starting from A than those starting from B. For each path starting from B, since the path cannot end at A, the lazy rook must visit A between visits to Y and Z. Suppose the lazy rook visits Y first. Then the path corresponds to the following one starting from A: move to Y, follow the original path in reverse to B, move to Z, and follow the original path to the end. If the lazy rook visits Z first, then start from A, move to Z, follow the original path in reverse to B, move to Y, and follow the original path to the end. The path in the diagram below does not correspond to any path starting from B because no path starting from B can end at Z unless it moves from A to Z. This justifies our claim.



2 World Wide Web

Information on the Tournament, how to enter it and its rules are on the World Wide Web. Information on the Tournament can be obtained from the Australian Mathematics Trust web site at

http://www.amt.edu.au

3 Books on Tournament Problems

There are four books on problems of the Tournament available. Information on how to order these books may be found in the Trust's advertisement elsewhere in this journal, or directly via the Trust's web page.

Please note the Tournament's postal address in Moscow:

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PO Box 68
Moscow 121108
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WFNMC Congress 5 – Cambridge July, 22–28, 2006

WFNMC is an Affiliated Study Group of ICMI whose interests are in competitions and related activities and their role in enriching the learning process. Every four years it holds a conference covering all aspects of mathematics competitions. The last conference was in Melbourne in 2002.

The WFNMC 2006 conference will be held in

Robinson College, Cambridge, England
from Saturday 22 to Friday 28 July 2006.

Full details and a registration form are available on the conference website www.wpr3.co.uk/wfnmc (or go to www.amt.edu.au, click on "Links", "WFNMC", then "WFNMC 5").

The venue has been carefully chosen to offer:

- a historic location with all the colleges and atmosphere of Cambridge on your immediate doorstep;
- all the conference facilities together on a single site, within a short distance of each other, so as to facilitate the personal interactions which characterise WFNMC conferences;
- a college with excellent facilities (two auditoriums, numerous mid-sized and small teaching rooms, chapel, bar and grounds); and a very reasonable all-in price (lower than 2002 and remarkable for the UK in July).

The all-inclusive registration fee covers accommodation and board and all official additional activities.

This should provide an ideal venue for the family atmosphere of a conference (for information about the venue see www.robinson.cam.ac.uk).

Access from airports: London has several different airports. London-Stansted is the closest, but Cambridge is accessible from them all by rail and/or coach. Details and advice will be provided to those registering or enquiring nearer the date. We will provide maps and do all we can to meet trains and coaches on the opening day.

Accompanying persons programme: Cambridge and its surrounds provide plenty for ordinary visitors to explore; and London is less than an hour away by train for those who have other ideas. We will facilitate and help to organise an informal Cambridge-based accompanying persons programme to suit the interests of those who attend.

The attractions of the venue are complemented by the quality of those who have agreed to give plenary talks.

- Simon Singh is not only the author of the bestsellers *Fermat's Last Theorem*, *The Code Book*, and most recently *Big Bang*, but is also an excellent presenter, who has worked tirelessly on behalf of mathematics, see www.simonsingh.com.

- Robin Wilson is the author and editor of more than 20 books, one of the most recent being *Four Colours Suffice* (Princeton University Press 2002), and is well-known for his entertaining, yet scholarly, lectures on the history of mathematics.

- A well-known Cambridge mathematician with an enviable track record as a lecturer for wider audiences will give a strictly mathematical lecture. (Insiders may well be able to fit names to this description; none of those approached have declined, but we have yet to sort out exactly which of those approached will be available!) We expect this talk will be held amidst the stunning architecture of the new “Centre for Mathematical Sciences” and the “Isaac Newton Institute”.

- The WFNMC President Petar Kenderov will open the conference; the WFNMC Vice-President Maria de Losada will extract lessons for us all from a brief history of mathematics competitions in Colombia; and Jozsef Pelikan (Chair, IMO Advisory Board) will draw on his extensive experience to reflect on issues arising from the present state of mathematics competitions, and in particular on the question of what determines whether those who are successful in competitions eventually become research mathematicians.

There is nothing harder, or more important, than a good beginning.

- As soon as delegates have registered and freshened up, they will be put in teams, given a map and a set of clues, and will be set to follow a "mathematics trail" around the very compact, but mostly hidden, city of Cambridge. Cars are largely excluded from the city centre, and following this "trail" should make it clear at the outset how much there is to see, and how accessible the city, its historic colleges and beautiful grounds are on foot. It will also introduce all delegates to some of the reference points for Robin Wilson's historical lecture the next day on "Cambridge Mathematical Figures".
- In the evening Petar Kenderov (WFNMC President) will begin the serious business of the meeting.
- Thereafter each day will have a more predictable structure. Each day will begin with delegates in small working groups focussing on "problem creation and improvement" in the spirit of the first WFNMC conference in Waterloo (1990). We plan to have groups to cover most of the obvious domains including some new ones: these will distinguish
  (i) different ages (primary (roughly Grades 3–5), Junior (Grades 6–8), Intermediate (Grades 9–10), and Senior (Grades 11–12)),
  (ii) target groups (popular multiple choice or "Olympiad"),
  (iii) content (traditional, or applied), and
  (iv) formats (individual timed written, take-home, team, or student problem journal).

  Those who have accepted to act as Chairs of these groups are: Primary (Peter Bailey), Junior popular (Gregor Dolinar), Intermediate popular (Ian VanderBurgh), Senior popular (Harold Reiter), Junior Olympiad (Bruce Henry), Intermediate Olympiad (Andrew Jobbings), Senior Olympiad (Gerry Leversha), Team (Steve Mulligan), Student problem journals (John Webb). Delegates should declare their preferences on the Registration Form.

- The collection of problems that result from this exercise will be circulated after the conference, provided delegates agree to embargo publication for 12–15 months, so that the problems can be used in national competitions during the ensuing period. We hope that an ad hoc jury will award prizes (donated by Cambridge University Press) for the best problems each day.

- The session before lunch provides an opportunity for delegates to present papers in parallel sessions. Those who wish to present such a paper should indicate this on their registration form and provide a title and abstract by the date specified. We anticipate that there will be sections covering competition types (as for the problem creation sessions); interactions between competitions and ordinary classroom teaching; new developments; research related to competitions; serious work relating to practical aspects of running mathematics competitions (administration, finance, sponsorship, how to organise problem setting groups and marking weekends, etc.).

- The programme needs to be flexible to fit the requirements of delegates. However, some of the themes listed in the previous paragraph, though important to all of us, are less likely to lend themselves to formal papers. We have therefore labelled two pre-lunch sessions as "Forums", to allow for two or three parallel "open debates" on themes of this kind, in relatively small groups, with each Forum beginning with one or two short contributions from delegates with relevant experience.

- Two afternoon sessions provide further slots for parallel sessions. One slot has been allocated for a "Team competition", in which delegates can experience this kind of event at first hand. Two afternoon sessions have been allocated for "Visits", where we plan to arrange a selection of guided tours to colleges and local sights (including "punting on the river Cam").

- After the first day, the early evening session will always be used for a plenary lecture.

- The evenings (apart from the conference dinner) will remain informal.

(i) On two evenings we will arrange a chamber recital in the very striking College Chapel (e.g. a piano quartet, an evening of Lieder, or a wind quintet).

(ii) Most evenings in July one can find open-air performances of Shakespeare in various college grounds.

(iii) And on one evening we plan a bit of modern English culture—a pub quiz (with a mathematical bent).

See you there!

Tony Gardiner

Chair, Organising Committee WFNMC 2006

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Useful Problem Solving Books from AMT Publications

These books are a valuable resource for the school library shelf, for students wanting to improve their understanding and competence in mathematics, and for the teacher who is looking for relevant, interesting and challenging questions and enrichment material.

To attain an appropriate level of achievement in mathematics, students require talent in combination with commitment and self-discipline. The following books have been published by the AMT to provide a guide for mathematically dedicated students and teachers.

Australian Mathematics Competition (AMC) Solutions and Statistics
Edited by DG Pederson

This book provides, each year, a record of the AMC questions and solutions, and details of medallists and prize winners. It also provides a unique source of information for teachers and students alike, with items such as levels of Australian response rates and analyses including discriminatory powers and difficulty factors.

Australian Mathematics Competition Book 1 (1978-1984)

Australian Mathematics Competition Book 2 (1985-1991)

Australian Mathematics Competition Book 3 (1992-1998)

An excellent training and learning resource, each of these extremely popular and useful books contains over 750 past AMC questions, answers and full solutions. The questions are grouped into topics and ranked in order of difficulty. Book 3 also available on CD (for PCs only).

Problem Solving Via the AMC
Edited by Warren Atkins

This 210 page book consists of a development of techniques for solving approximately 150 problems that have been set in the Australian Mathematics Competition. These problems have been selected from topics such as Geometry, Motion, Diophantine Equations and Counting Techniques.

Methods of Problem Solving, Book 1
Edited by JB Tabov, PJ Taylor

This introduces the student aspiring to Olympiad competition to particular mathematical problem solving techniques. The book contains formal treatments of methods which may be familiar or introduce the student to new, sometimes powerful techniques.

Methods of Problem Solving, Book 2
JB Tabov & PJ Taylor
After the success of Book 1, the authors have written Book 2 with the same format but five new topics. These are the Pigeon-Hole Principle, Discrete Optimisation, Homothety, the AM-GM Inequality and the Extremal Element Principle.

Mathematical Toolchest
Edited by AW Plank & N Williams

This 120 page book is intended for talented or interested secondary school students, who are keen to develop their mathematical knowledge and to acquire new skills. Most of the topics are enrichment material outside the normal school syllabus, and are accessible to enthusiastic year 10 students.

International Mathematics — Tournament of Towns (1980-1984)

International Mathematics — Tournament of Towns (1984-1989)

International Mathematics — Tournament of Towns (1989-1993)

International Mathematics — Tournament of Towns (1993-1997)

International Mathematics — Tournament of Towns (1997-2002)
Edited by PJ Taylor

The International Mathematics Tournament of Towns is a problem solving competition in which teams from different cities are handicapped according to the population of the city. Ranking only behind the International Mathematical Olympiad, this competition had its origins in Eastern Europe (as did the Olympiad) but is now open to cities throughout the world. Each book contains problems and solutions from past papers.

Challenge! 1991 – 1995
Edited by JB Henry, J Dowsey, A Edwards, L Mottershead, A Nakos, G Vardaro

The Mathematics Challenge for Young Australians attracts thousands of entries from Australian High Schools annually and involves solving six in depth problems over a 3 week period. In 1991-95, there were two versions – a Junior version for Year 7 and 8 students and an Intermediate version for Year 9 and 10 students. This book reproduces the problems from both versions which have been set over the first 5 years of the event, together with solutions and extension questions. It is a valuable resource book for the class room and the talented student.

USSR Mathematical Olympiads 1989 – 1992
Edited by AM Slinko

Arkadii Slinko, now at the University of Auckland, was one of the leading figures of the USSR Mathematical Olympiad Committee during the last years before democratisation. This book brings together the problems and solutions of the last four years of the All-Union Mathematics Olympiads. Not only are the problems and solutions highly expository but the book is worth reading alone for the fascinating history of mathematics competitions to be found in the introduction.

Australian Mathematical Olympiads 1979 – 1995
H Lausch & PJ Taylor

This book is a complete collection of all Australian Mathematical Olympiad papers since the first competition in 1979. Solutions to all problems are included and in a number of cases alternative solutions are offered.

Chinese Mathematics Competitions and Olympiads 1981-1993 and 1993-2001
A Liu

These books contain the papers and solutions of two contests, the Chinese National High School Competition and the Chinese Mathematical Olympiad. China has an outstanding record in the IMO and these books contain the problems that were used in identifying the team candidates and selecting the Chinese teams. The problems are meticulously constructed, many with distinctive flavour. They come in all levels of difficulty, from the relatively basic to the most challenging.

Asian Pacific Mathematics Olympiads 1989-2000
H Lausch & C Bosch-Giral

With innovative regulations and procedures, the APMO has become a model for regional competitions around the world where costs and logistics are serious considerations. This 159 page book reports the first twelve years of this competition, including sections on its early history, problems, solutions and statistics.

Polish and Austrian Mathematical Olympiads 1981-1995
ME Kuczma & E Windischbacher

Poland and Austria hold some of the strongest traditions of Mathematical Olympiads in Europe even holding a joint Olympiad of high quality. This book contains some of the best problems from the national Olympiads. All problems have two or more independent solutions, indicating their richness as mathematical problems.

Seeking Solutions
JC Burns

Professor John Burns, formerly Professor of Mathematics at the Royal Military College, Duntroon and Foundation Member of the Australian Mathematical Olympiad Committee, solves the problems of the 1988, 1989 and 1990 International Mathematical Olympiads. Unlike other books in which only complete solutions are given, John Burns describes the complete thought processes he went through when solving the problems from scratch. Written in an inimitable and sensitive style, this book is a must for a student planning on developing the ability to solve advanced mathematics problems.

101 Problems in Algebra from the Training of the USA IMO Team
Edited by T Andreescu & Z Feng

This book contains one hundred and one highly rated problems used in training and testing the USA International Mathematical Olympiad team. These problems are carefully graded, ranging from quite accessible towards quite challenging. The problems have been well developed and are highly recommended to any student aspiring to participate at National or International Mathematical Olympiads.

Hungary Israel Mathematics Competition
S Gueron

This 181 page book summarizes the first 12 years of the competition (1990 to 2001) and includes the problems and complete solutions. The book is directed at mathematics lovers, problem solving enthusiasts and students who wish to improve their competition skills. No special or advanced knowledge is required beyond that of the typical IMO contestant and the book includes a glossary explaining the terms and theorems which are not standard that have been used in the book.

Bulgarian Mathematics Competition 1992-2001
BJ Lazarov, JB Tabov, PJ Taylor, AM Storozhev

The Bulgarian Mathematics Competition has become one of the most difficult and interesting competitions in the world. It is unique in structure, combining mathematics and informatics problems in a multi-choice format. This book covers the first ten years of the competition complete with answers and solutions. Students of average ability and with an interest in the subject should be able to access this book and find a challenge.

Mathematical Contests – Australian Scene
Edited by AM Storozhev, JB Henry & DC Hunt

These books provide an annual record of the Australian Mathematical Olympiad Committee's identification, testing and selection procedures for the Australian team at each International Mathematical Olympiad. The books consist of the questions, solutions, results and statistics for: Australian Intermediate Mathematics Olympiad (formerly AMOC Intermediate Olympiad), AMOC Senior Mathematics Contest, Australian Mathematics Olympiad, Asian-Pacific Mathematics Olympiad, International Mathematical Olympiad, and Maths Challenge Stage of the Mathematical Challenge for Young Australians.

WFNMC — Mathematics Competitions
Edited by Warren Atkins

This is the journal of the World Federation of National Mathematics Competitions (WFNMC). With two issues each of approximately 80-100 pages per year, it consists of articles on all kinds of mathematics competitions from around the world.

Parabola incorporating Function

This Journal is published in association with the School of Mathematics, University of New South Wales. It includes articles on applied mathematics, mathematical modelling, statistics, and pure mathematics that can contribute to the teaching and learning of mathematics at the senior secondary school level. The Journal's readership consists of mathematics students, teachers and researchers with interests in promoting excellence in senior secondary school mathematics education.

ENRICHMENT STUDENT NOTES

The Enrichment Stage of the Mathematics Challenge for Young Australians (sponsored by the Dept of Education, Science and Training) contains formal course work as part of a structured, in-school program. The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. We are making these Notes available as a text book to interested parties for whom the program is not available.

Newton Enrichment Student Notes
JB Henry
Recommended for mathematics students of about Year 5 and 6 as extension material. Topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

Dirichlet Enrichment Student Notes
JB Henry
This series has chapters on some problem solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory. It is designed for students in Years 6 or 7.

Euler Enrichment Student Notes
MW Evans and JB Henry
Recommended for mathematics students of about Year 7 as extension material. Topics include elementary number theory and geometry, counting, pigeonhole principle.

Gauss Enrichment Student Notes
MW Evans, JB Henry and AM Storozhev
Recommended for mathematics students of about Year 8 as extension material. Topics include Pythagoras theorem, Diaphantine equations, counting, congruences.

Noether Enrichment Student Notes
AM Storozhev
Recommended for mathematics students of about Year 9 as extension material. Topics include number theory, sequences, inequalities, circle geometry.

Pólya Enrichment Student Notes
G Ball, K Hamann and AM Storozhev
Recommended for mathematics students of about Year 10 as extension material. Topics include polynomials, algebra, inequalities and geometry.

T-SHIRTS

T-shirts celebrating the following mathematicians are made of 100% cotton and are designed and printed in Australia. They come in white, and sizes Medium (Polya only) and XL.

Carl Friedrich Gauss T–shirt

The Carl Friedrich Gauss t-shirt celebrates Gauss' discovery of the construction of a 17-gon by straight edge and compass, depicted by a brightly coloured cartoon.

Emmy Noether T–shirt

The Emmy Noether t-shirt shows a schematic representation of her work on algebraic structures in the form of a brightly coloured cartoon.

George Pólya T–shirt

George Pólya was one of the most significant mathematicians of the 20th century, both as a researcher, where he made many significant discoveries, and as a teacher and inspiration to others. This t-shirt features one of Pólya's most famous theorems, the Necklace Theorem, which he discovered while working on mathematical aspects of chemical structure.

Peter Gustav Lejeune Dirichlet T–shirt

Dirichlet formulated the Pigeonhole Principle, often known as Dirichlet's Principle, which states: "If there are p pigeons placed in h holes and p>h then there must be at least one pigeonhole containing at least 2 pigeons." The t-shirt has a bright cartoon representation of this principle.

Alan Mathison Turing T-shirt

The Alan Mathison Turing t-shirt depicts a colourful design representing Turing's computing machines which were the first computers.

ORDERING

All the above publications are available from AMT Publishing and can be purchased on-line at:

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The Trust, of which the University of Canberra is Trustee, is a non-profit organisation whose mission is to enable students to achieve their full intellectual potential in mathematics. Its strengths are based upon:

- a network of dedicated mathematicians and teachers who work in a voluntary capacity supporting the activities of the Trust;
- the quality, freshness and variety of its questions in the Australian Mathematics Competition, the Mathematics Challenge for Young Australians, and other Trust contests;
- the production of valued, accessible mathematics materials;
- dedication to the concept of solidarity in education;
- credibility and acceptance by educationalists and the community in general whether locally, nationally or internationally; and
- a close association with the Australian Academy of Science and professional bodies.